POLARIZATION IN eRHIC ELECTRON STORAGE RING AN ERGODIC APPROACH

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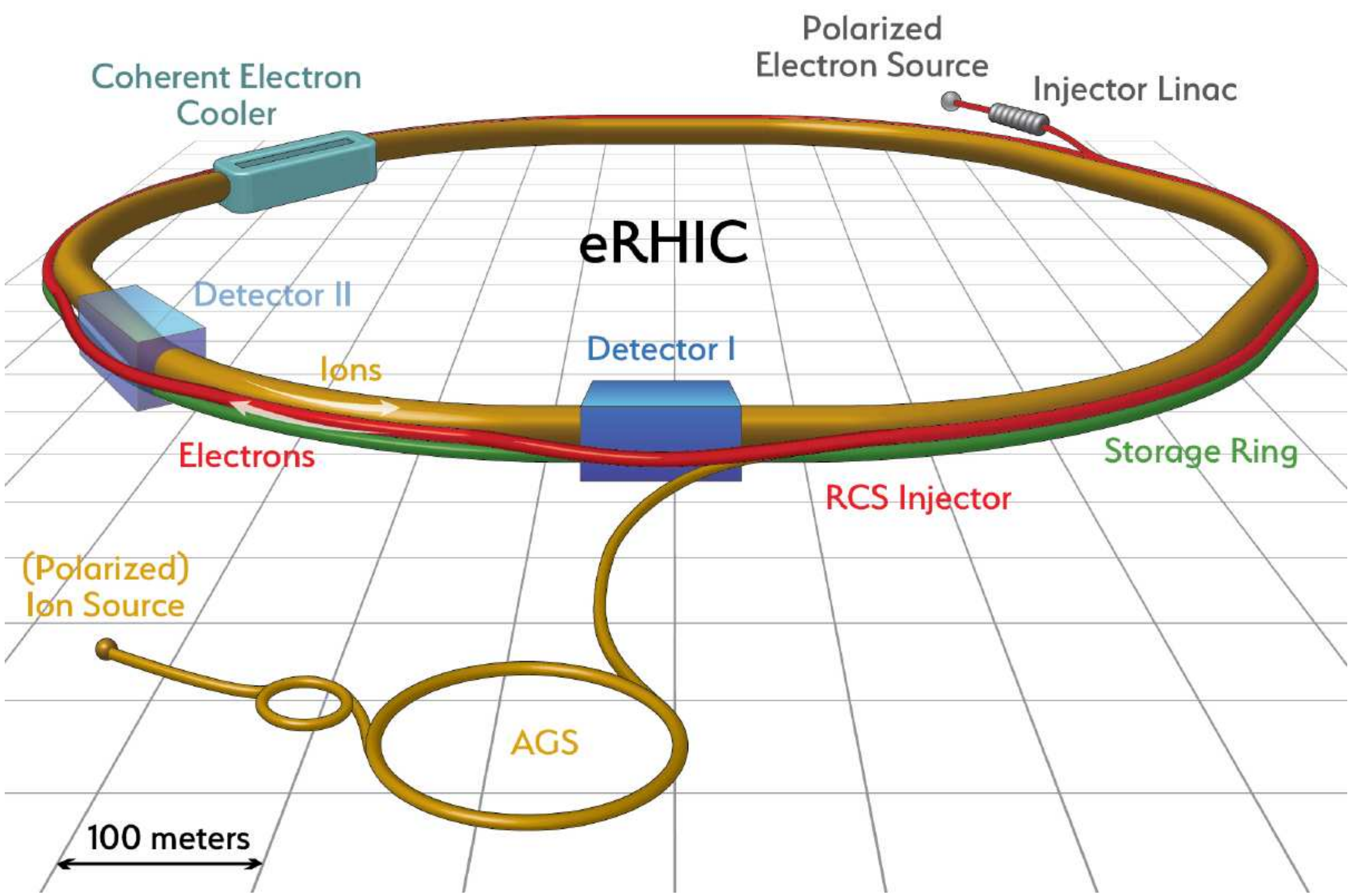


