An Interplay of Beam Dynamics and HEP
The Muon g-2 Experiment

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Fermilab

*for the Muon **g**-2 Collaboration*

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Fundamental Particle Spin

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- For a spin ½ point particle, classically the expectation is *g* = 1
- With Stern-Gerlach and atomic spectroscopy experiments in the 1920s, it became apparent that ***g*e = 2** .
  - » i.e., an electron precesses twice as fast (muon as well)
- Solution to the *g* problem appeared in 1926 with a relativistic treatment by Thomas
- Incorporated into Dirac's famous equation by 1928

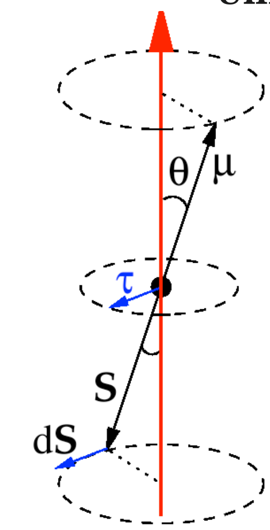
$$\left(\frac{1}{2m}(\vec{P}+e\vec{A})^2+\frac{e}{2m}\vec{\sigma}\cdot\vec{B}-eA^0\right)\psi_A=(E-m)\psi_A$$

So, for an elementary spin ½ particle in Dirac's theory, *g*=2!

spin $\vec{S}=\frac{\hbar}{2}\vec{\sigma}$

magnetic moment $\vec{\mu}=g\frac{q\hbar}{4mc}\vec{\sigma}$

$$\vec{\mu}=g\frac{q}{2mc}\vec{S}$$



Then, in 1933 (Stern and Estermann):

$g_p \approx 5.6$ (*proton*)

The first '**anomalous**' magnetic moment!

Then, same year, Rabi inferred g_n = -3.8 from measurements on the deuteron!

Set $g=2(1+a)$

anomaly: $a\equiv\frac{g-2}{2}$

The Thomas BMT Equation and the Magic Momentum

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- For electromagnetic fields in the lab frame, the precession of the spin vector in the rest frame of the particle is given by the Thomas-BMT eq.*:

$$\frac{d\vec{S}}{dt} = \vec{\omega}_s \times \vec{S} = -\frac{e}{\gamma m}\left[(1+a\gamma)\vec{B}_\perp + (1+a)\vec{B}_\parallel + \left(a\gamma + \frac{\gamma}{\gamma+1}\right)\frac{\vec{E}\times\vec{\beta}}{c}\right]\times\vec{S}$$

- The momentum vector of the particle will precess with

$$\frac{d\vec{p}}{dt} = \vec{\omega}_c \times \vec{p} = -\frac{e}{\gamma m}\left[\vec{B}_\perp + \frac{\gamma^2}{\gamma^2-1}\frac{\vec{E}\times\vec{\beta}}{c}\right]\times\vec{p}$$

* citations:
— Thomas L H 1927 Philos. Mag. 3 1–22
— Bargmann V, Michel L and Telegdi V L, 1959 Phys. Rev. Lett. 2 435–6

- Hence, relative to the motion of the particle, the precession frequency will be

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -\frac{e}{m}\left[a\vec{B}_\perp + \frac{1+a}{\gamma}\vec{B}_\parallel + (a - \frac{1}{\gamma^2-1})\frac{\vec{E}\times\vec{\beta}}{c}\right]$$

- For ideal condition of **pure** vertical magnetic field and electric fields:

$$\vec{\omega}_a = -\frac{e}{m}\left[a\vec{B}_0 + (a - \frac{1}{\gamma^2-1})\frac{\vec{E}\times\vec{\beta}}{c}\right]$$

The Thomas BMT Equation and the Magic Momentum

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- As we need to provide vertical focusing, if we operate at the “magic momentum” where the last term goes to zero, then can use *electrostatic* quadrupoles for this task

$$\vec{\omega_a} = -\frac{e}{m}\left[a\vec{B_0} + \cancel{(a - \frac{1}{\gamma^2 - 1})}\frac{\vec{E}\times\vec{\beta}}{c}\right]$$

- Thus, a detector at one point in the ring would see frequency:

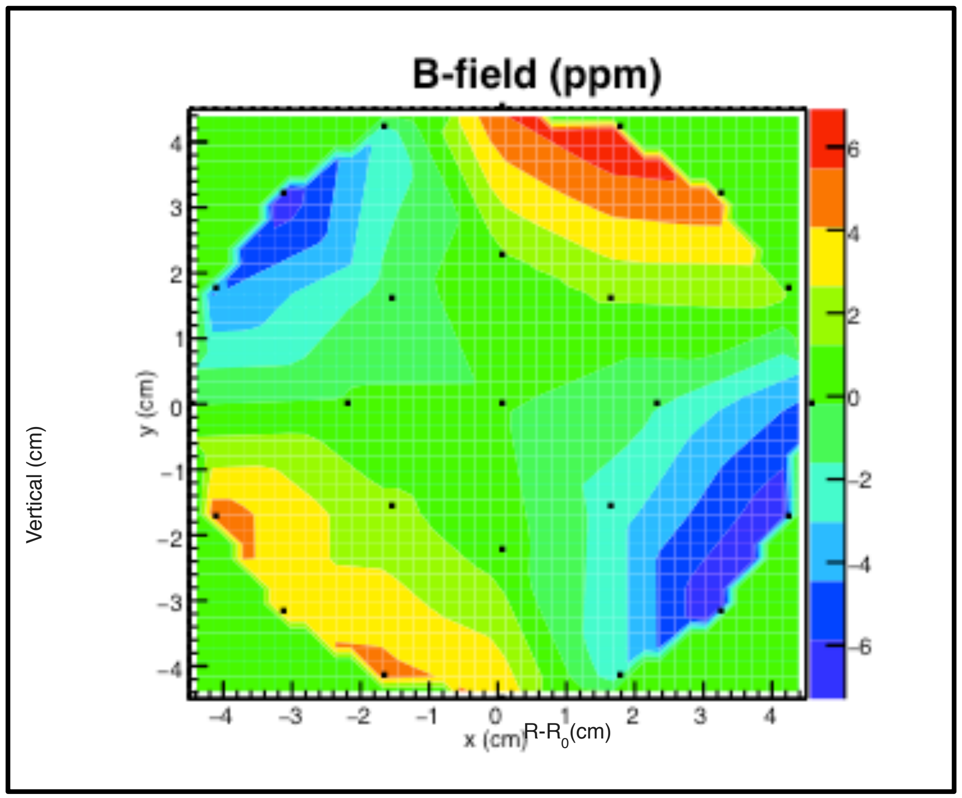
$$\omega_a = \frac{eB_0}{m}\cdot a$$

- So, provide a highly polarized beam of muons at the magic momentum into a highly uniform magnetic field, focus with electrostatic fields, and place detectors around circumference to detect positrons from the muon decays — kinematics show that the positrons with highest energies will emerge in the direction of the original muon’s spin
- The rate of decay at these higher energies will oscillate with ω_a

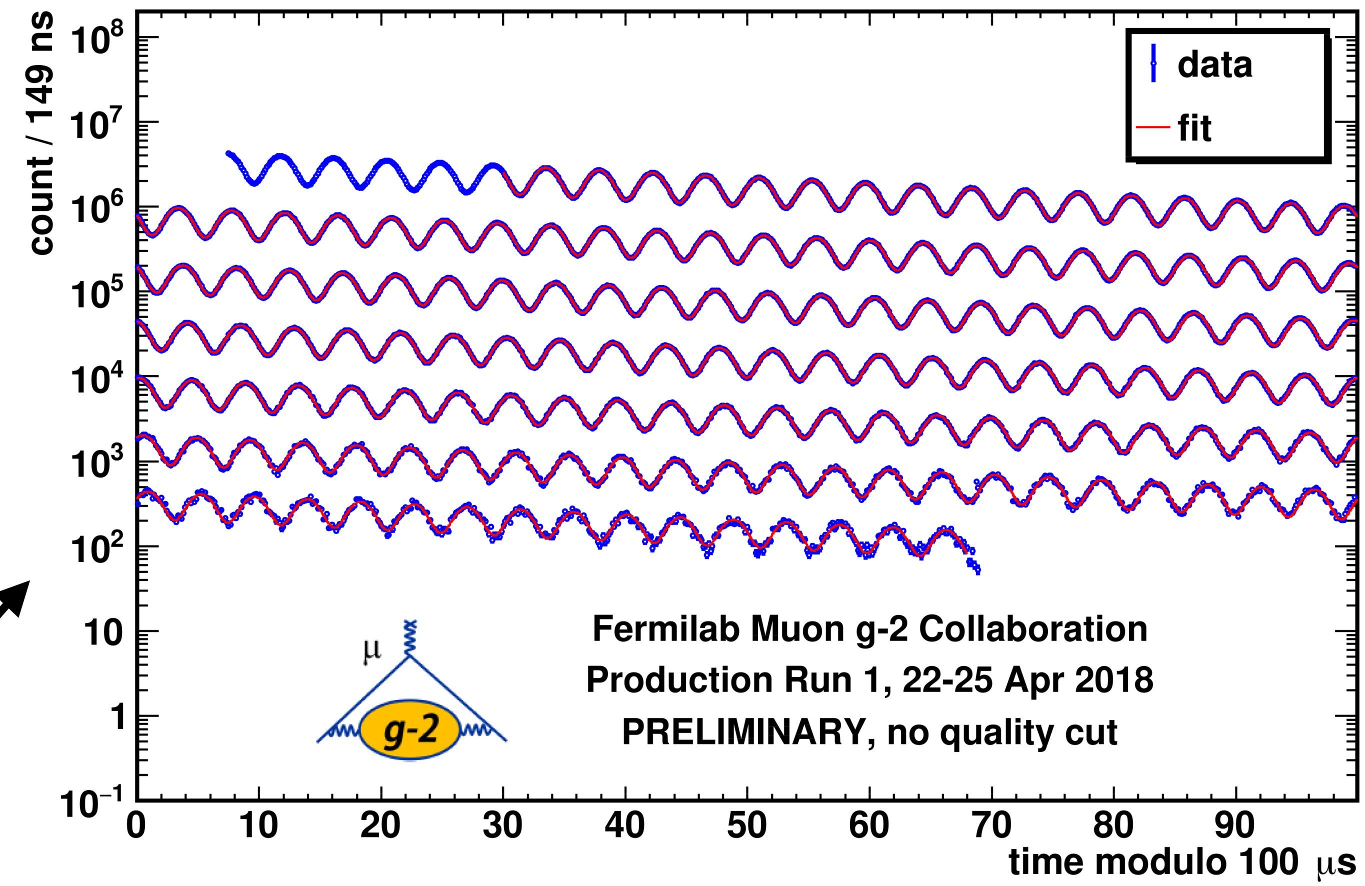
Wiggle Plots

- Fixed detector in the ring would observe the rate of muon decay "wiggle" with a frequency given by

$$\omega_a = \frac{eB_0}{m} \cdot a$$



high-quality B



- Muon g-2 Experiment uses 24 detector systems around the circumference, measuring positron trajectories, energies, etc.
- repeat the wiggle plot thousands of times…

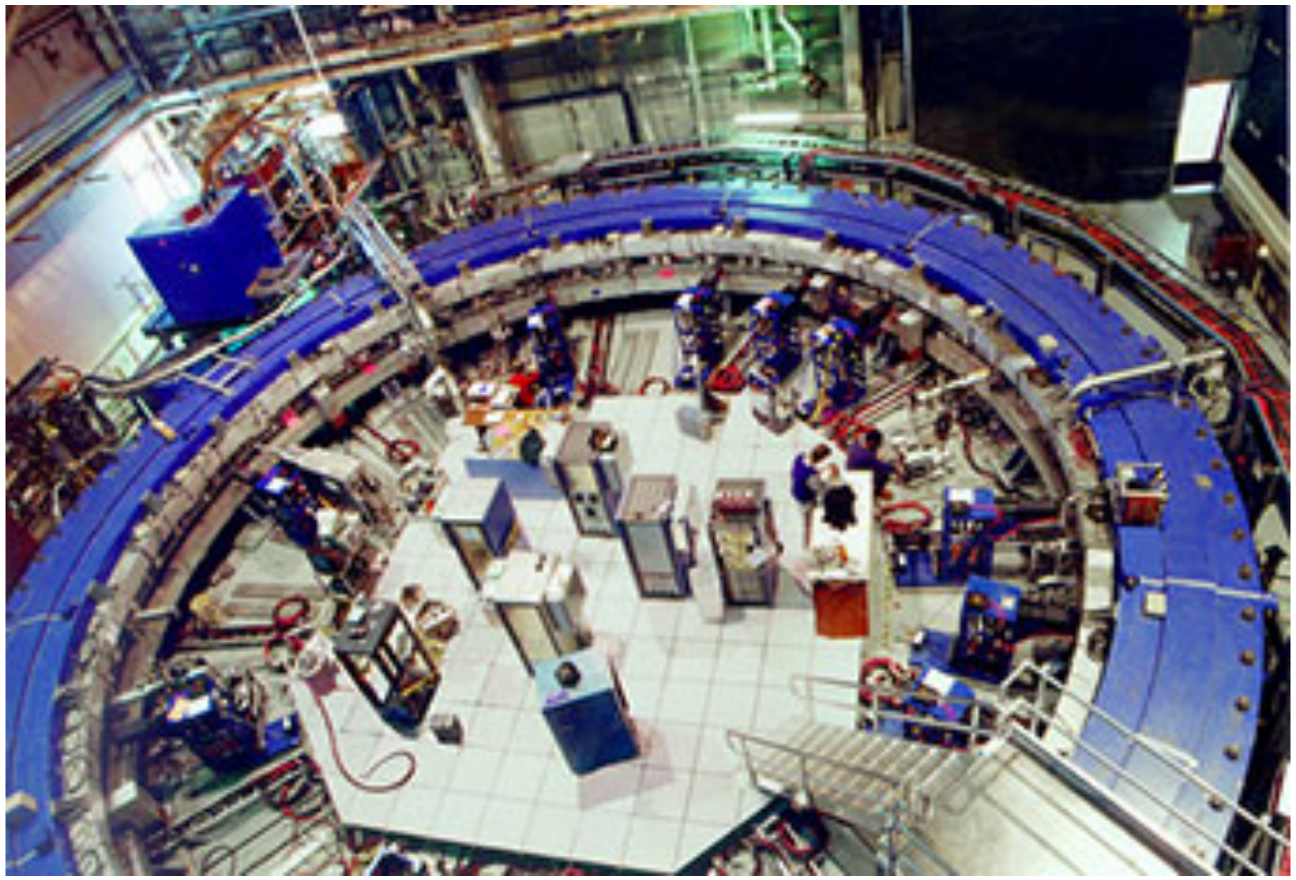
Short History of the *g*-2 Experiment

- Started out at CERN
  - 1959 (Lederman, et al.), using Synchrocyclotron — 2% result published in 1961, followed by more precise result — 0.4% error — confirming QED calculations at the time
  - 1966, using the CERN Proton Synchrotron (PS)
    - » 25x more accurate, showed inconsistency between experiment and the theory of the day
  - 1969-1979, third iteration of the experiment (still with PS) gave much more accuracy
    - » theory was confirmed to precision of 0.0007%
  - As time went on, theory continued to improve
  - In 1980s, new experiment formed in U.S.
    - » led to BNL g-2 Experiment E821
    - » on toward more precise measurement

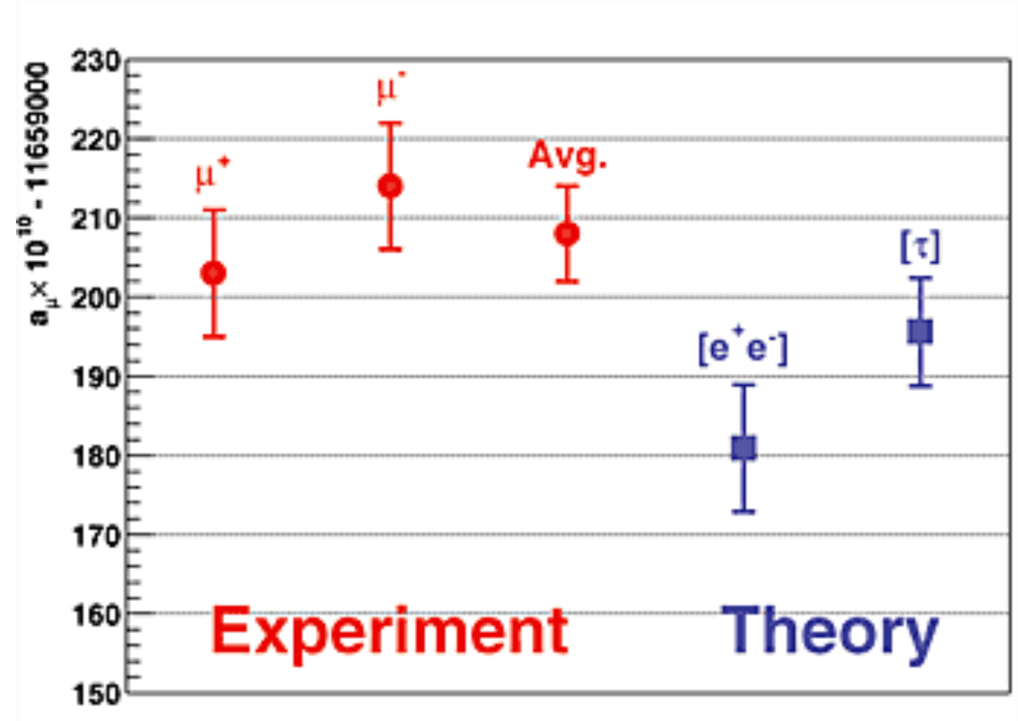
CERN g-2 storage ring, 1974

Short History of the *g*-2 Experiment

- BNL Experiment E821 began running in 1997, published final result in 2004



a_μ (exp) = 11 659 208 (6) × 10^{-10} (0.5 ppm)



- Since then, theory has improved further
- Presently: ~3.5σ discrepancy, between E821 and Standard Model calculations

Errors in E821? or Something missing in the Standard Model?