Electron polarization

- From source: 85%, longitudinal
- Into RCS: 85%, vertical, alternately $\uparrow$ and $\downarrow$

  Tilted to vertical at RCS injection
- RCS transmission to store energy: >95%

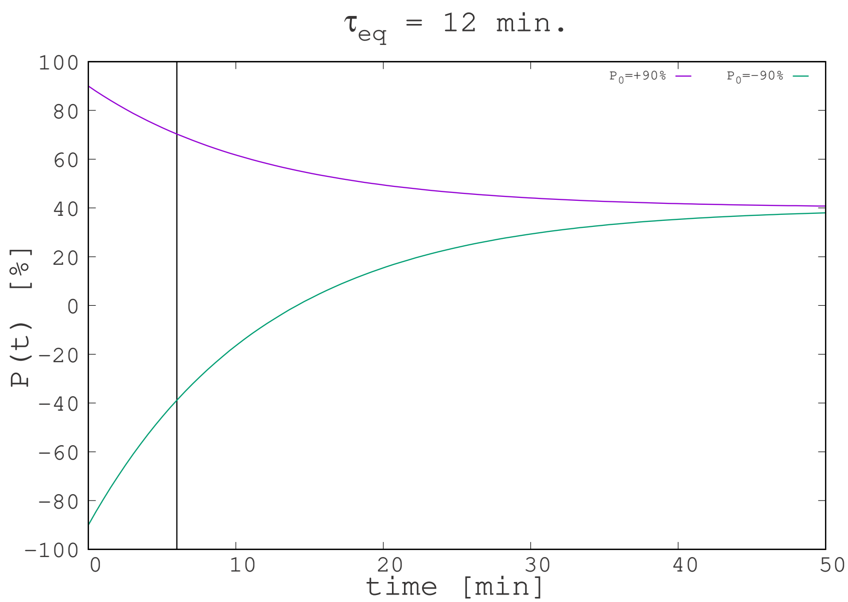
  Provided quadrupole/BPM alignment $< 0.2\,\mathrm{mm}$ *rms, and orbit control* $< 0.5\,\mathrm{mm}$
- Bunch into storage ring: >80%, $\uparrow$ and $\downarrow$

  Operation energies: 5, 10 or 18 GeV
- Stored polarization is affected by synchrotron radiation effects, two processes

Self-polarization (Sokolov-Ternov effect)

- **Synchrotron radiation causes electron spins to slowly self-polarize towards anti-$\vec{B}//$, equilibrium P_{ST},**

$$\mathbf{P(t) = P_{ST} + (P(0) - P_{ST})e^{-t/\tau_{ST}}}$$



$$\diamond\ \tau_{ST} = \left[\frac{5\sqrt{3}}{8}\frac{\hbar r_e}{m}\frac{\gamma^5}{\mathcal{C}} \times \oint_{dipoles}\frac{ds}{|\rho|^3}\right]^{-1} \xrightarrow{iso-B} 99\frac{\rho^2 R}{E^5_{[GeV]}}[sec]$$

- **In particular: bunches injected $\vec{B}//$ slowly loose their polarization by spin-flip to anti-$\vec{B}//$**

- **Goal polarization at store : $< 70\% >$**

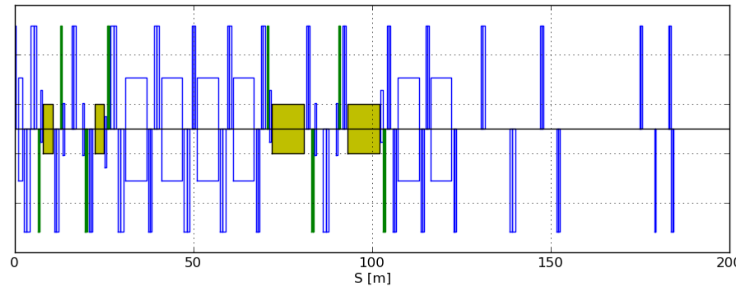
- **At 18 GeV, a short $\tau_{ST} \approx 30$ minute requires replacement of "wrong" polarization bunches every 6 minutes (180 bunches /360)**

Spin diffusion

- Stems from the stochastic change of the spin precession direction due to synchrotron radiation, $\partial\vec{\mathbf{n}}_\delta/\partial\gamma$

(in a similar way that the chromatic closed-orbit jumps upon SR)

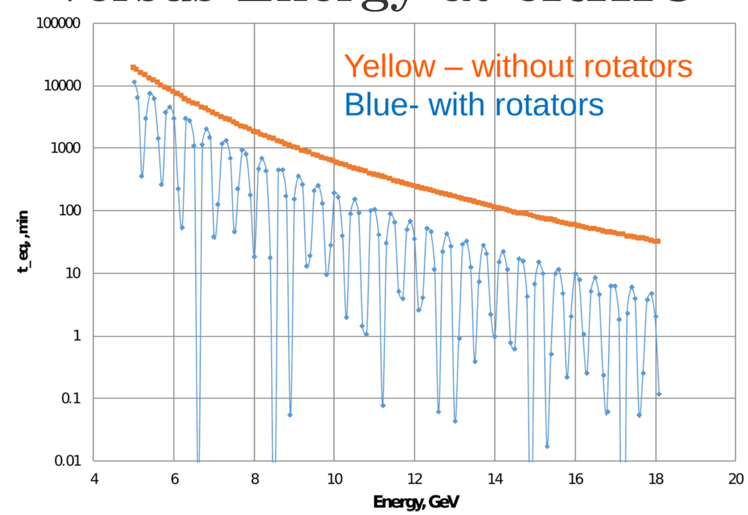
◇ Two solenoid + two bending magnet sections, interleaved



- Dominates bunch depolarization:
- shortens polarization time, $\tau_{\text{eq}} < \tau_{\text{ST}}$,
- reduces asymptotic polarization, $\text{P}_{\text{eq}} < \text{P}_{\text{ST}}$.

◇ Strength of depolarizing effect $\propto E^7$

◇ Equilibrium time τ_{eq} versus Energy at eRHIC



- Spin rotators at eRHIC enhance diffusion as they introduce *vertical orbit, vertical dispersion, coupling...*

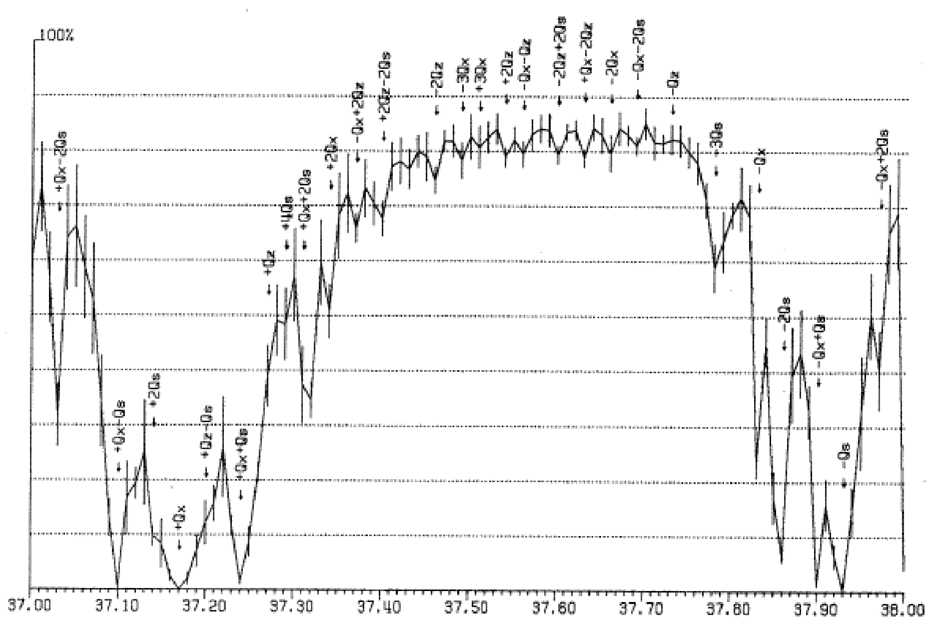
Assessment of polarization performance

- Typically:

As produced using SITROS at DESY *[J. Kewisch, DESY Rep. 83-032, 1983]*

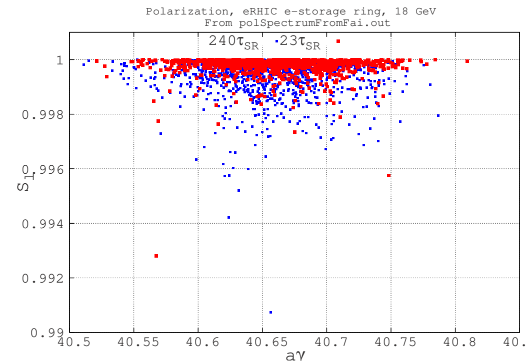
(i) track particles and spins, including Monte Carlo SR,

(ii) produce polarization landscape, *i.e.*, P_{eq} versus ring rigidity setting ($a\gamma$ units, here):



HERA-e, linear, res. non-corrected

- Spins tracked over 240 damping times in eRHIC, 18 GeV



- Polarization decay with time:

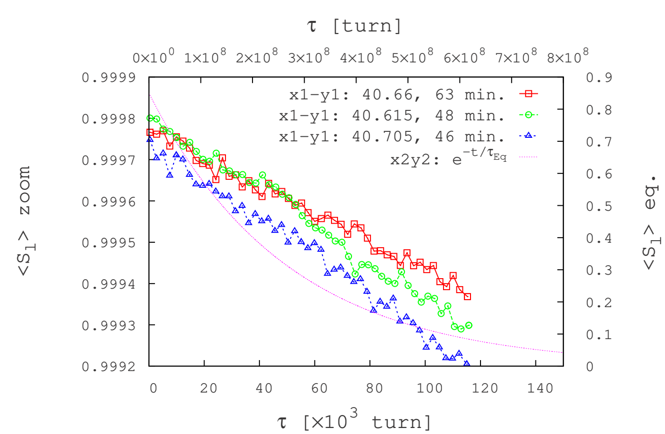
The diffusion time constant τ_D is obtained from linear regression $P/P_0 = \exp(-t/\tau_D) \approx 1 - t/\tau_D$.

Then, P_{eq} stems from

$$\boxed{P_{eq} = P_{ST} \times \tau_{eq}/\tau_{ST}},$$

given that $\tau_{eq} = (1/\tau_{ST} + 1/\tau_D)^{-1}$.

Polarization vs. time at store, 18 GeV



This is very much HPC consuming

- **Typically:**

◇ **tens of bunches, each its own ring energy**

◇ **of the order of a thousand particles per bunch**

◇ **tracking is over several damping times,**

damping time:

- **500 turns at 18 GeV**
- **3000 turns at 10 GeV**

◇ **around a large ring - many optical elements (eRHIC circumference 3.833 km).**