Fermilab E989 — Next Incarnation

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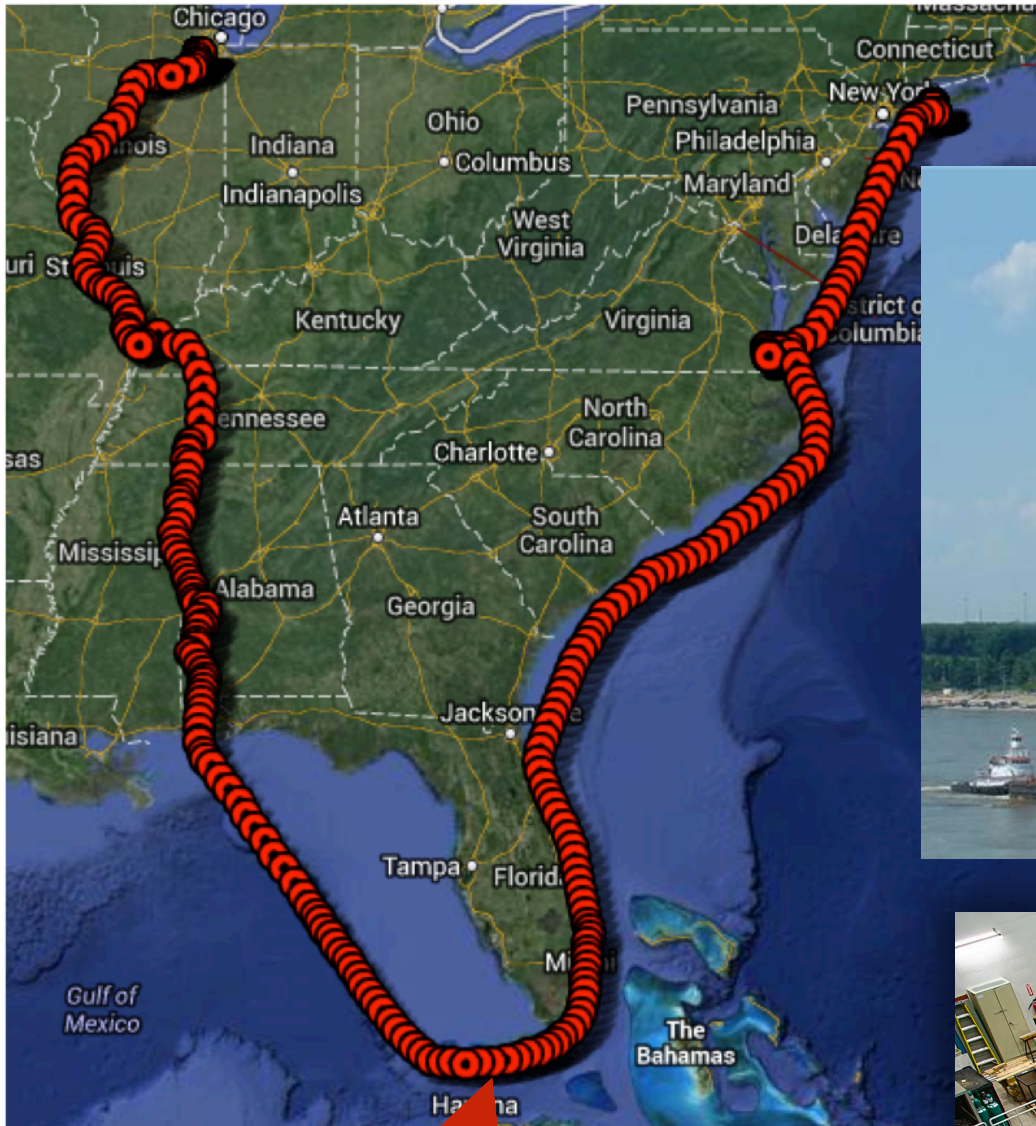
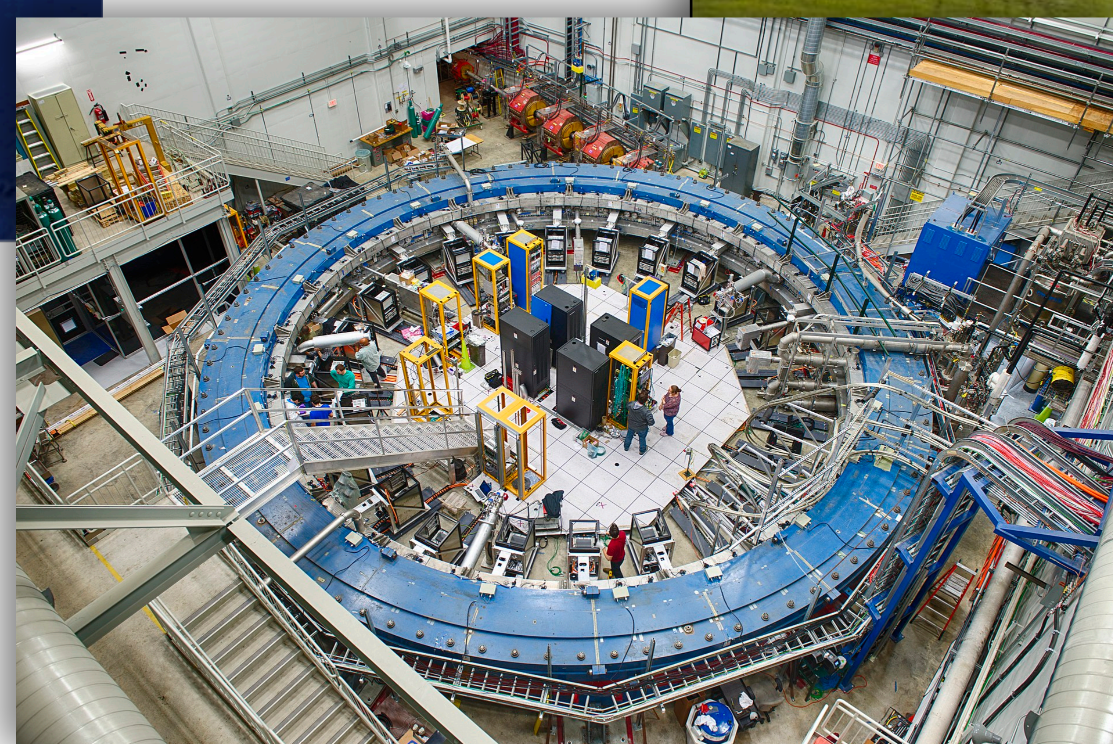


You are Here!

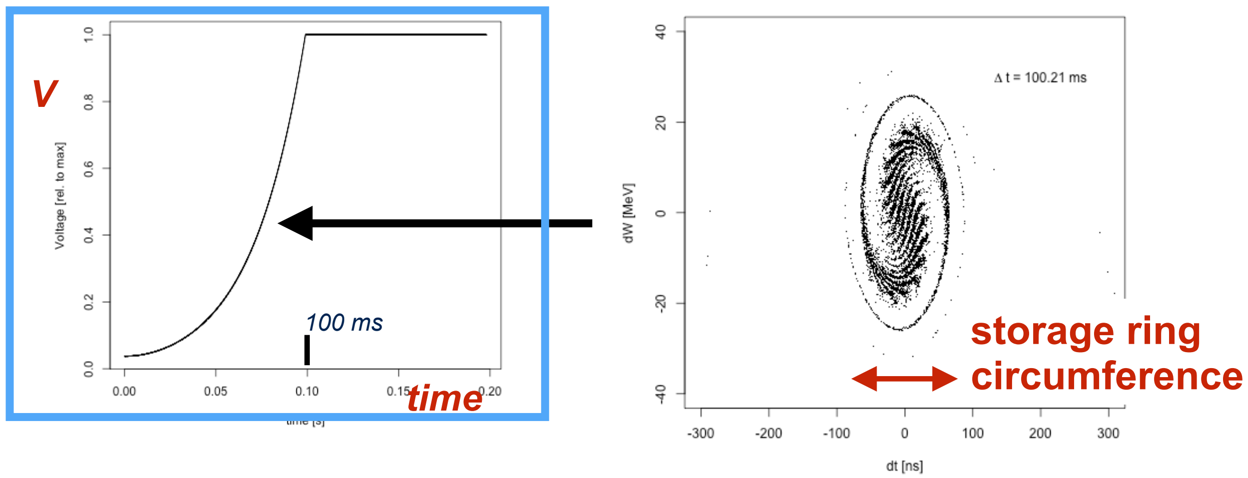
moved magnet to Fermilab in 2013…

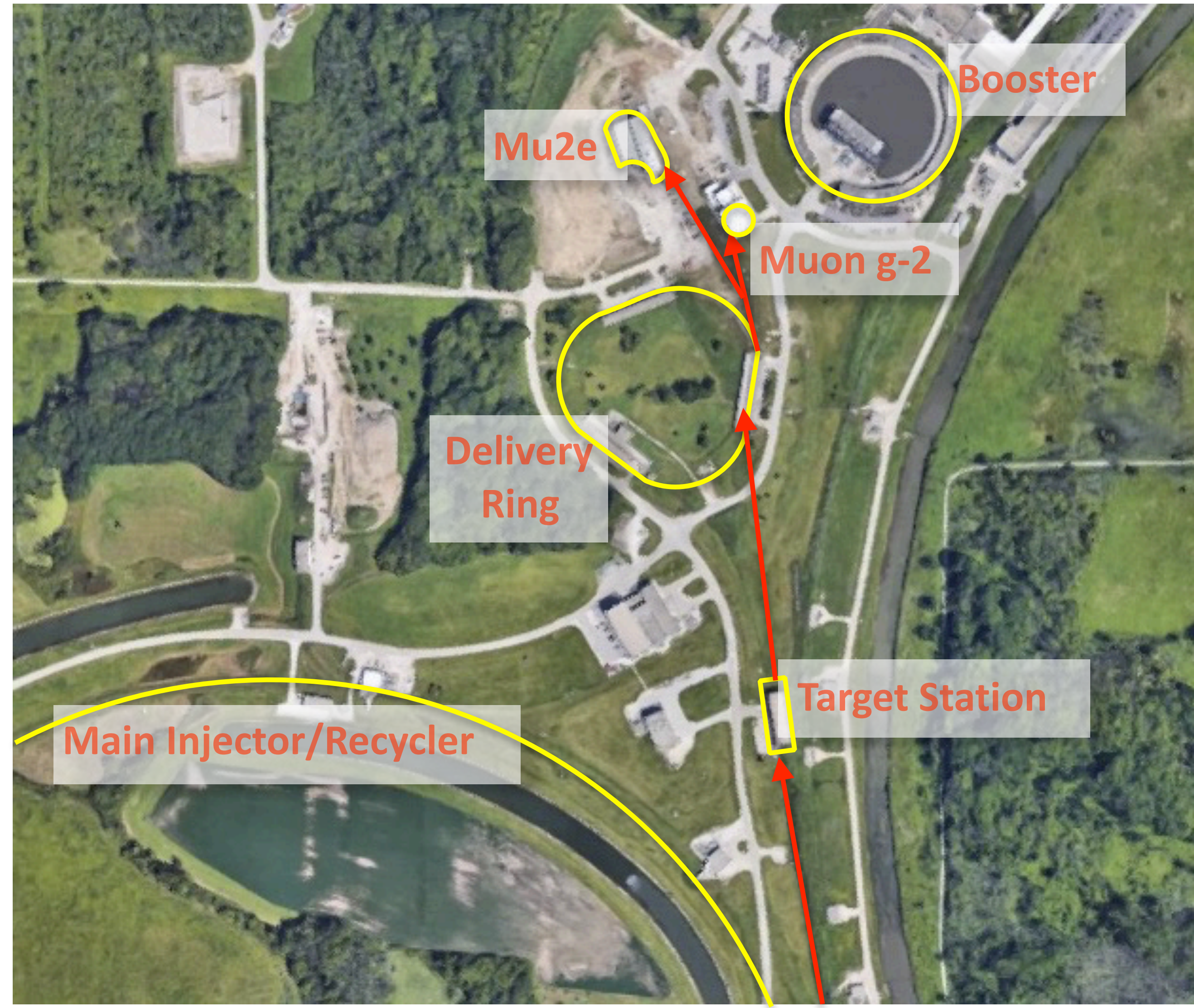


Fermilab data taking started in 2017…

Fermilab Implementation — E989

- Fermilab re-purposed its antiproton rings to create the The Muon Campus
- Bunch formation in the Recycler





- System delivers 8 pulses / 1.4 s
- 10^{12} protons on target / pulse
- Roughly 10^{6} muons / pulse to ring
  - ~10^{4} magic muons stored / pulse
- Goal: **20x** the statistics compared to BNL

see D. Tarazona, next talk…

- *Heavy reliance on modeling of beam production, transport, ring injection and beam storage to reduce systematic errors in the determination of anomalous magnetic moment*

Beam/Ring Modeling Efforts

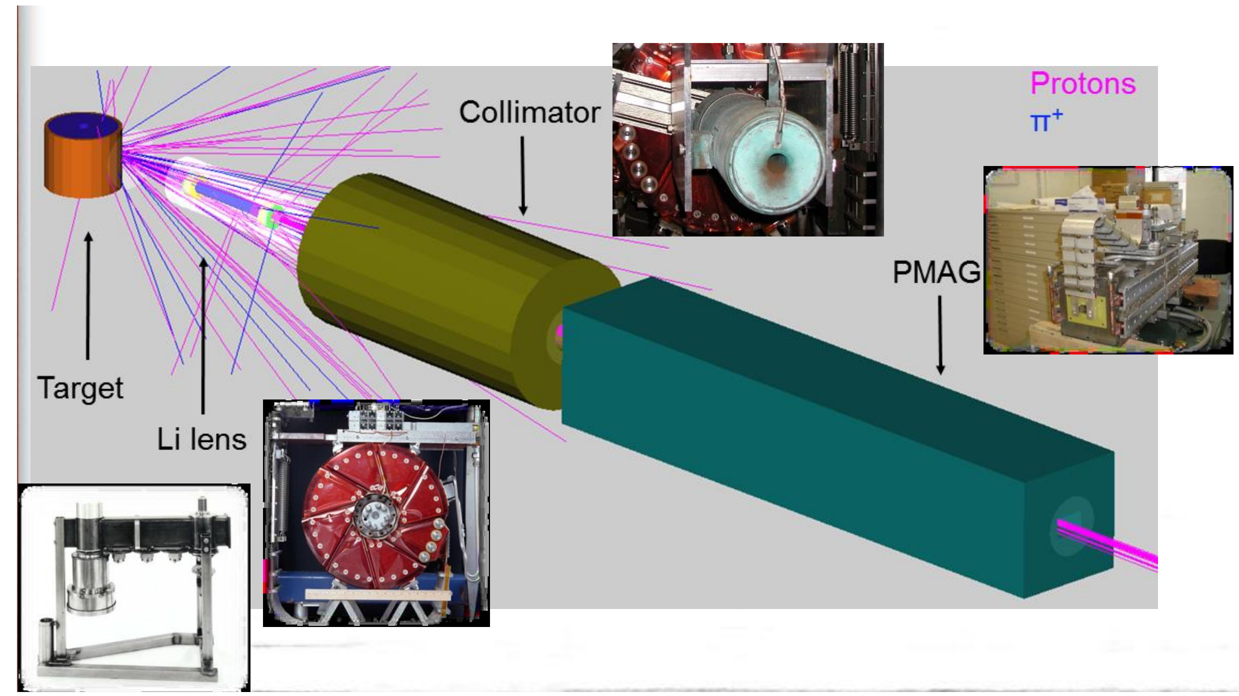
- Bunch Formation
  - ESME
- Targeting
  - MARS, GEANT, g4beamline
- Pion, Muon Capture
  - bmad, g4beamline
- Beam Line Transport
  - MAD, bmad, g4beamline, COSY
- Injection
  - bmad, gm2ringsim
- Storage Ring Properties
  - MAD, bmad, COSY, …
- Storage Ring Dynamics, Tracking
  - gm2ringsim, COSY, bmad, g4beamline

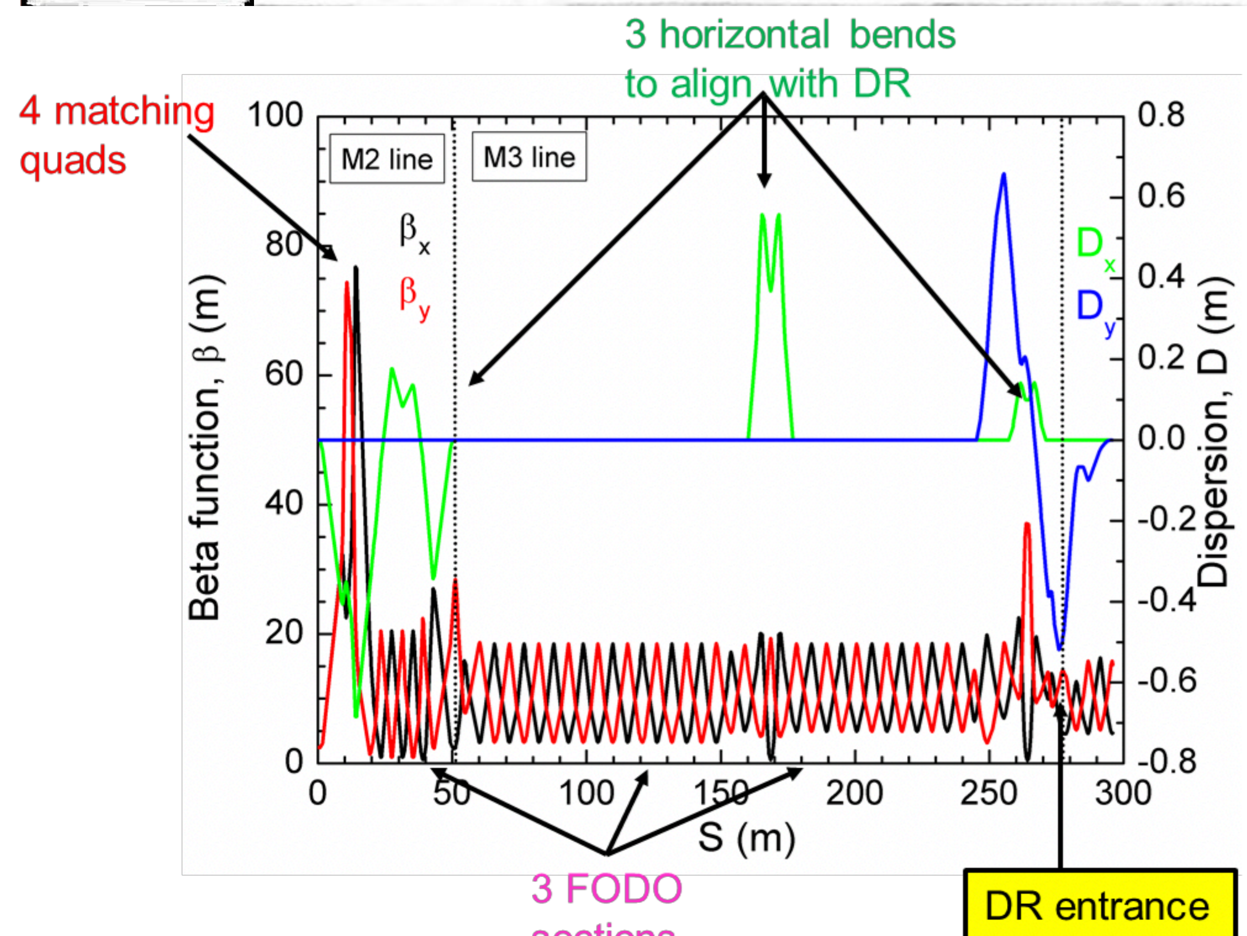
plus individual codes, analyses, ..

- Major beam dynamics modeling contributors:
  - Cornell U.
  - Brookhaven
  - Fermilab
  - U. Kentucky
  - U. Lancaster (UK)
  - U. Liverpool (UK)
  - Michigan State U.
  - Northern Illinois U.
- plus, analysts of the detector data, from all collaborating institutions!

Muon Beam Production

- Simulations of targeting of 8GeV proton beams onto the Fermilab A0 target
  - Inconel target, followed by Lithium lens and pulsed momentum-selection magnet
- Followed by simulation of the transport from target through 300 m of beam line, 4 turns about the 500 m circumference Delivery Ring, and through another 200 m of beam line to the g-2 storage ring
- From pion decay at high energies, the polarization of the muon beam will be >95%, necessary for measuring the spin precession to high precision



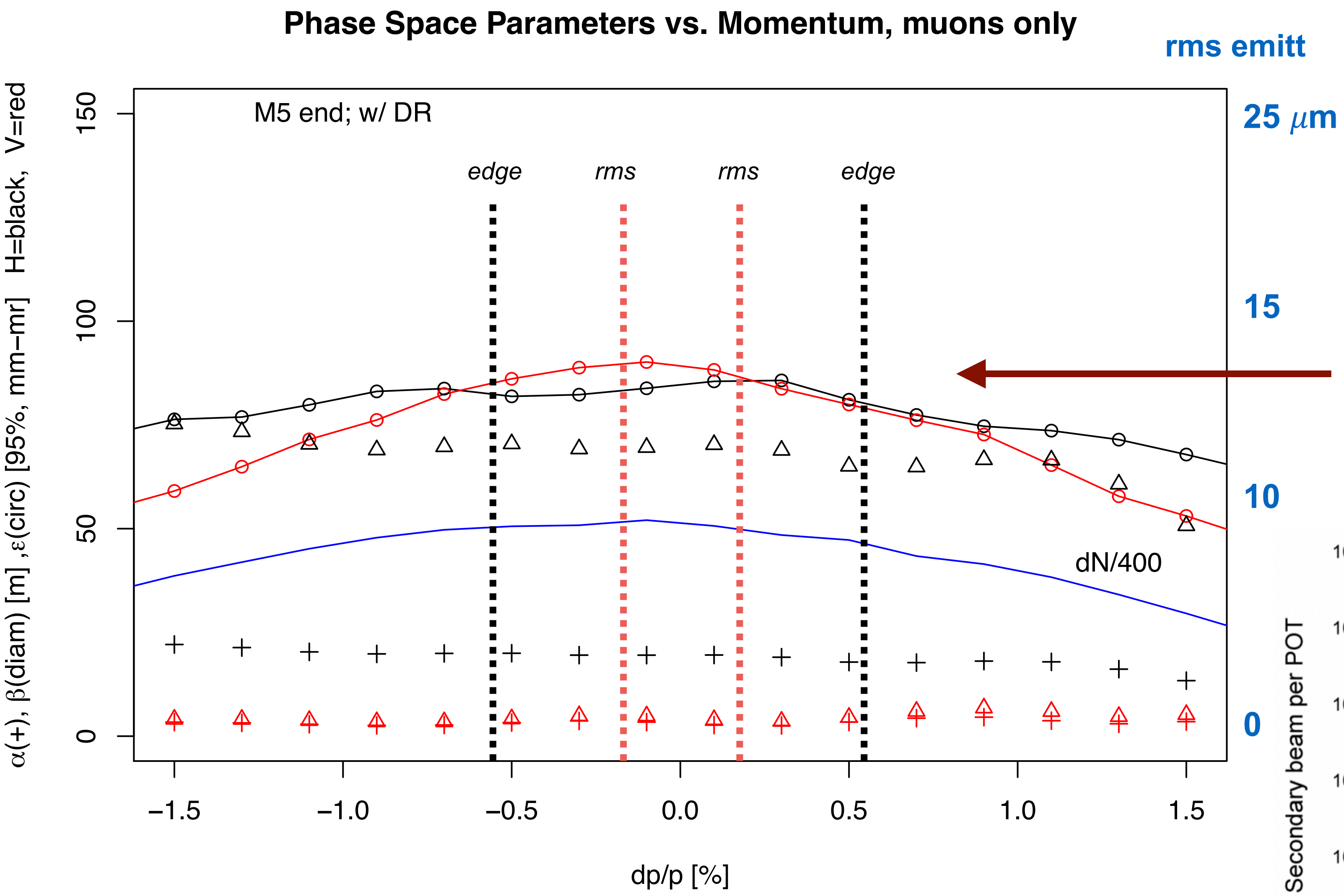


Muon Beam Production

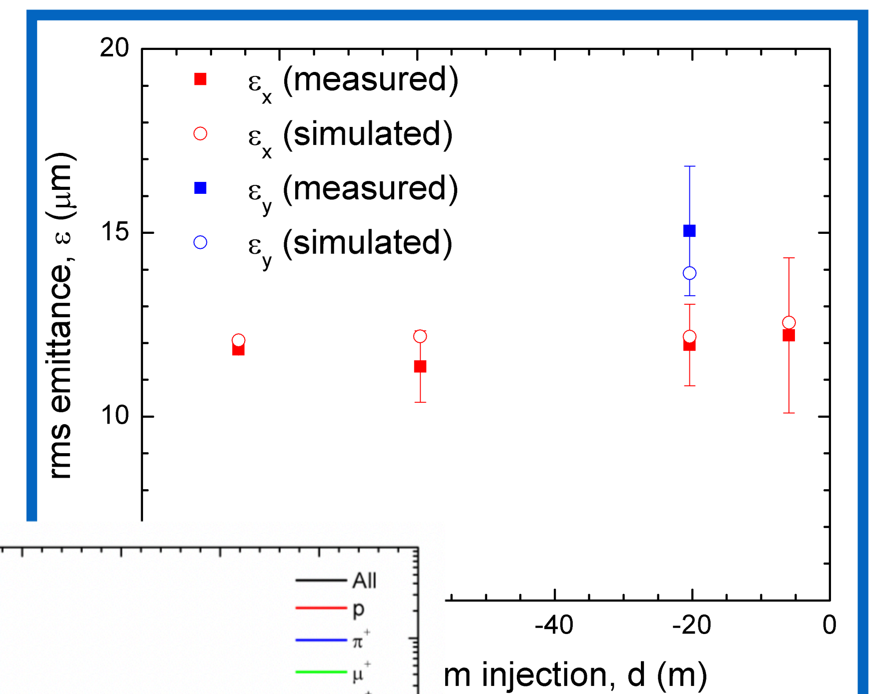
- In this application the beam arriving at the storage ring is very large and the ring acceptance is comparatively small; thus, only the ***core*** of the muon beam is admitted and stored

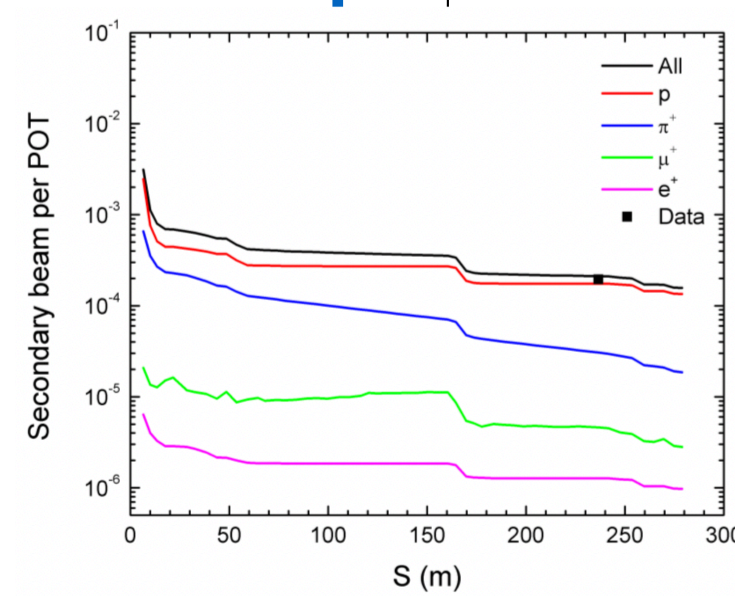
Simulation: *g4beamline*

measured beam properties agree very well with the predictions from derived targeting and transport simulations



Measurements:

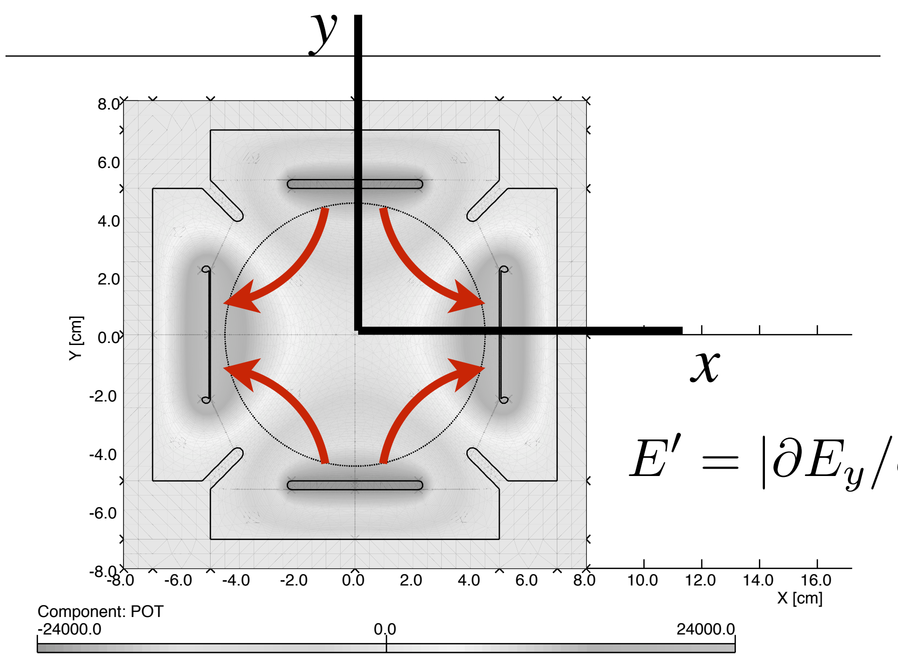




Stratakis, et al.

The Muon *g*-2 Storage Ring

- Implementation of weak focusing (discrete electrostatic quads)

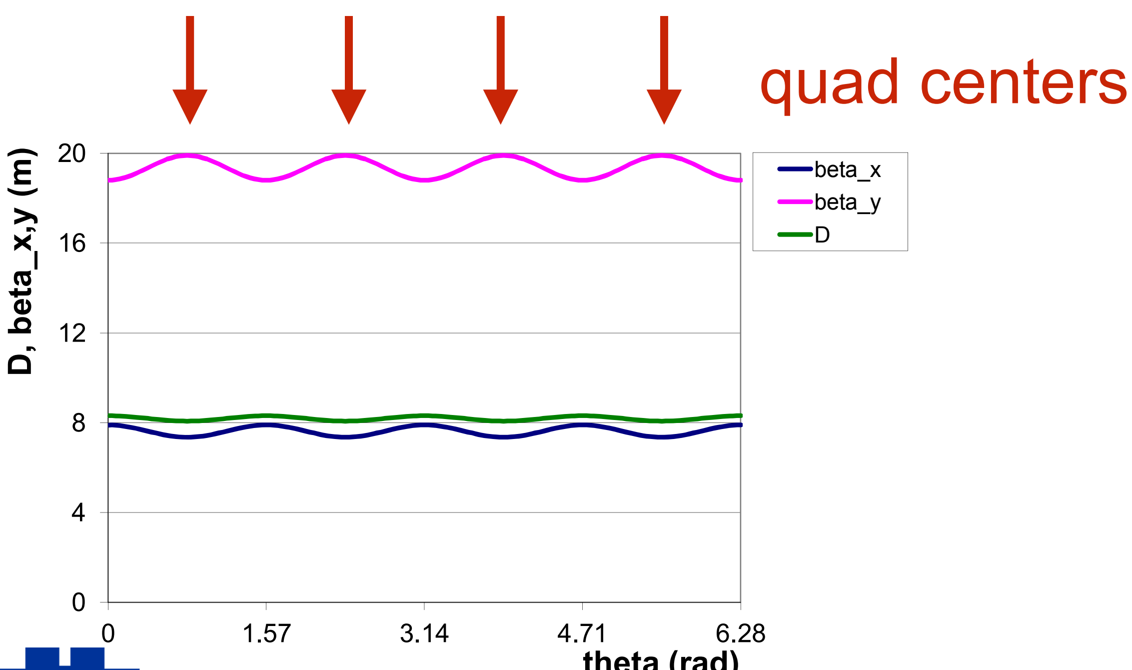


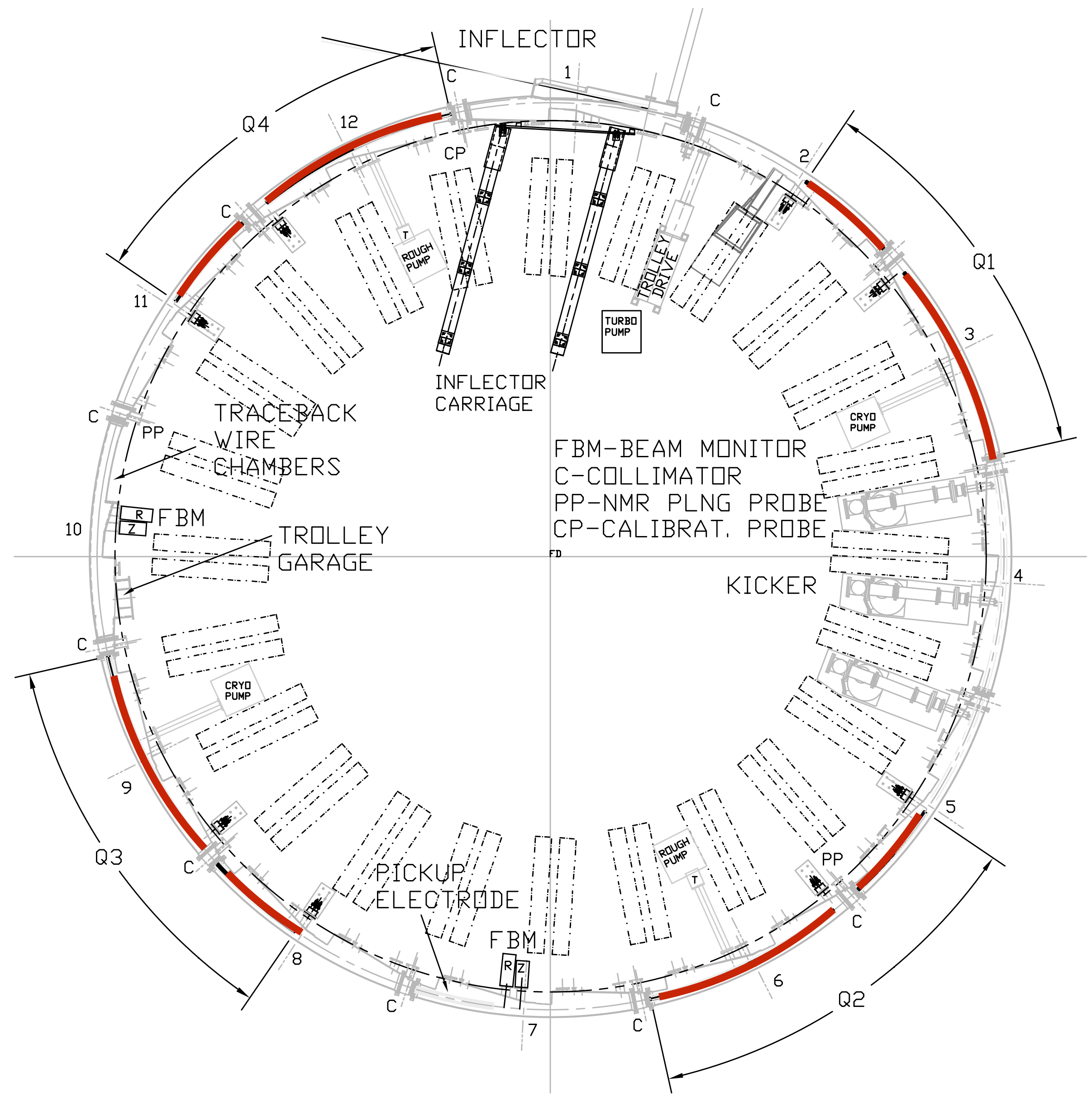
$$E' = |\partial E_y/\partial y| \approx 2V/a^2$$

$$x'' + K_x(s)x = 0$$
$$y'' + K_y(s)y = 0$$

where

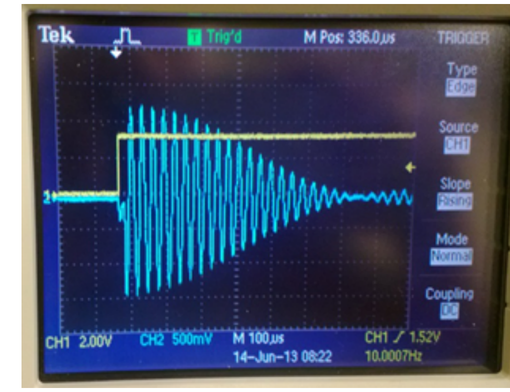
$$K_x(s) = \frac{1}{R_0^2} - \frac{E'(s)}{vB_0R_0}$$
$$K_y(s) = + \frac{E'(s)}{vB_0R_0}$$

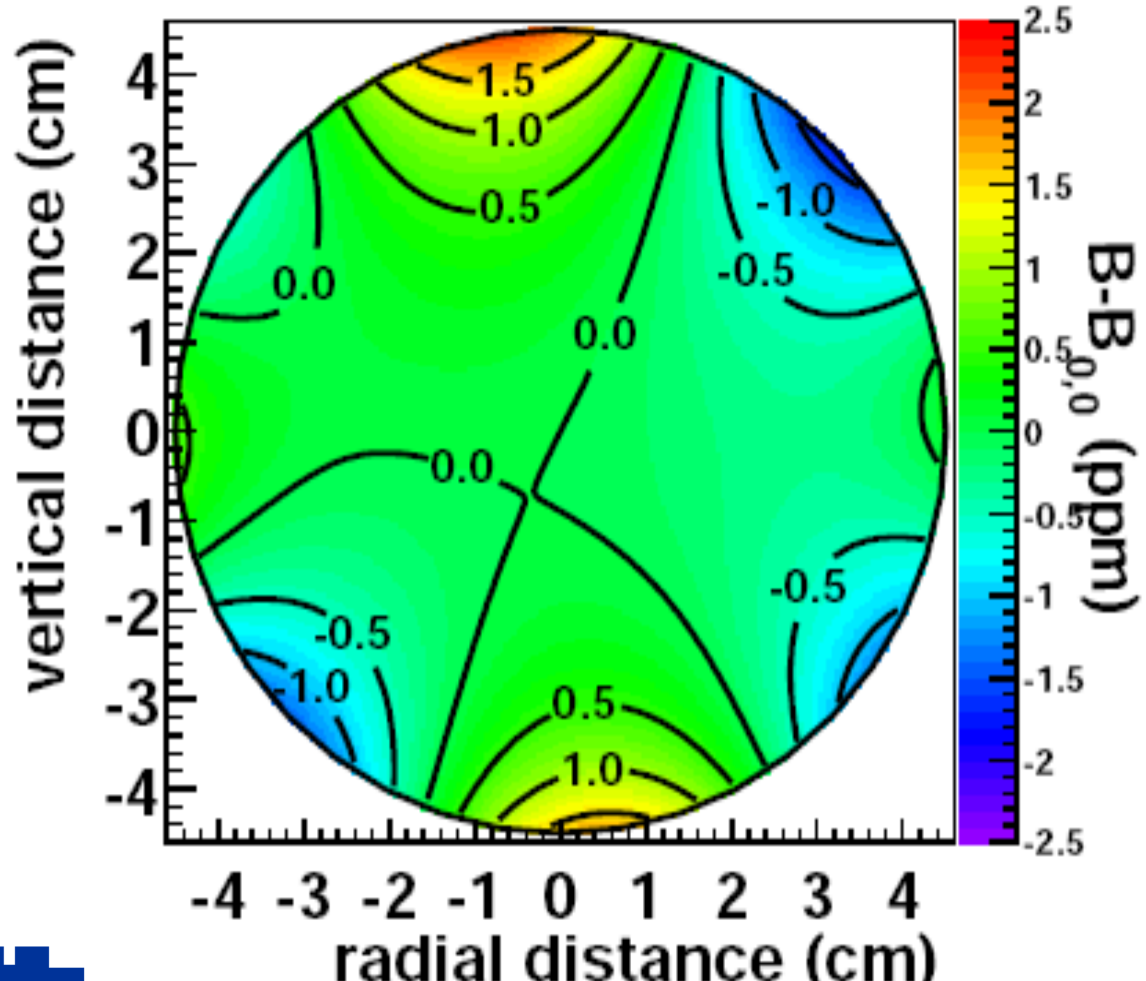


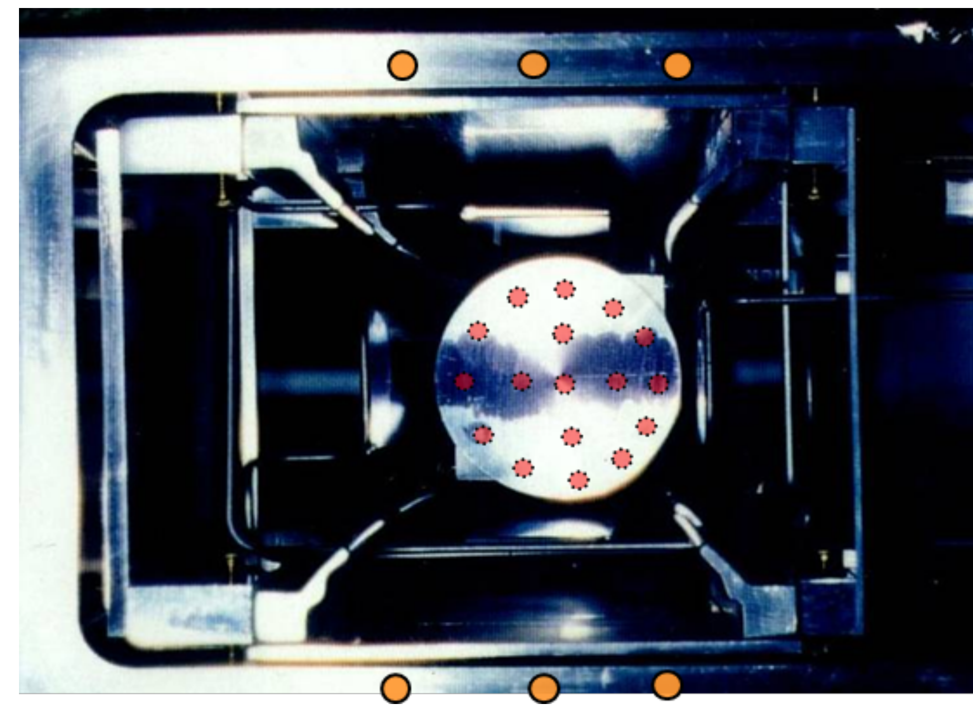


The Storage Ring and Field Precision

- Collection of NMR probes on a railed system map out the field quality periodically over time
- Hall probe system, as well as vertical orbit distortion detection, provides information on local radial field distortions