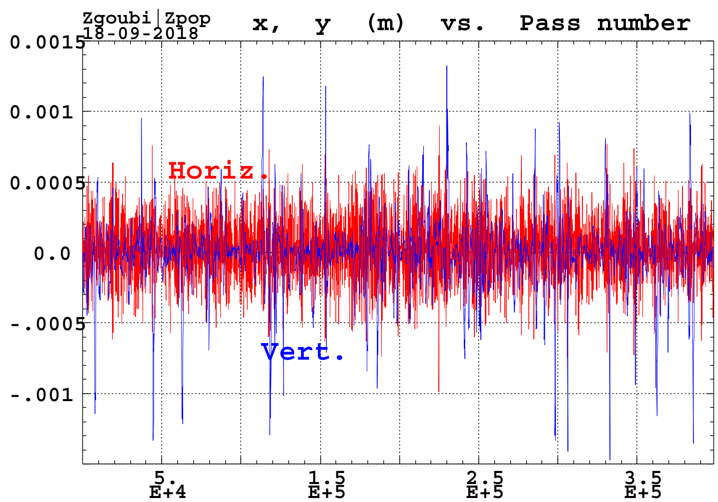
Try some ergodic hypotheses, instead

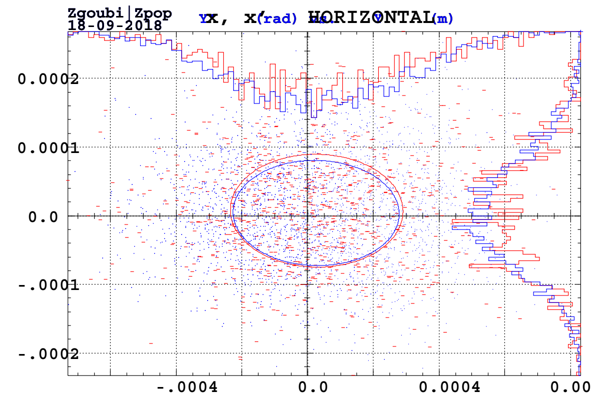
• The dynamical system of an electron bunch in the presence of synchrotron radiation, at equilibrium, is ergodic.

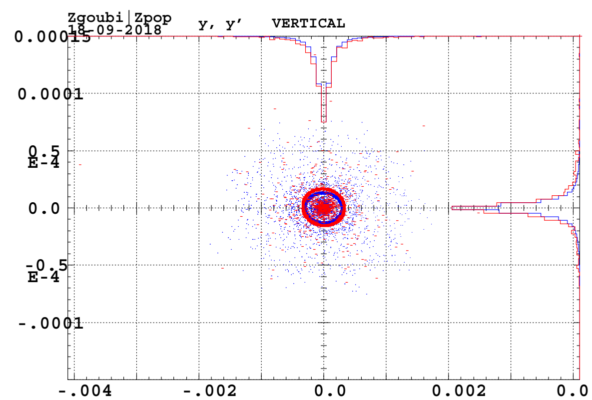
$$\lim_{T\to\infty}\int_{t_0}^{t_0+T} f(\vec{X}(t))dt = \int f(\vec{X})\rho(\vec{X})d^N\vec{X}\Big|_{time=t}$$

TRANSVERSE PHASE SPACES.

Transverse particle excursion:

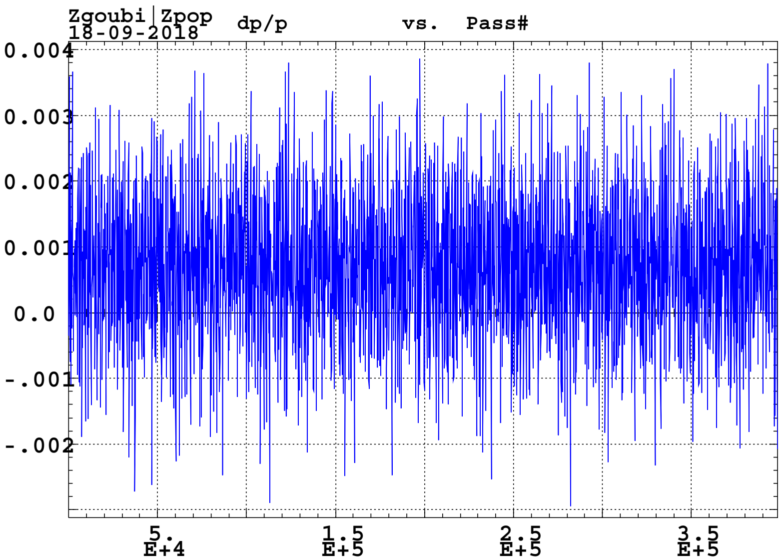




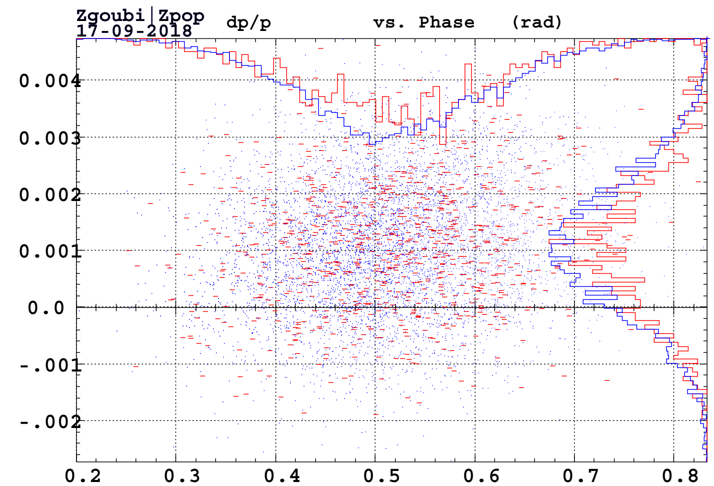


Blue: single particle motion of the left figure, and matching ellipses. (a 27% coupling, by the solenoid based spin rotator in eRHIC IR6).
Red: for comparison, case of a 10^3 particle bunch, observed at time $t = 10^3\tau_{SR}$; *rms* matching ellipses.

LONGITUDINAL PHASE SPACE

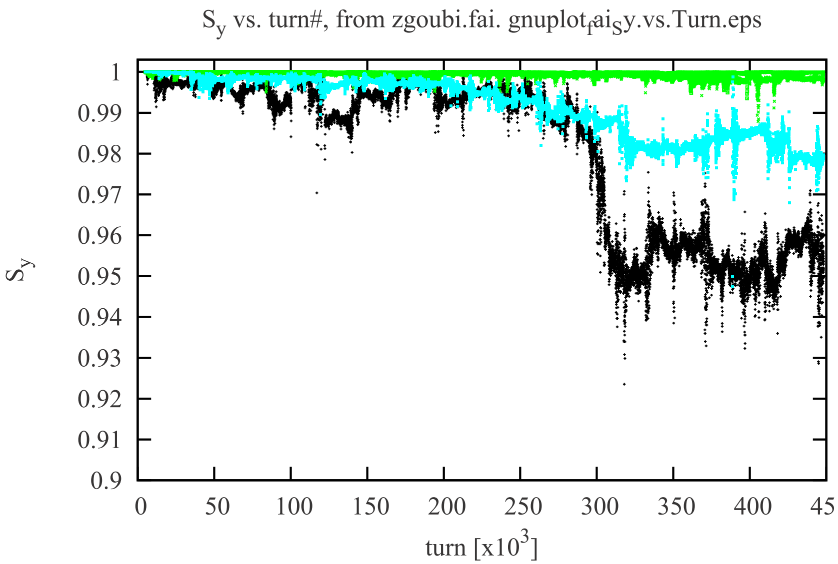


Stochastic energy excursion over time interval $t/\tau_{SR} : 1 \to 10^3$. $<\ \delta p/p\ > = 1.11\,10^{-3}$ and $\sigma_{\delta p/p} = 1.14\,10^{-3}$.



Blue: projection of the single particle motion of the left plot;
$< \delta p/p > = 1.11\,10^{-3}$
$\sigma_{\delta p/p} = 1.14\,10^{-3}$
$< \phi > = 0.519$
$\sigma_\phi = 0.091$;
Red: a 10^3 particle bunch observed at $t = 10^3 \tau_{SR}$;
$< \delta p/p > = 1.07\,10^{-3}$
$\sigma_{\delta p/p} = 1.13\,10^{-3}$
$< \phi > = 0.519$
$\sigma_\phi = 0.091$.

Spin motion



• Stochastic spin motion, single particle, observed at IP8.

• Motion is not at equilibrium. However:

◇ only ring settings that feature very slow polarization decay are of interest

- if the electron motion neighbors depolarizing resonances, these will be revealed by fast decay as $\tau_{\mathrm{D}} \sim (\mathrm{a}\gamma_{\mathrm{Res.}} - \mathrm{a}\gamma)^2 \tau_{\mathrm{ST}}$, $\mathrm{P}_{\mathrm{eq}} \sim (\mathrm{a}\gamma_{\mathrm{Res.}} - \mathrm{a}\gamma)\mathrm{P}_{\mathrm{ST}}$, meaning a ring configuration which is not viable

• In a short time interval, an electron will have explored the all 6D phase-space