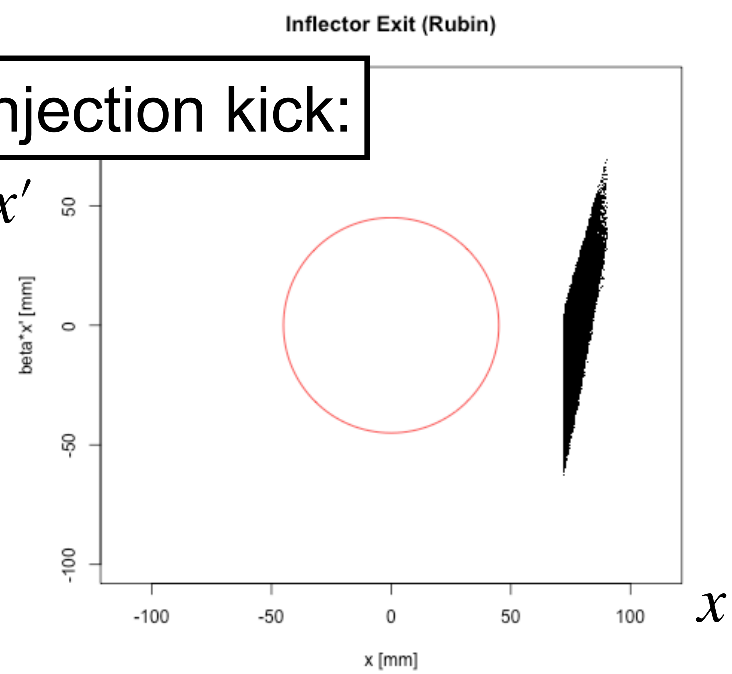


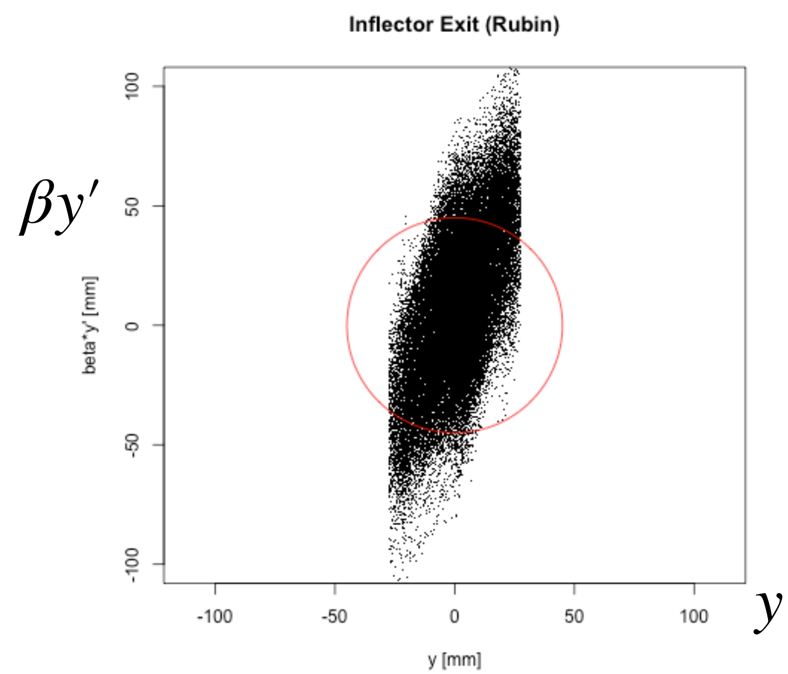


Modeling the Injection Process

- Incoming particle distribution, momentum distribution

prior to injection kick:





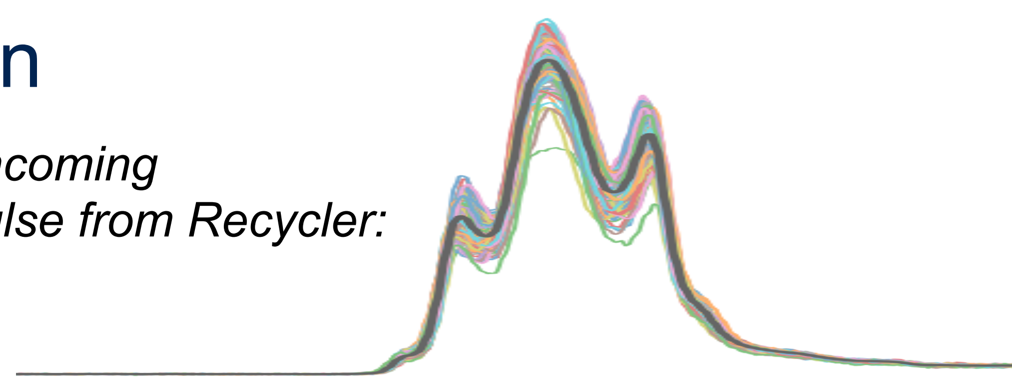
From >400,000 **simulated** particles off of the target, 54,000 arrive at ring (here, *bmad simulation)*

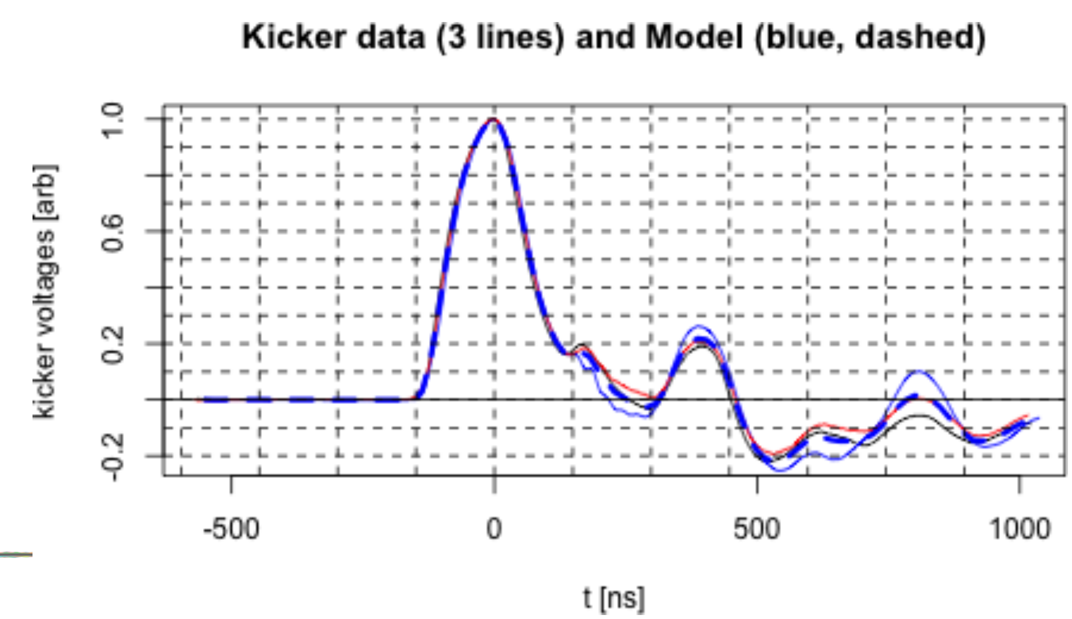
D. Stratakis, FNAL,
D. *Rubin, Cornell*

momentum distribution is very uniform

- Injection Kicker strength and pulse shape:
- Time distribution

typical incoming beam pulse from Recycler:

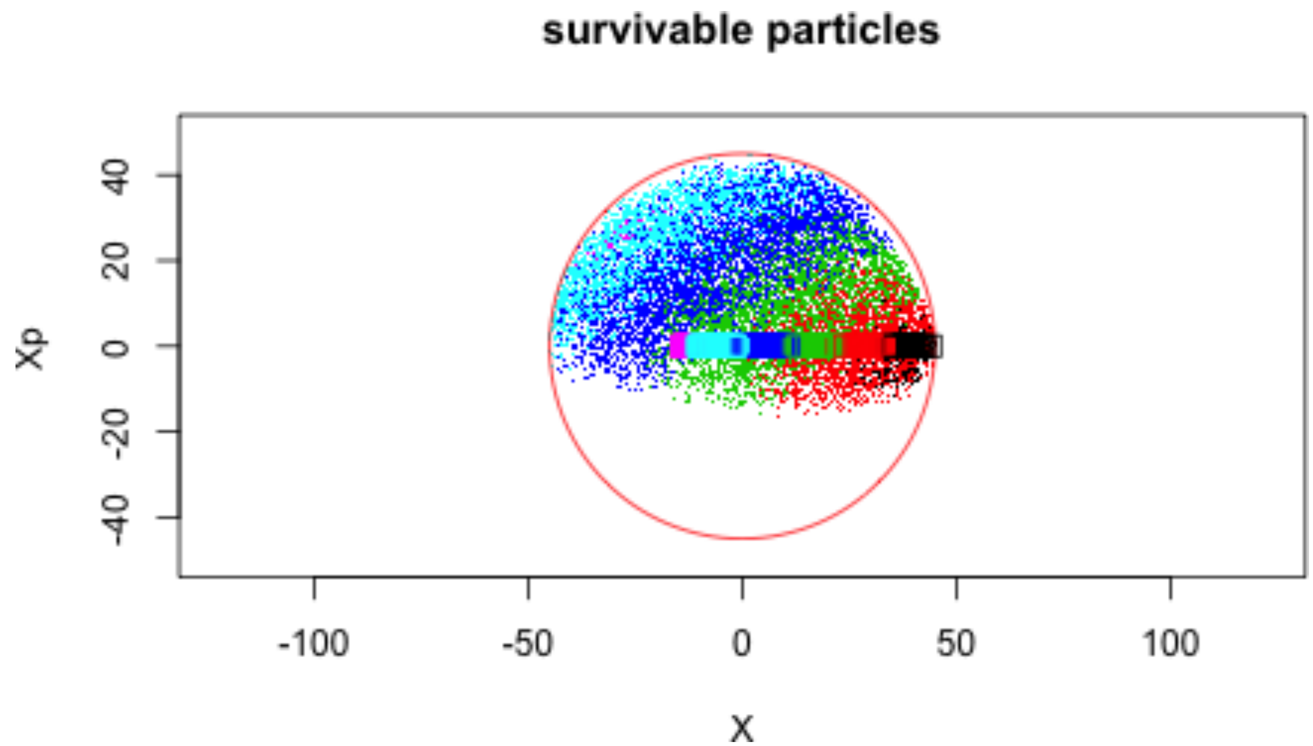


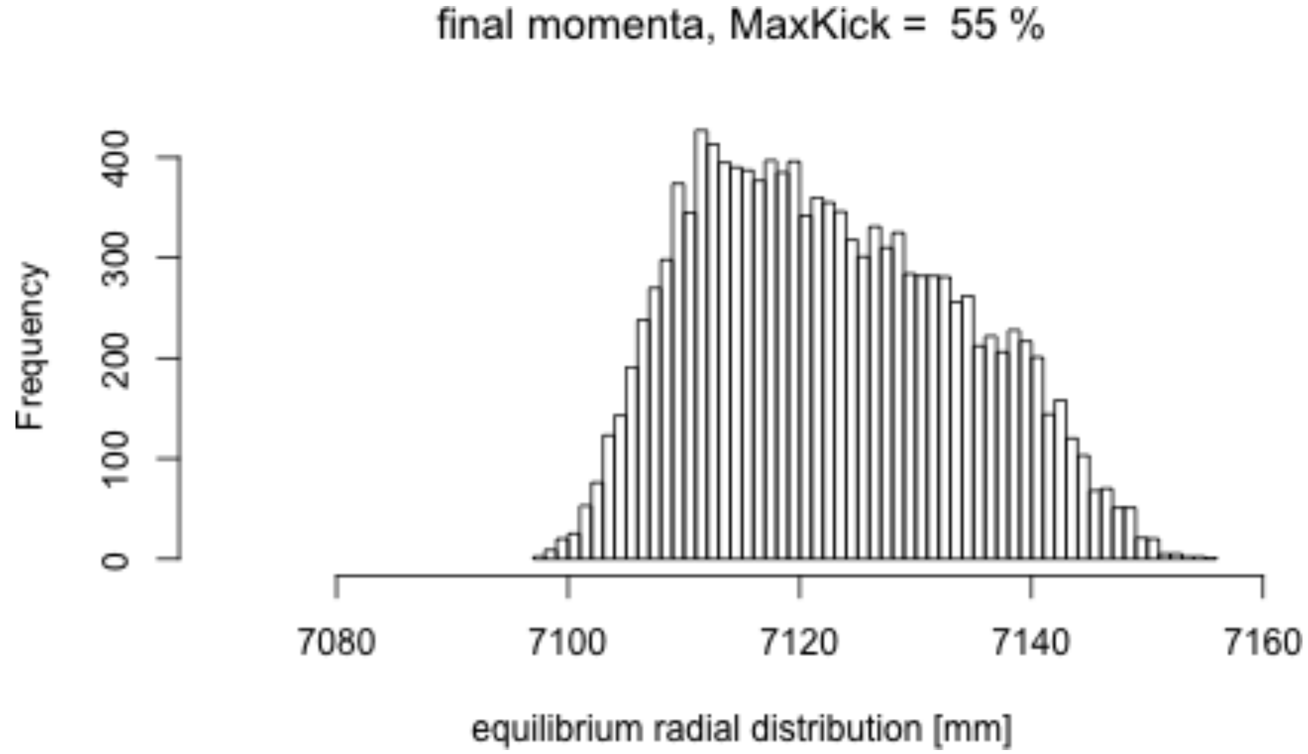


- Putting it all together…

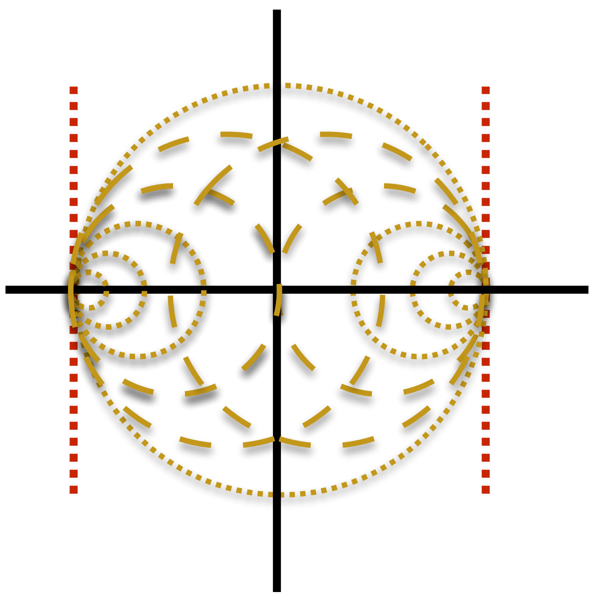
The Momentum Distribution

look at momentum distribution of the particles that can survive long-term:

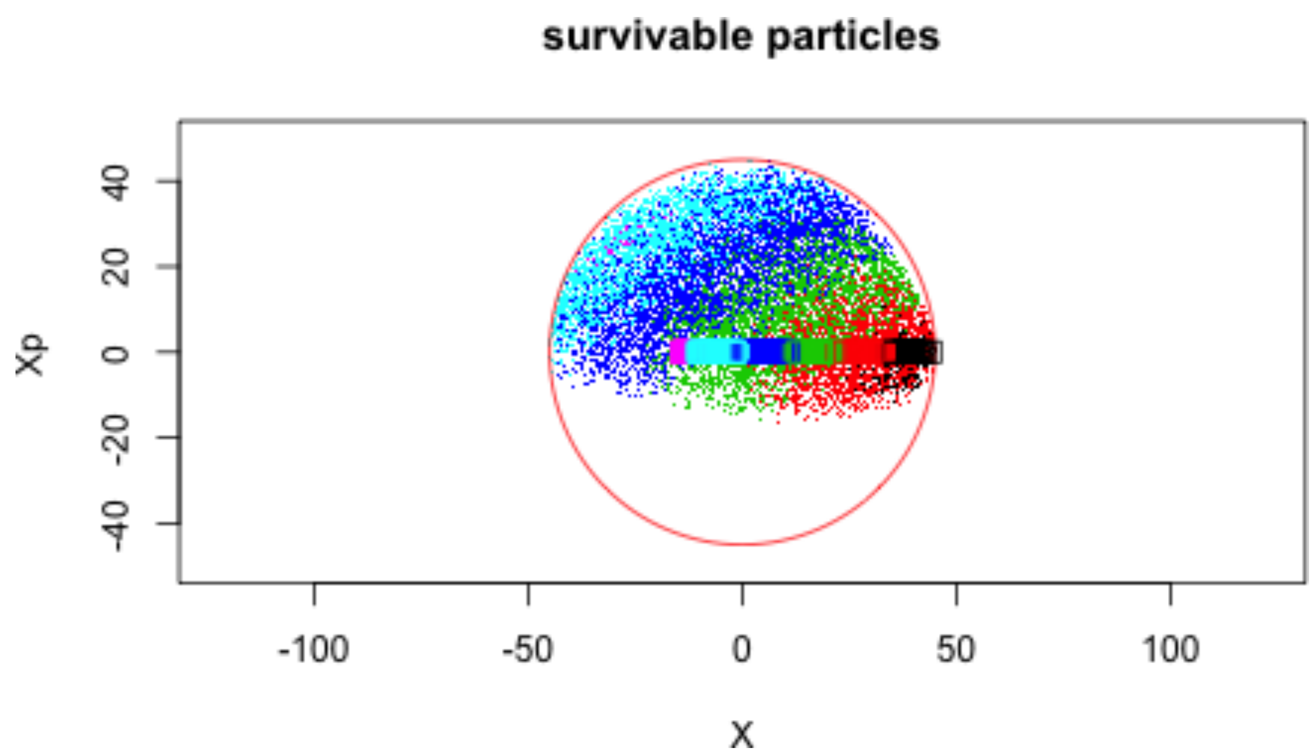




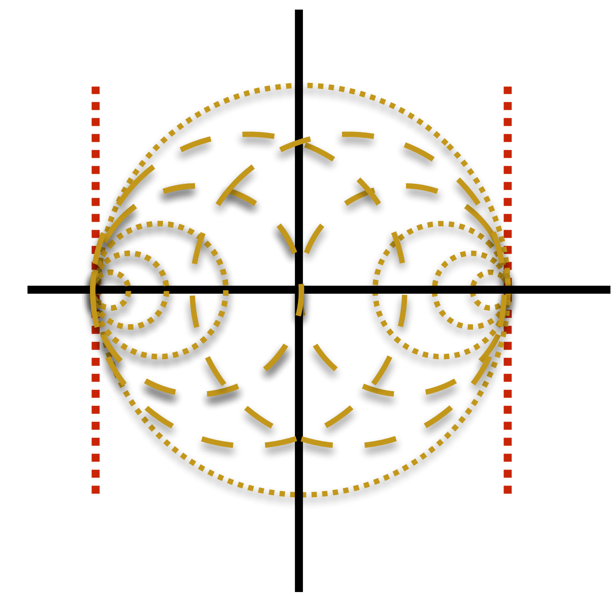
$a_\beta < r_{ap} - D\delta$



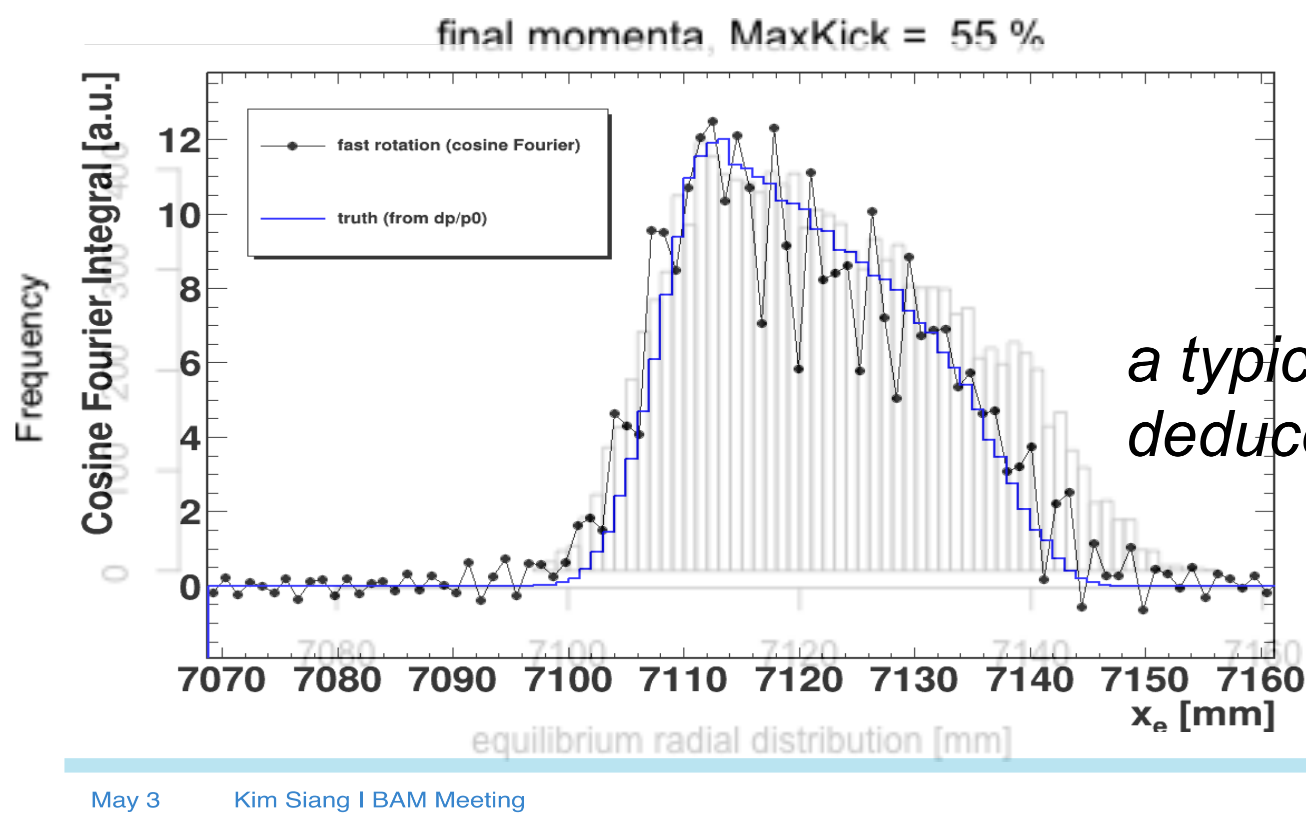
The Momentum Distribution



$a_\beta < r_{ap} - D\delta$

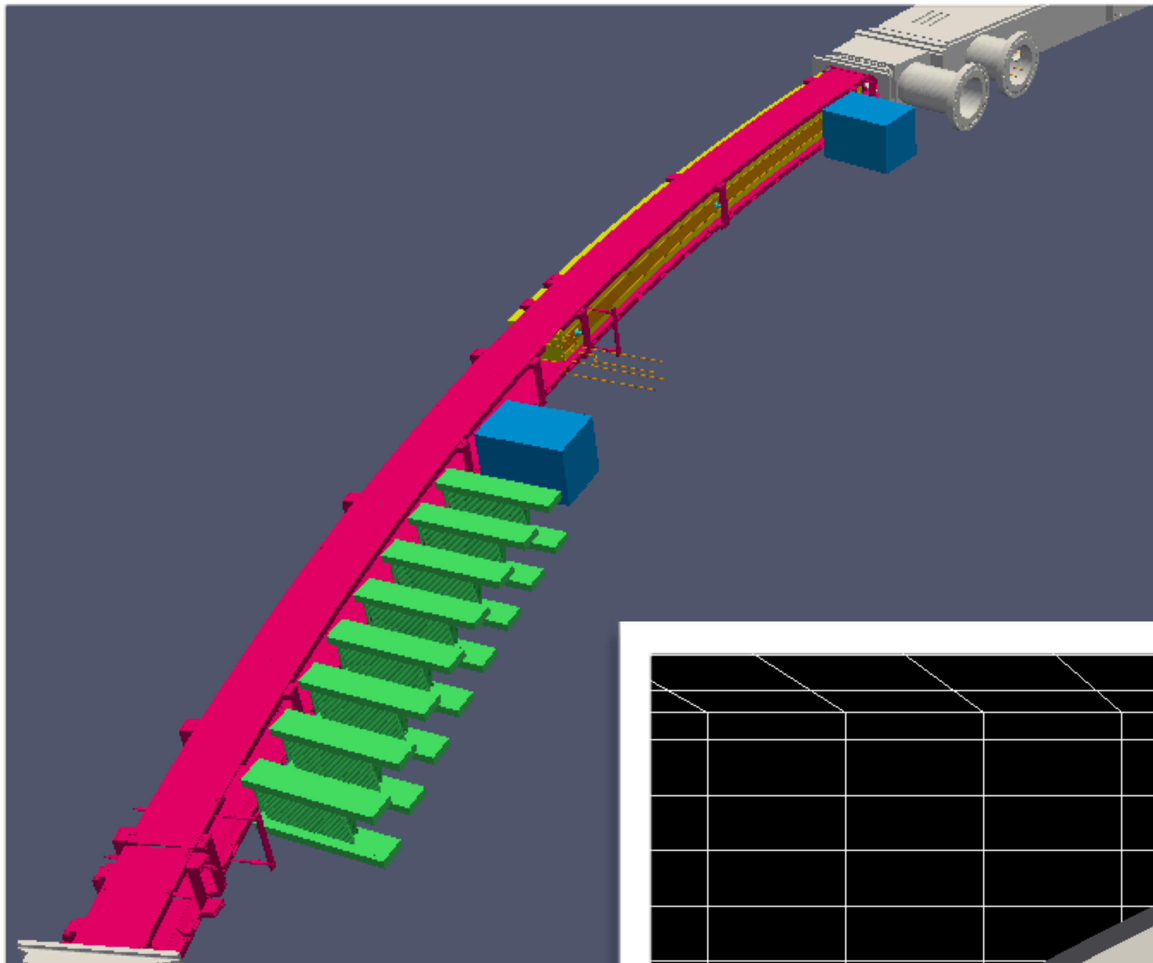


look at momentum distribution of the particles that can survive long-term:



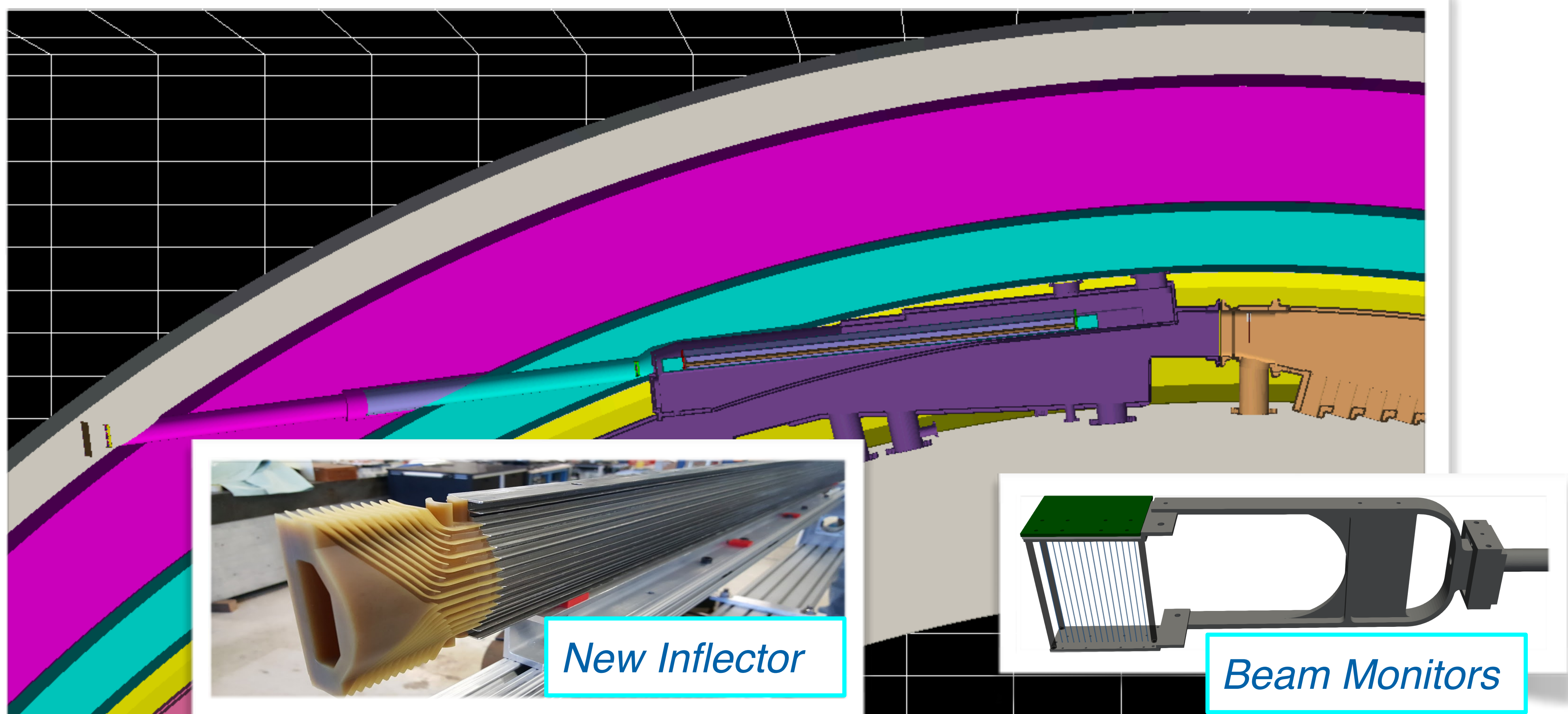
a typical distribution, deduced from data analysis

Modeling with *gm2ringsim*



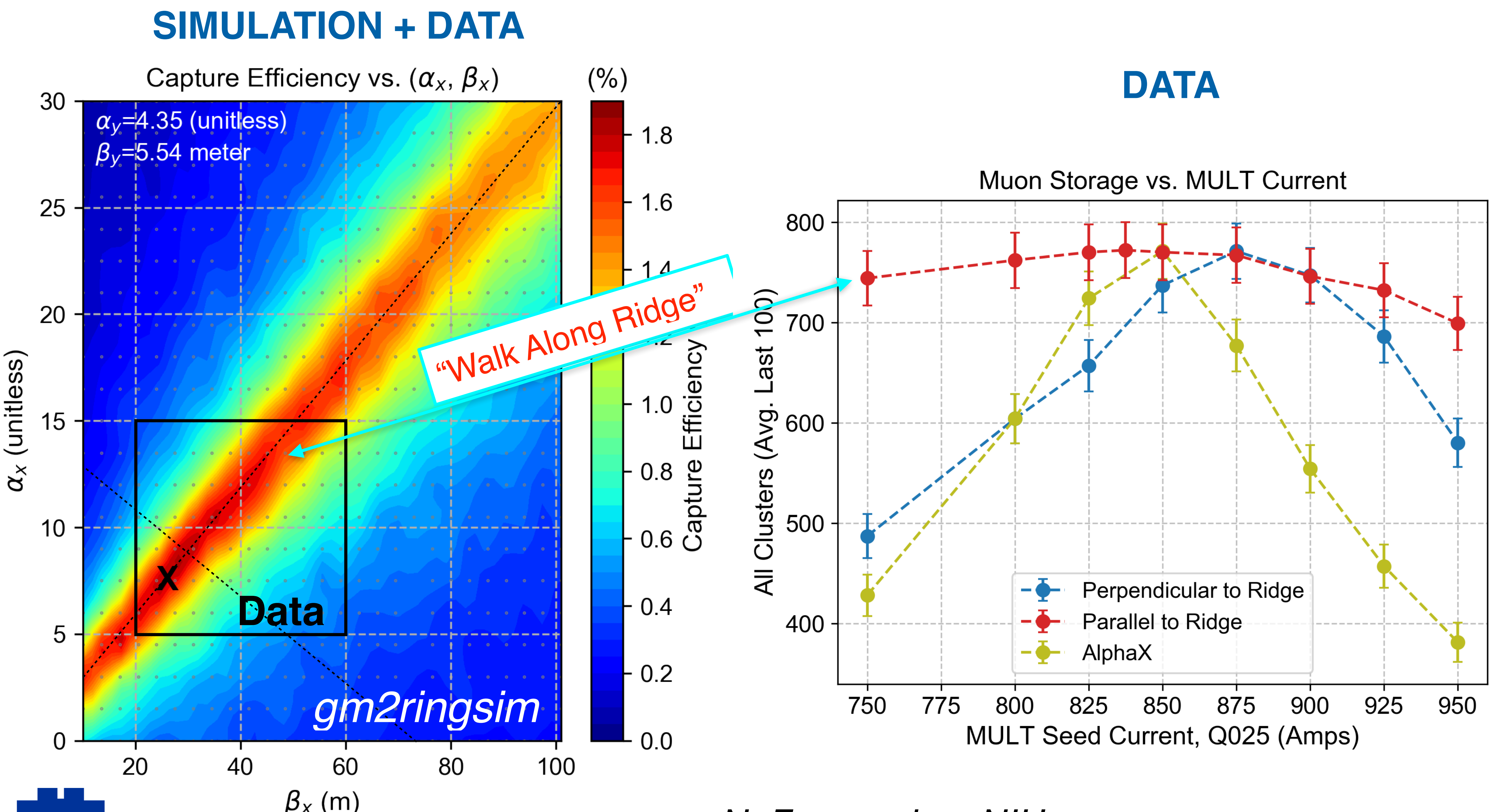
- Geant4-based (C++) tracking simulation
- Used to model muon injection, capture, storage, EM fields, beam dynamics, spin/edm tracking, muon decay, materials effects, detector response, systematics, etc. Very detailed physics.

N. Froemming, NIU



Gm2ringsim: Beam, Injection Tuning

- Plays a critical role in understanding injection, optimization, beam tuning



N. Froemming, NIU

Systematic: E-Field Contribution

- Important to tabulate and tackle the various **systematic errors** present in the data analysis in the determination of *a*
- Example: Not all (any?) muons are at the *magic momentum*, so a momentum offset or an asymmetry in the momentum distribution can generate a systematic error:

how well is this cancelled?

$$\vec{\omega_a} = -\frac{e}{m}\left[a\vec{B}_0 + \left(a - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{E} \times \vec{\beta}}{c}\right]$$

$$x_e = D \cdot \frac{\Delta p}{p}$$

$$\frac{\Delta\omega}{\omega} = -2n(1-n)\beta^2\frac{\langle x_e^2\rangle}{R_0^2} \equiv C_E$$

here, n = "field index" of the weak-focusing system

- Hence, precise determination of the momentum distribution for each store is important

Systematic: Pitch Correction

- Betatron oscillations lead to terms in the spin precession in which

$$\frac{d\vec{S}}{dt} = \vec{\omega}_s \times \vec{S} = -\frac{e}{\gamma m}\left[(1+a\gamma)\vec{B}_\perp + (1+a)\vec{B}_{||} + \left(a\gamma + \frac{\gamma}{\gamma+1}\right)\frac{\vec{E}\times\vec{\beta}}{c}\right]\times\vec{S}$$

$$\vec{B}\cdot\vec{\beta} \neq 0 \qquad (B_{||} \text{ terms})$$

- in particular, the vertical oscillations can contribute to the spin precession in the horizontal plane

$$\vec{\omega_a} \approx -\frac{q}{m}\left[a\vec{B} - a\left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta}\right]$$

- Due to vertical betatron oscillations we would have

$$\vec{\omega'_a} \approx -\frac{q}{m}\left[a\hat{y}B_y - a\left(\frac{\gamma}{\gamma+1}\right)\beta_y B_y(\hat{s}\beta_s + \hat{y}\beta_y)\right]$$

$$\approx -\frac{q}{m}aB_y(1 - \frac{1}{2}\hat{y'}^2\cos^2\omega_y t)$$

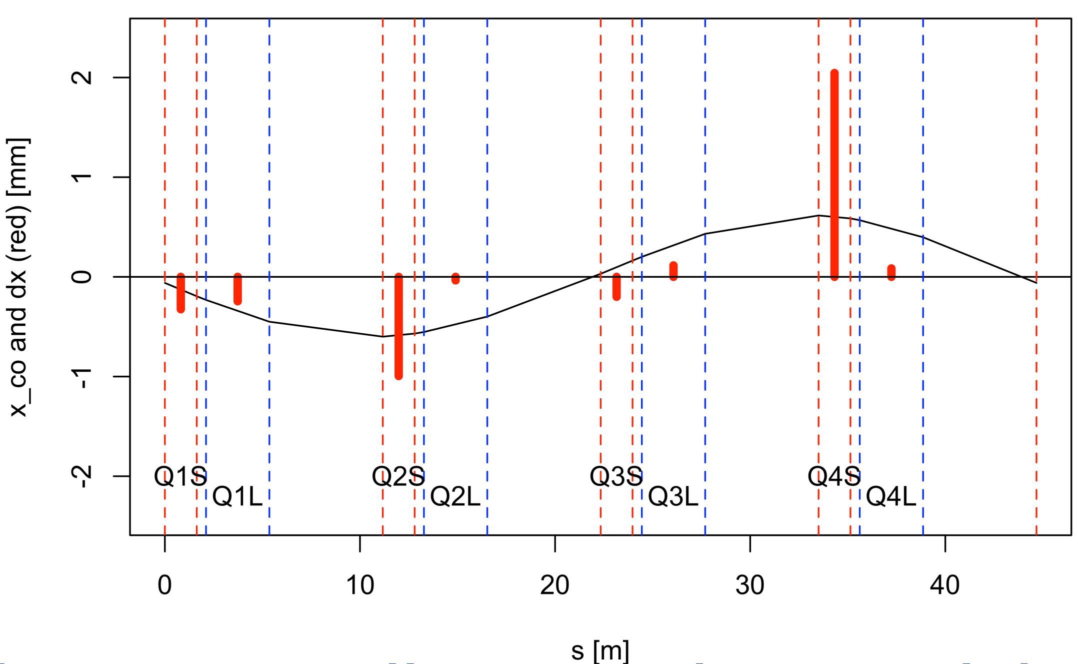
or, $$\vec{\omega'_a} \approx -\frac{q}{m}\left[a\hat{y}B_y - a\left(\frac{\gamma}{\gamma+1}\right)\beta_y B_y(\hat{s}\beta_s + \hat{y}\beta_y)\right]$$

$$\frac{\Delta\omega}{\omega} = -\frac{\langle y'^2\rangle}{2} = -\frac{\langle \hat{y'}^2\rangle}{4} = -\frac{n}{4}\frac{\langle y_0^2\rangle}{R_0^2} \equiv C_P$$

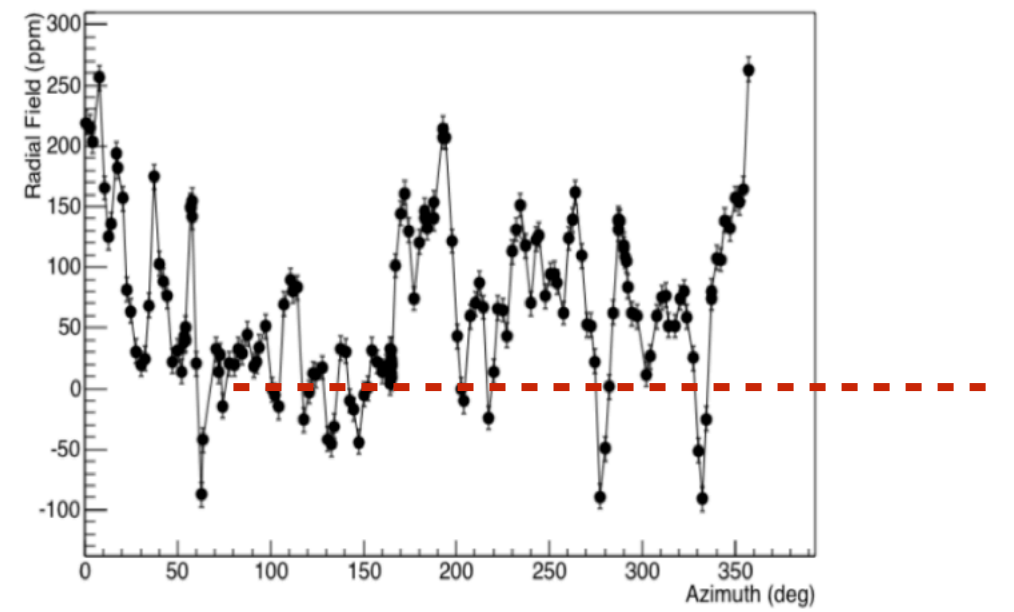
precise particle tracking required

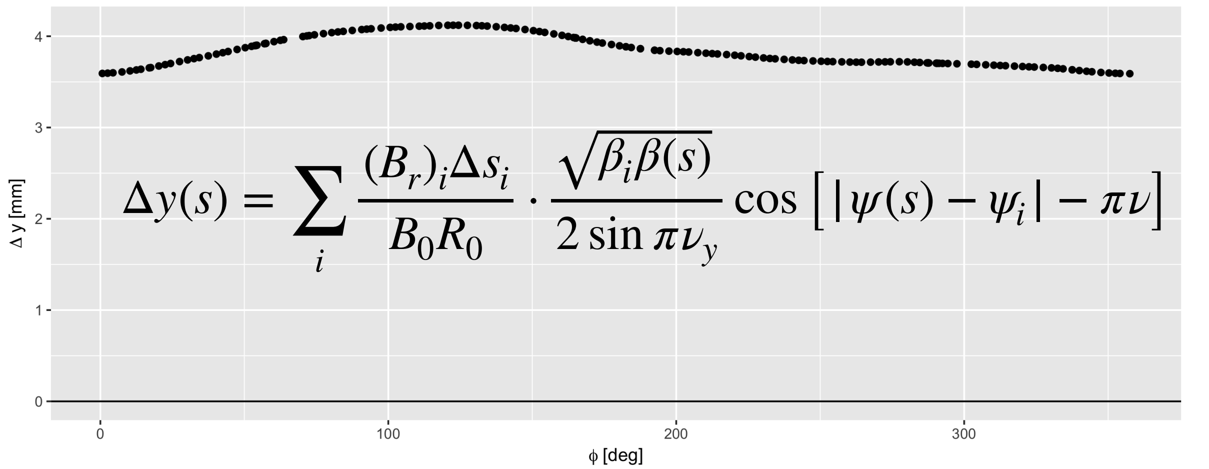
And All the Usual Suspects…

- Residual magnetic field errors and alignment errors of the electrostatic focusing quadrupole plates
  - Tune shifts
  - Orbit distortions
  - Beta function beating
  - Frequency shifts
    - » time-varying fields (on-going kicker fields, etc.)
  - …



Radial Fields





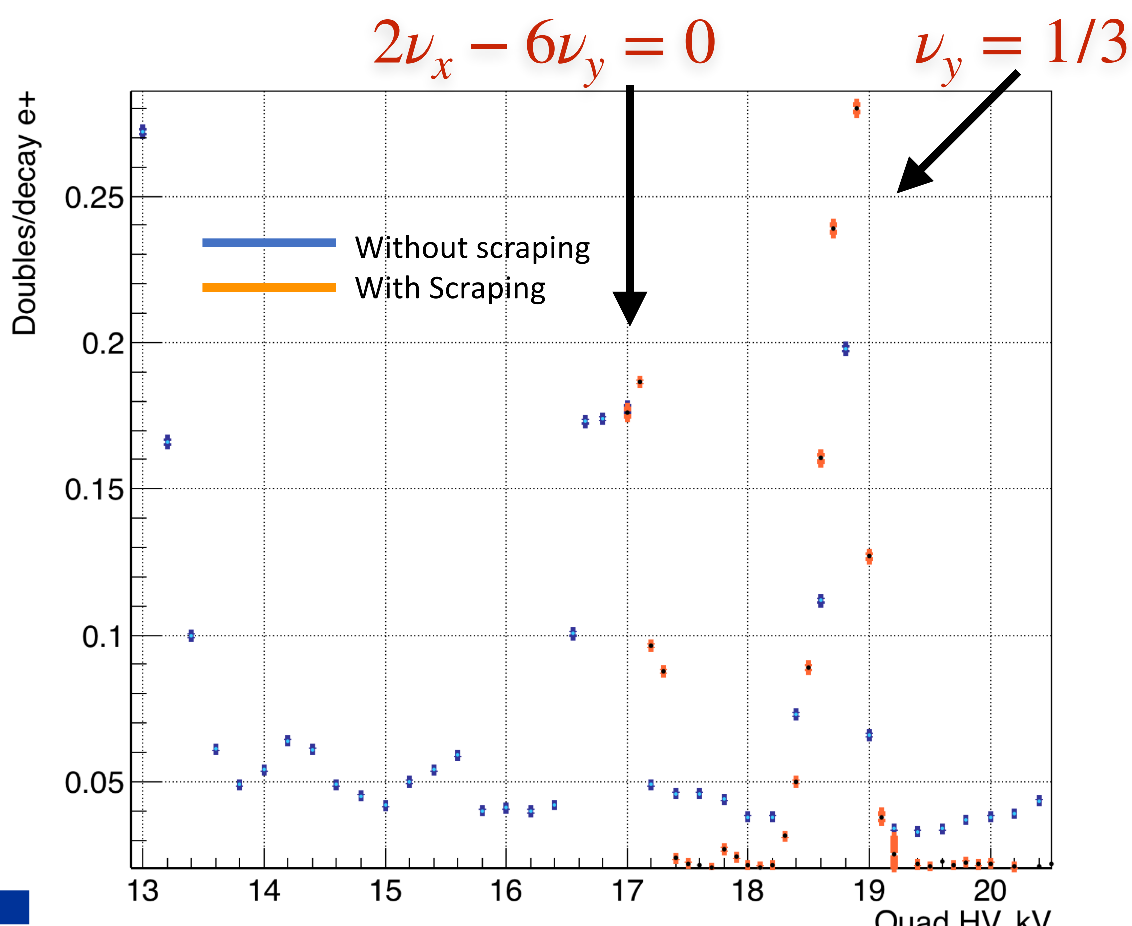
$$\Delta y(s) = \sum_i \frac{(B_r)_i \Delta s_i}{B_0 R_0} \cdot \frac{\sqrt{\beta_i \beta(s)}}{2 \sin \pi \nu_y} \cos\left[|\psi(s) - \psi_i| - \pi\nu \right]$$

- ALL can contribute to the precision of the final determined value of the anomalous magnetic moment

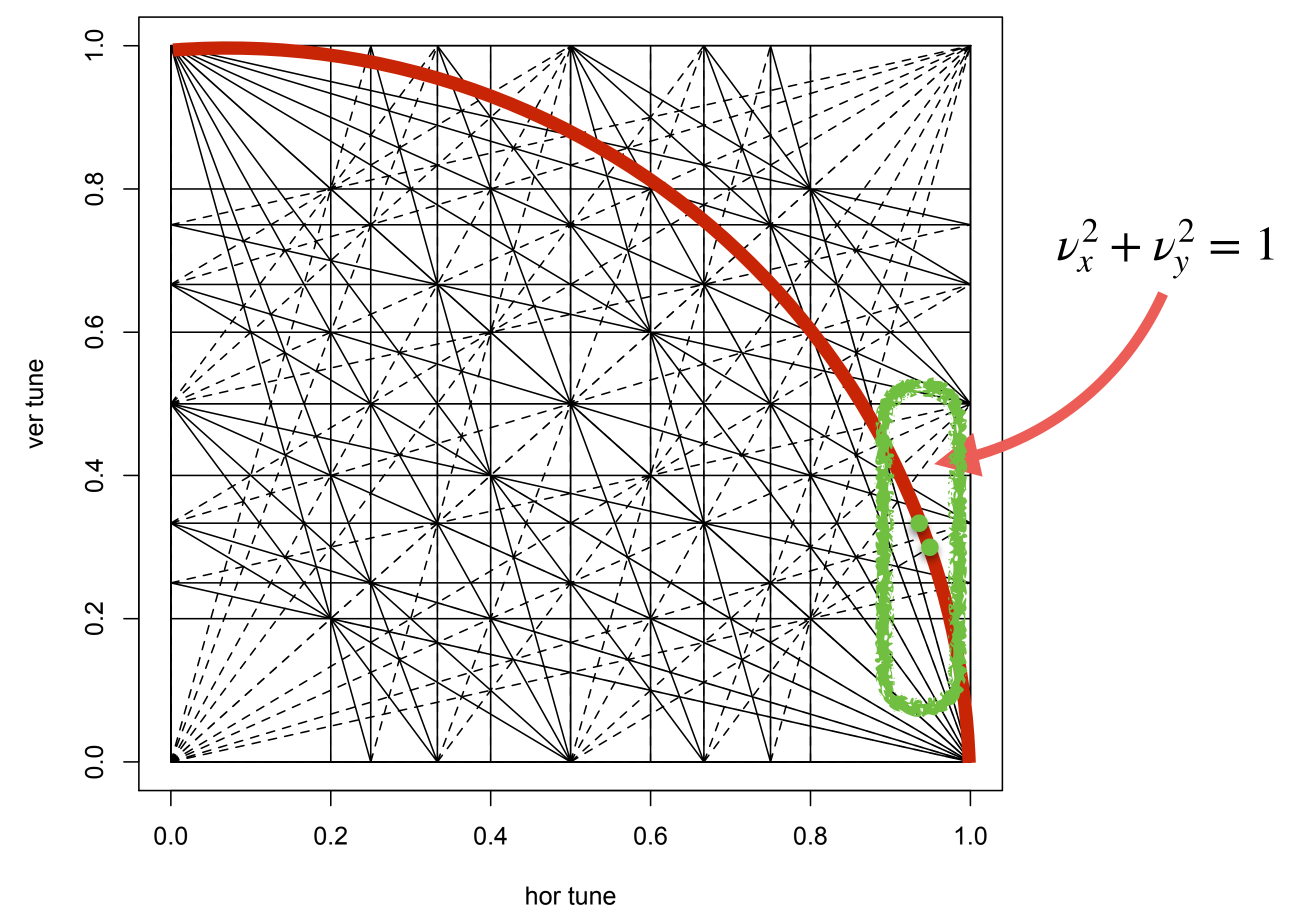
Muon Losses Prior to Decay

- So-called “lost muons” are identified as particular “hits” in the detector system that occur simultaneously on 2 or 3 consecutive detectors, assumed to be a single muon as opposed to 2 or 3 coincidental positrons
  - beam-gas scattering, field fluctuations, resonance conditions, …
  - Muon loss rate not due to decays must be taken into account in the analysis — will run at high and low vertical tunes, away from resonances
- Scan the quadrupole high voltage:
  - note: weak focusing — tunes are coupled: $\nu_x^2 + \nu_y^2 = 1$

$$m\,\nu_x \pm n\,\nu_y = k$$



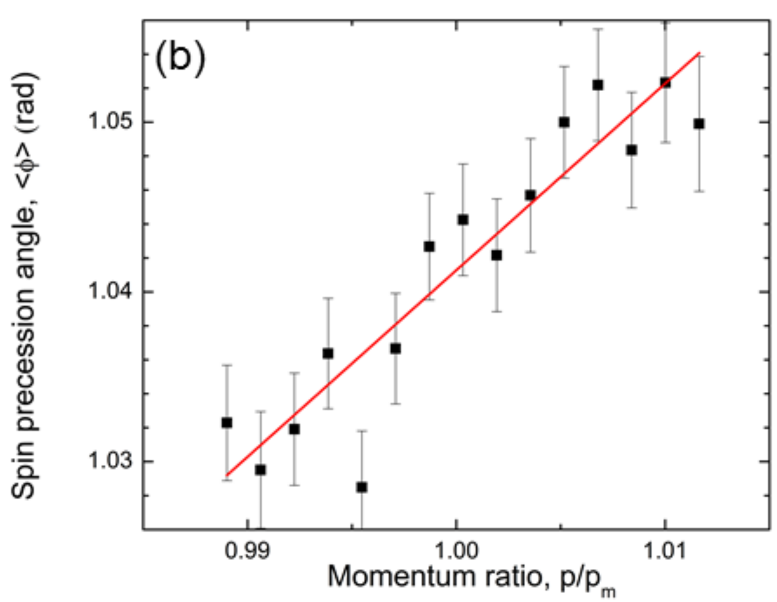
Lost muon doubles vs. quad HVPS set-points for t > 30µs after injection, with scraping and without scraping.



Polarization Simulations

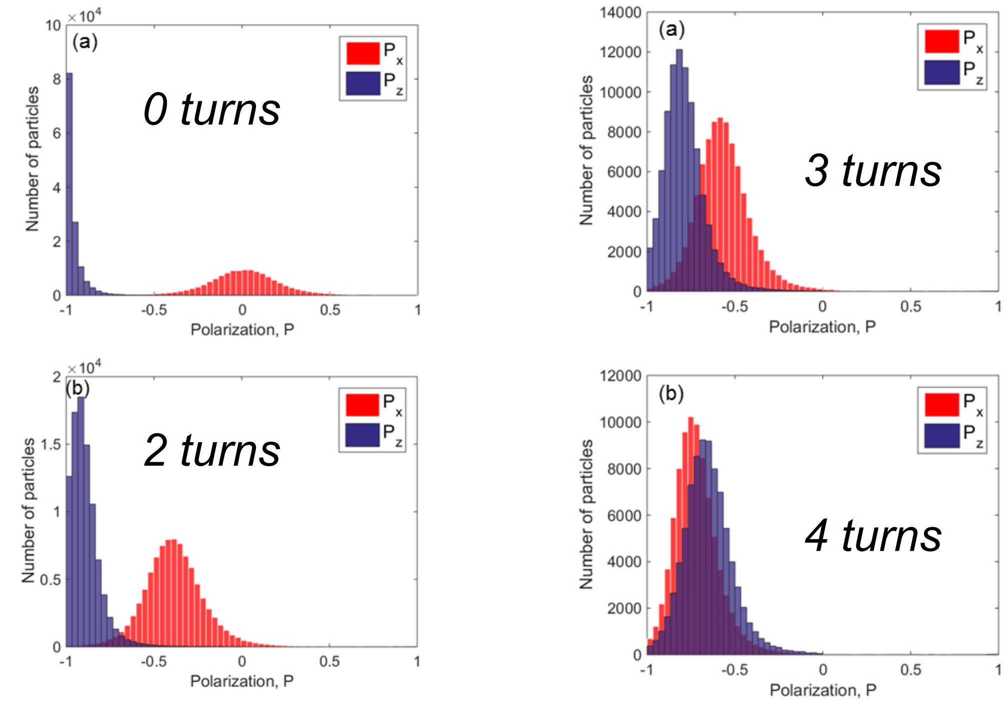
- It's all about the ***spin***
- For each decay, determine the initial spin angle of the muon from the measured trajectory of the emitted positron
- Understand spin correlations to improve the systematic error on the initial spin angle
  - spin direction *vs*. momentum
  - spin direction *vs*. time in the bunch
  - spin direction *vs*. betatron amplitudes
  - *etc*.

After 4 turns in Delivery Ring:



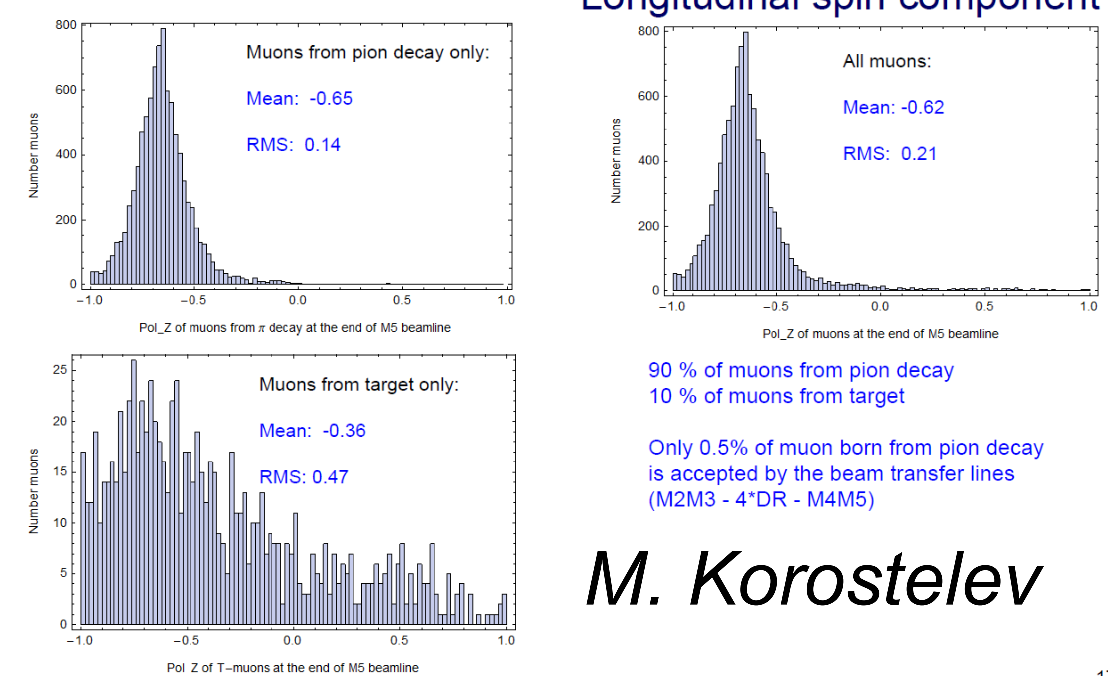
Polarization:

Stratakis, et al.



Distribution of muons at the end of M5 beamline:

Longitudinal spin component



M. Korostelev

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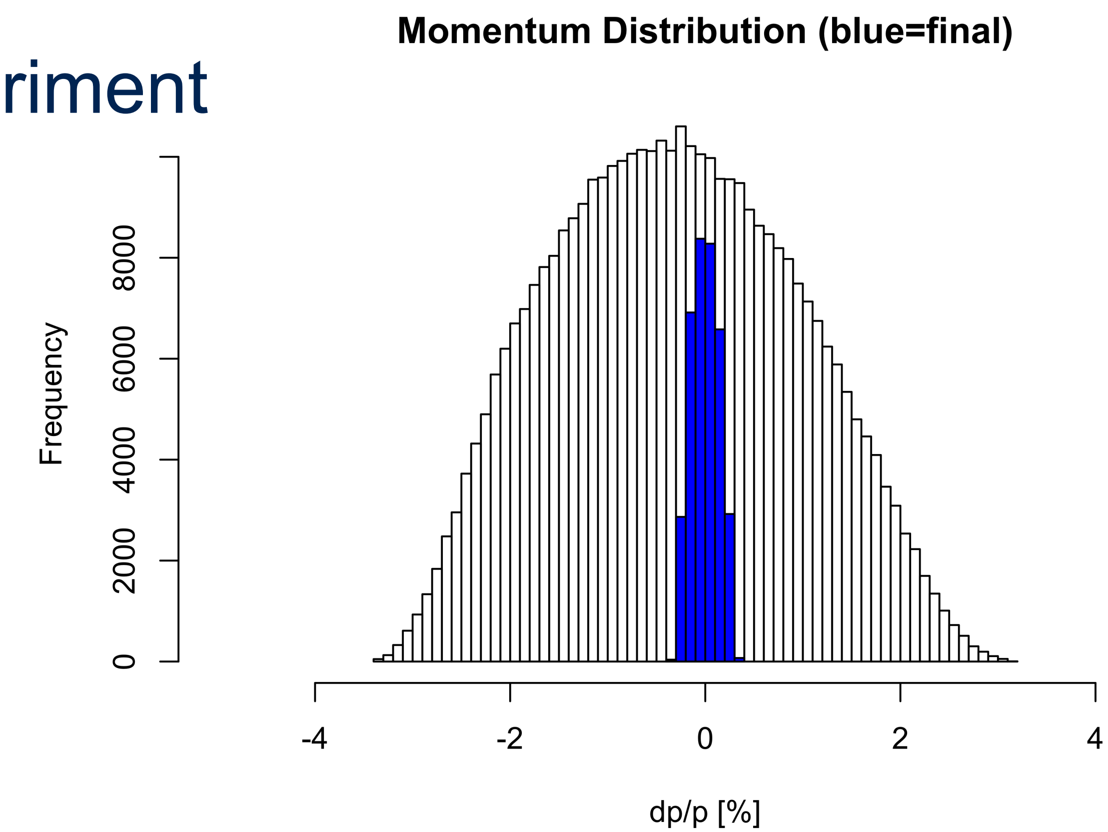
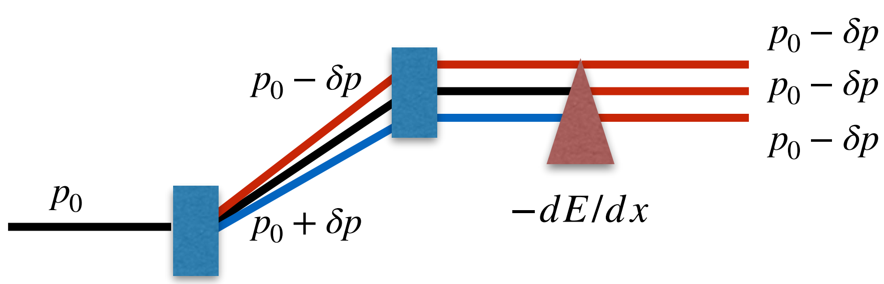
Outlook

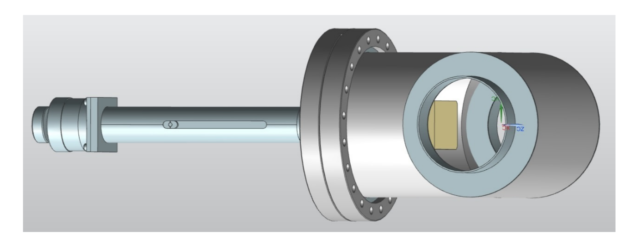
- This important HEP measurement not only relies upon high flux to the apparatus, but also ***heavily*** on particle beam dynamics, including spin dynamics
  - important contribution to high-profile experiment



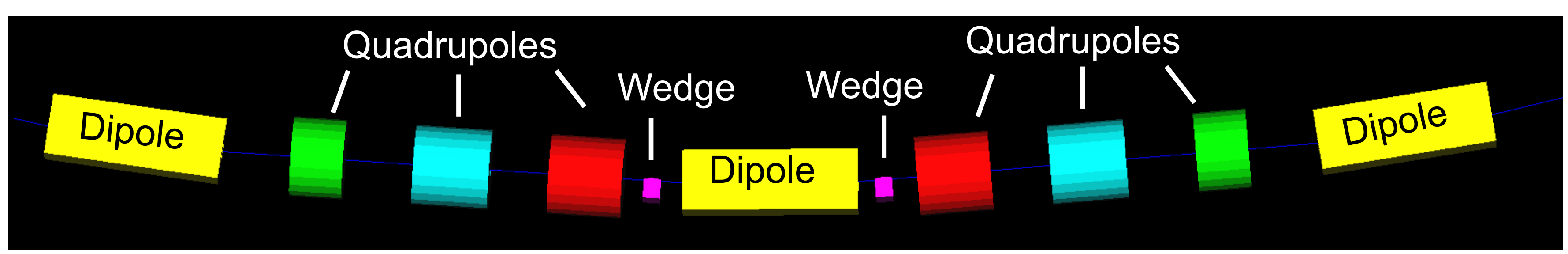
- Have generated ~2x BNL data set
  - looking for factor of 20 or more
  - Approx. 1-2 years more to run
- How to improve the muon flux?
  - Momentum Cooling using *wedges*



Fermilab LDRD Funding
D. Stratakis (PI)
M. Syphers (co-PI)
J. Morgan (coordinator)



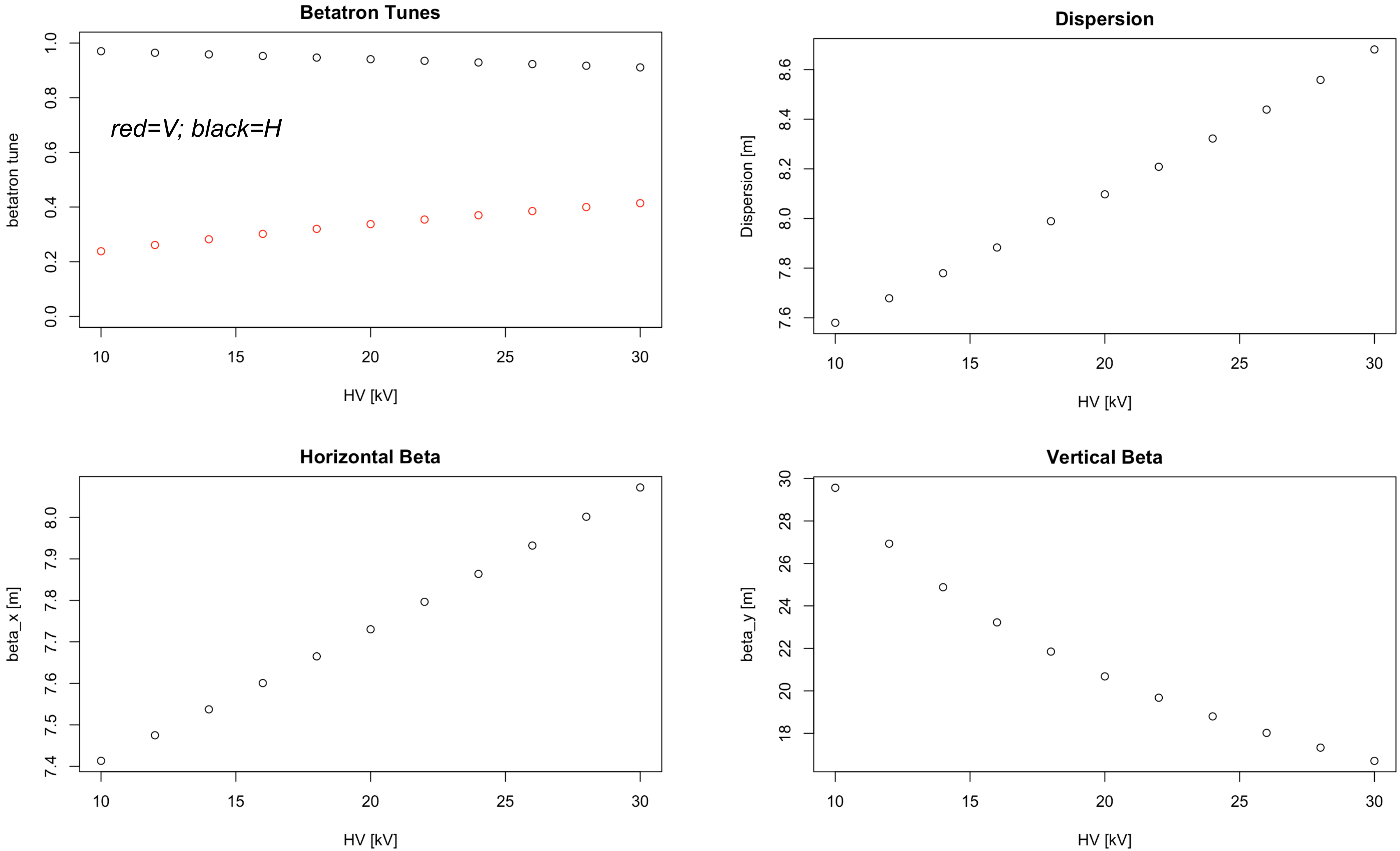
  - System in place for upcoming run



  - ***Continue running***!

Back-Up Slides

Variations with High Voltage

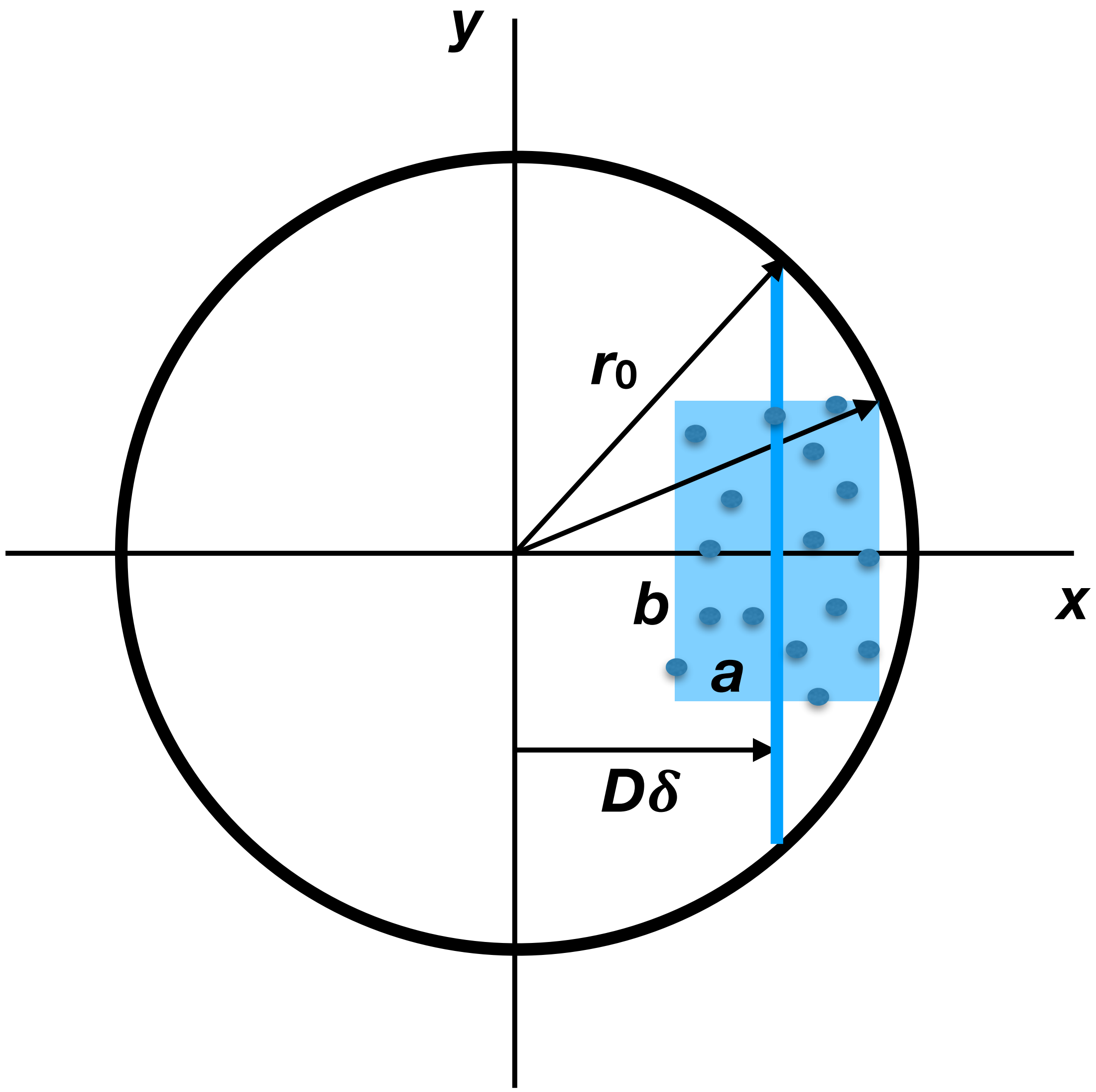


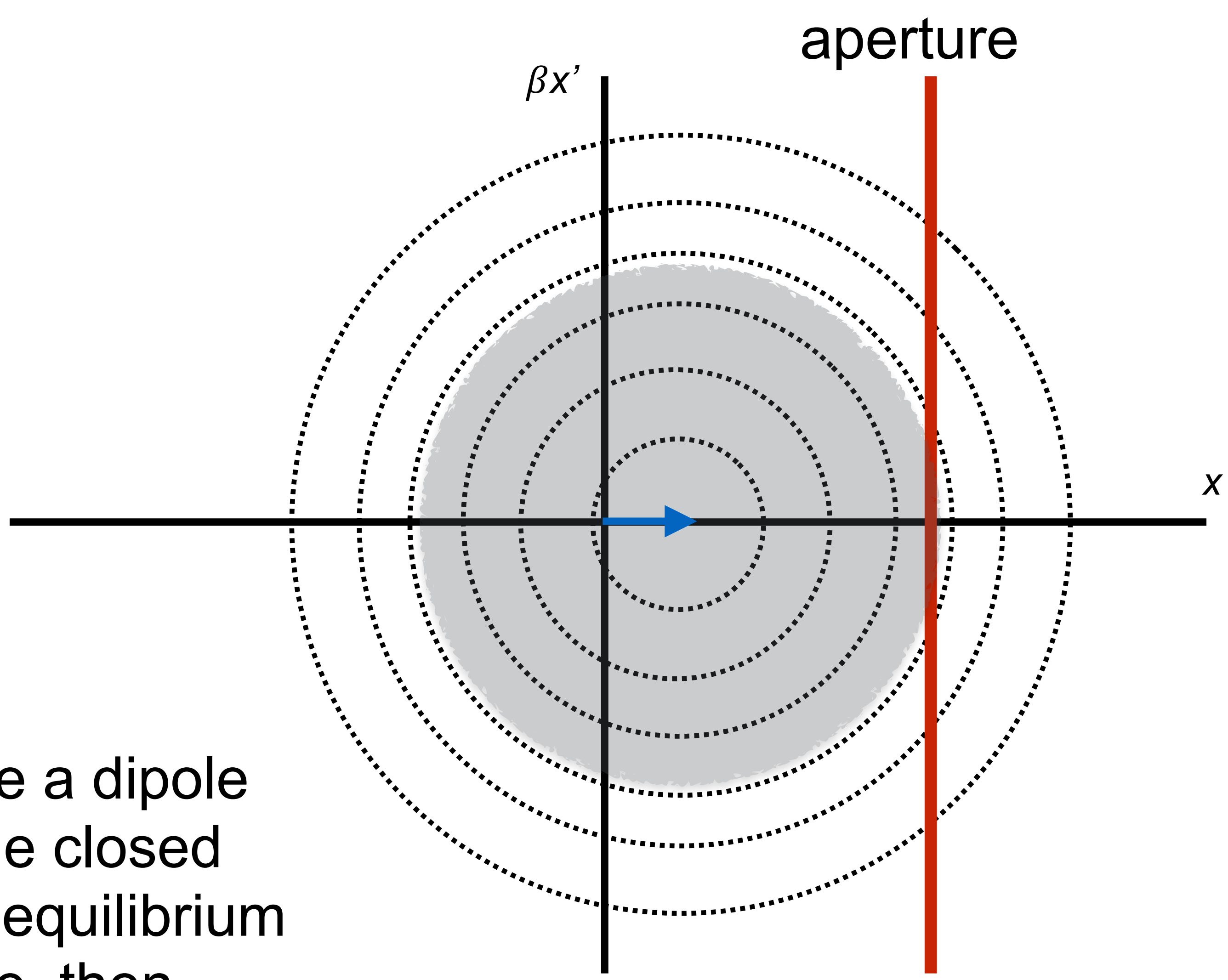
Losing Muons Prematurely

- When discussing *Lost Muons*, consider what the mechanism(s) might be:
  - Simply the time for a particle to eventually "find the aperture"?
    - » the "scraping" process (if the mountain won't come ...)
- Is it muons scattering off of the residual gas molecules in the vacuum chamber?
- We know that the rates go up when near "resonances" in the tune space; how do we interpret this in terms of phase space dynamics?

Scraping

- Some small fraction of particles will live “on the edge”; will eventually encounter the collimator aperture

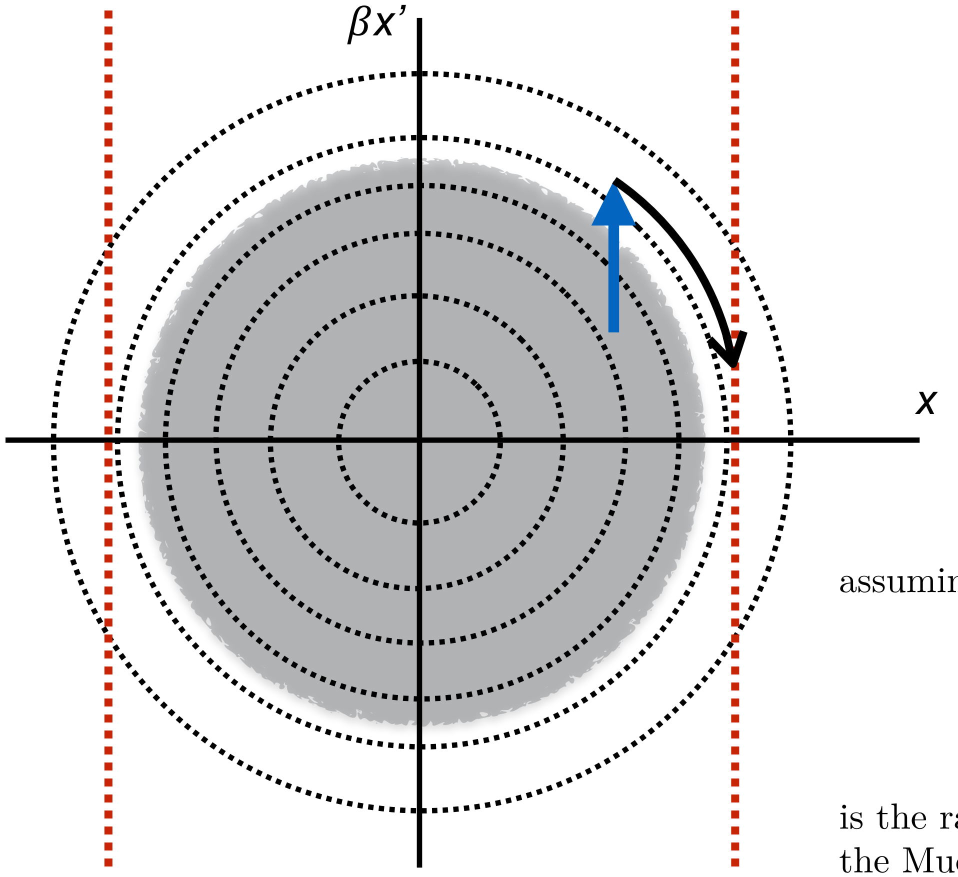




Use quad plates to create a dipole electric field, distorting the closed orbit (hor + ver) to move equilibrium orbits toward the aperture, then “back off”

Beam-Gas Scattering

- As muons Coulomb scatter off of residual gas molecules, their betatron amplitudes will grow (on average) and particles can eventually reach the aperture and be lost.



If entire distribution were well within the aperture, then would observe an emittance growth of rate *R*:

$$R \equiv \frac{d}{dt}\langle W \rangle = \frac{d}{dt}\langle \pi a^2/\beta \rangle = \pi\beta\frac{d}{dt}\langle x'^2 \rangle = \pi\beta \cdot \langle \dot{\theta^2} \rangle$$

assuming scattering events only alter x' and not x, and where

$$\langle \dot{\theta^2} \rangle \approx \left(\frac{13.6 \text{ MeV}}{pv}\right)^2 \frac{v}{L_{rad}}$$

is the rate of increase of the variance of the scattering angle, θ, due to multiple Coulomb interactions. For the Muon g-2 Storage Ring and using "air" as our scattering material, we find that

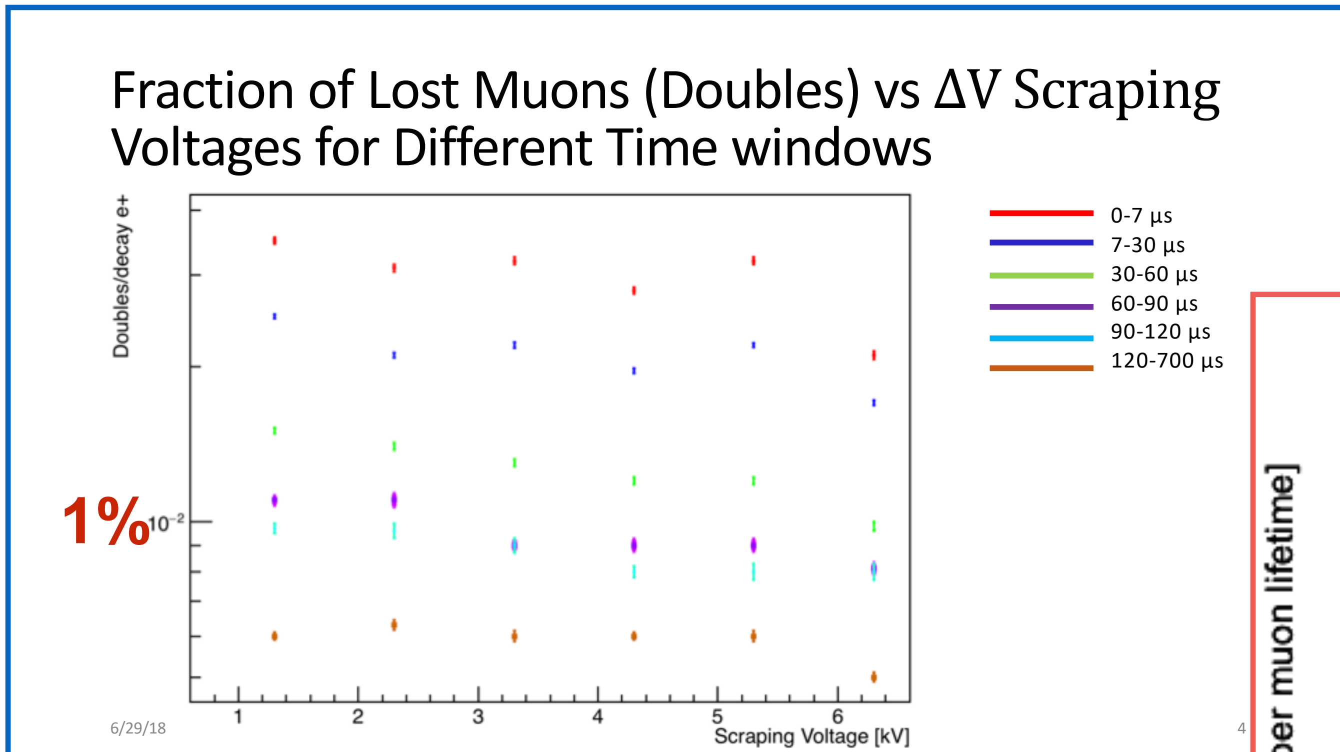
$$\langle \dot{\theta^2} \rangle \approx \left(\frac{13.6}{3094}\right)^2 \frac{3 \cdot 10^8 \text{ m/s}}{\frac{36.6 \text{ g/cm}^2}{1.205 \text{ g}/\ell}\frac{10^3 \text{cm}^3}{\ell}} \cdot \frac{100 \text{ cm}}{\text{m}} \cdot \frac{P_{\mu torr} \cdot 10^{-6}}{760}$$

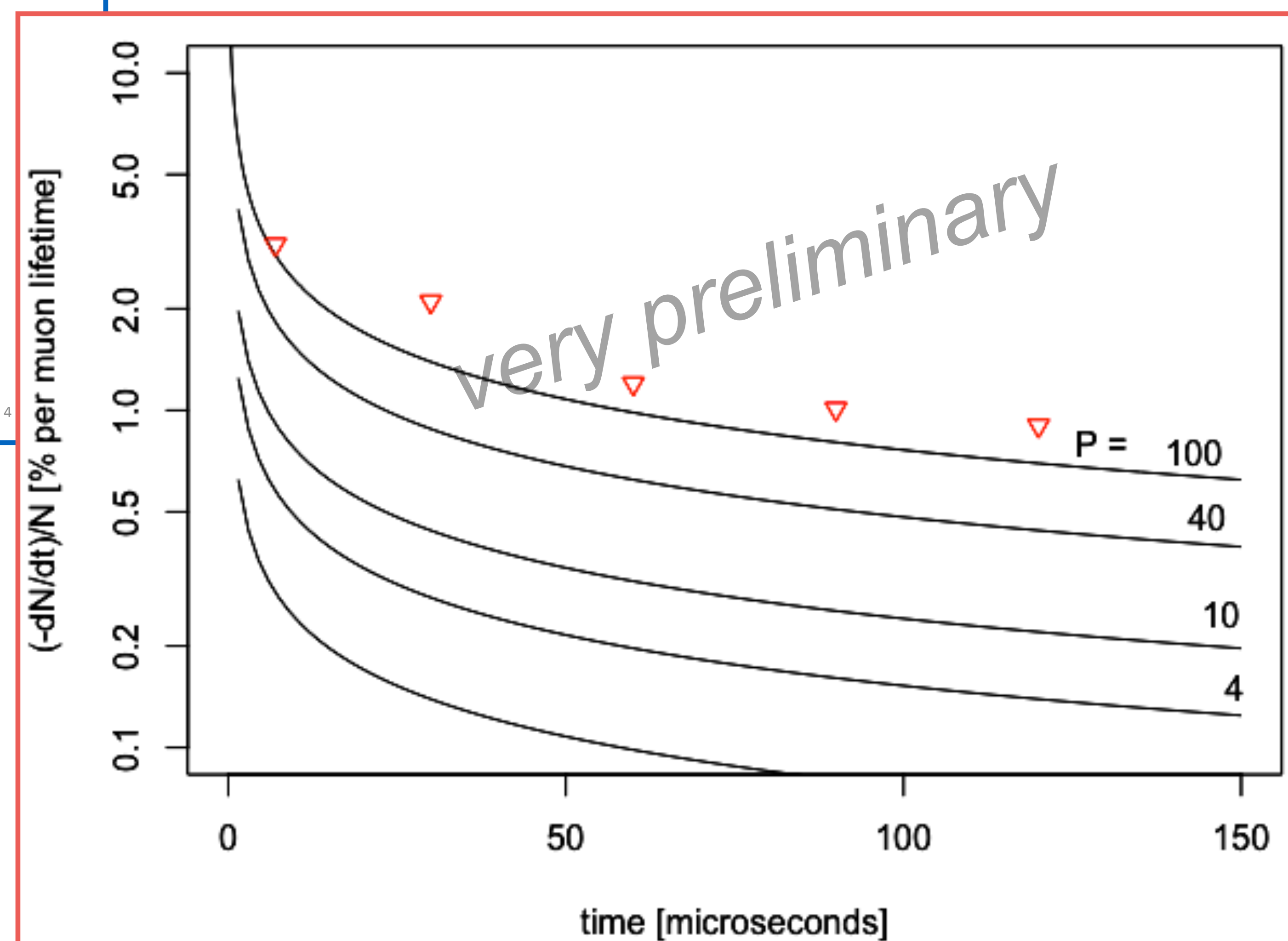
$$= (0.16 \text{ mr})^2/\text{s} \cdot P_{\mu torr}$$

where $P_{\mu torr}$ is the residual gas pressure in units of microtorr.

Beam-Gas Scattering

- As muons Coulomb scatter off of residual gas molecules, their betatron amplitudes will grow (on average) and particles can eventually reach the aperture and be lost



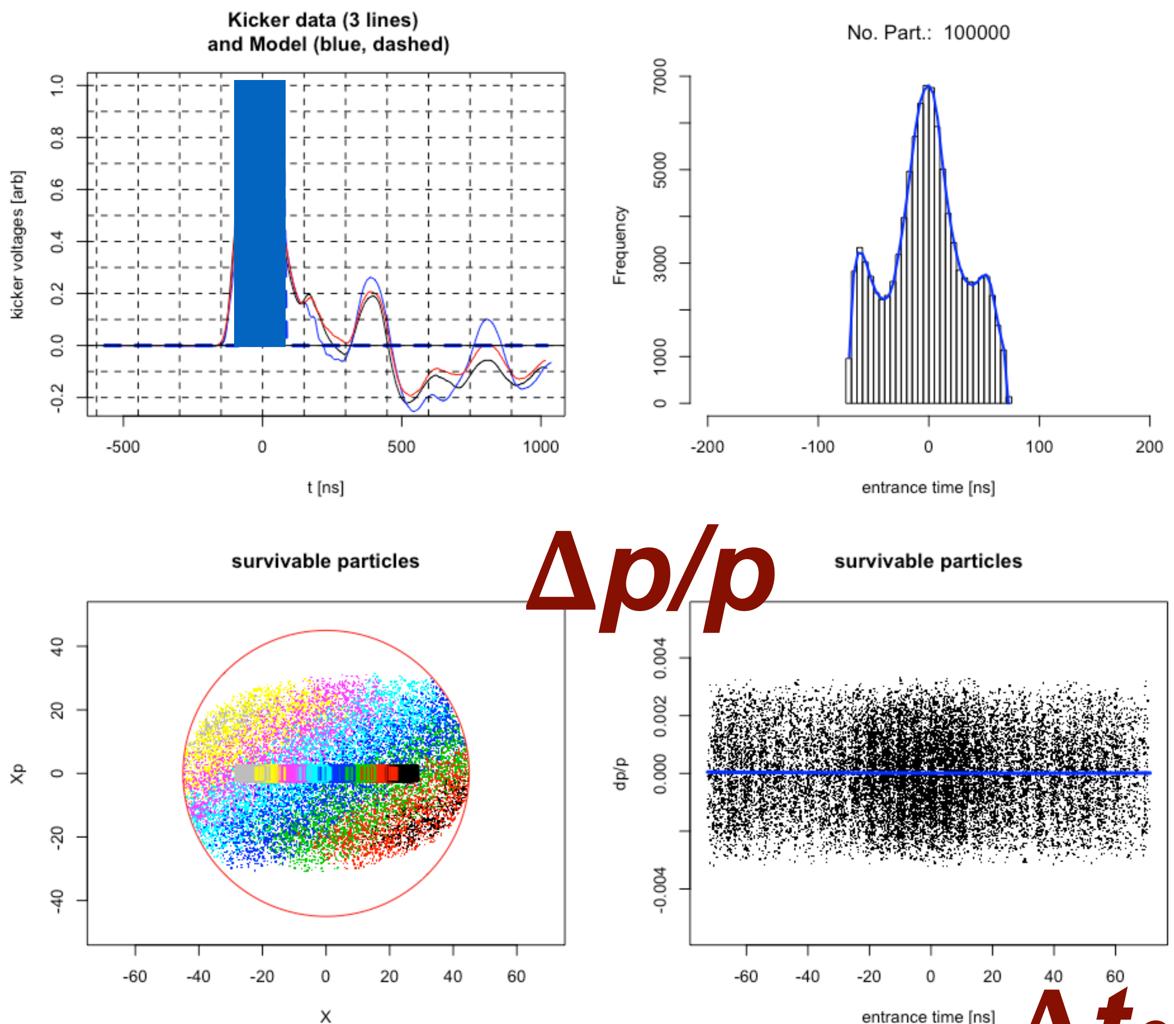


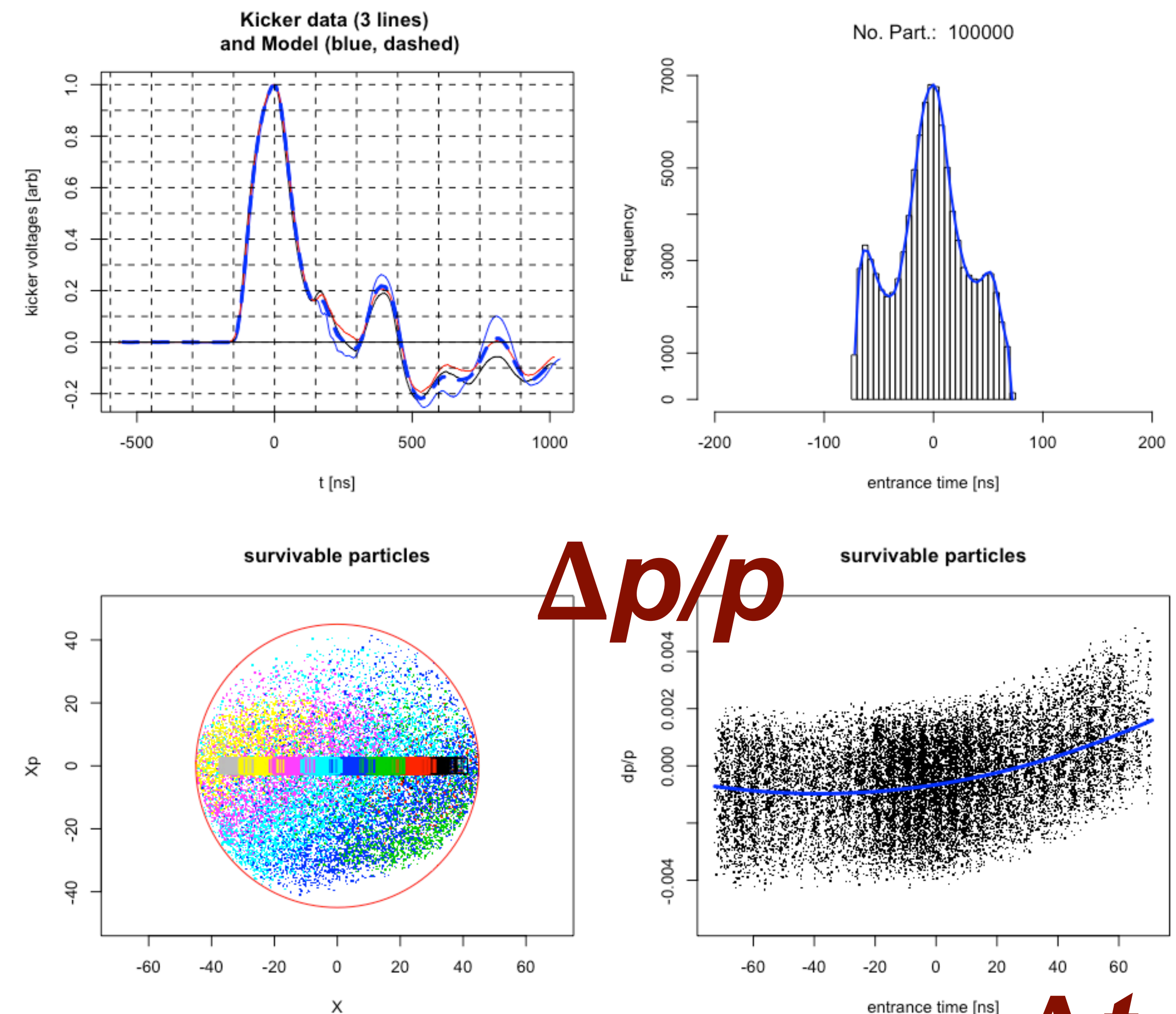
Momentum - Δt_0 Correlation

Northern Illinois University

- Square Pulse

Ringing Pulse





$\Delta p/p$

Δt_0

$\Delta p/p$

Δt_0

Gm2ringsim: Design Guidance

- Used to make several important design decisions (Q1outer + standoffs, kickers, collimators, new superconducting inflector magnet, trackers, etc.)

Old Design (bad)	New Concept (@gm2ringsim)	New Reality (~40% more μ^+)