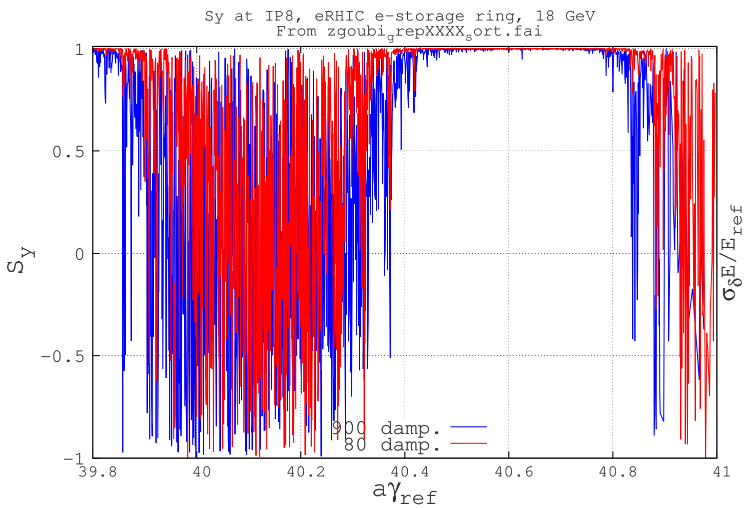
• In a similar way that τ_{SR} can be obtained from the observation of the damped motion of a single electron far from equilibrium, τ_{D} can be obtained from long enough observation of spin motion out of equilibrium.

• Track a single particle per bin

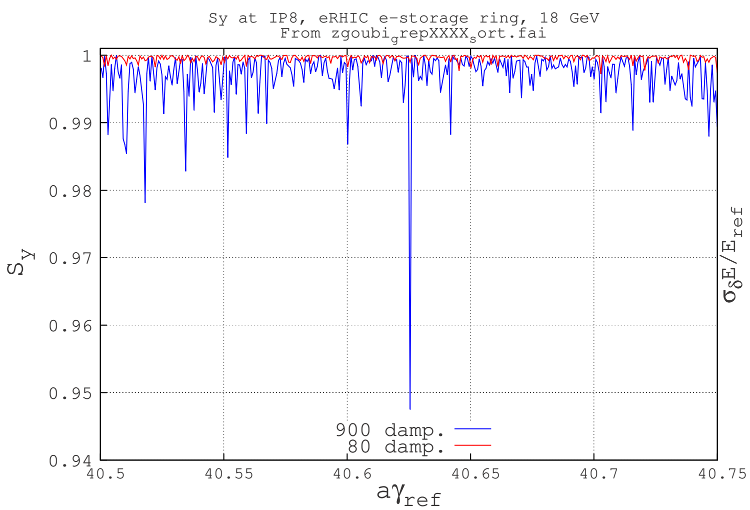
◇ 1 bin is 1 ring, set for a specific rigidity (here, "$a\gamma_{ref}$")

◇ about 1000 rings (bins) here

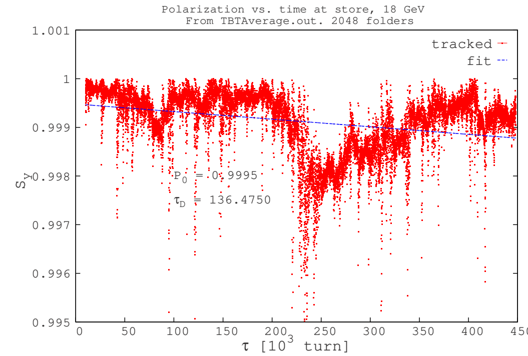
Spins at 80 and 900$\times\tau_{SR}$

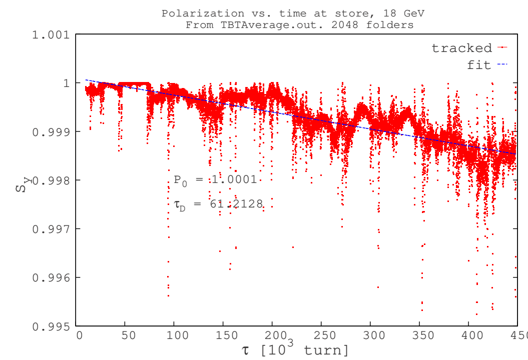


Zoom-in:

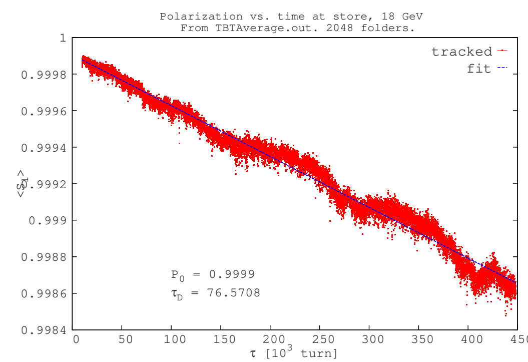


• Monitor individual spins: A linear regression on $P/P_0 = \exp(-t/\tau_D) \approx 1 - t/\tau_D$ provides τ_D.





• Possibly, average over reduced $a\Delta\gamma_{ref}$ interval, *i.e.*, a few rings/bins ($\Delta\gamma$: 40.60 – 40.62, here):

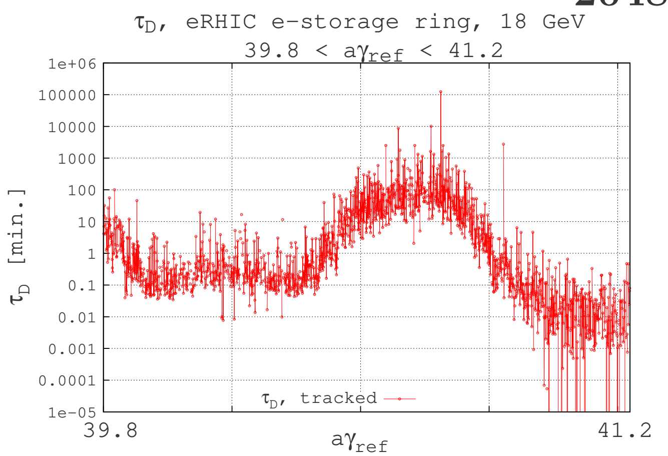


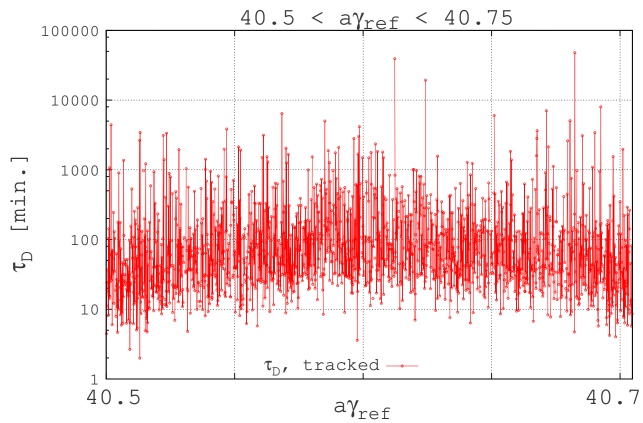
A metric

- This is in order to compare polarization performance, when optimizing the spin rotator, injecting errors and their compensation, etc.

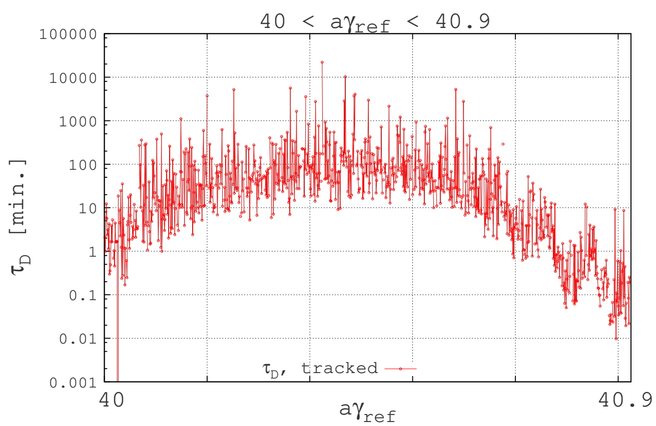
- A possibility: distance between τ_D distributions (or $P(t)$ distributions at given t)

2048 bins





1024 bins



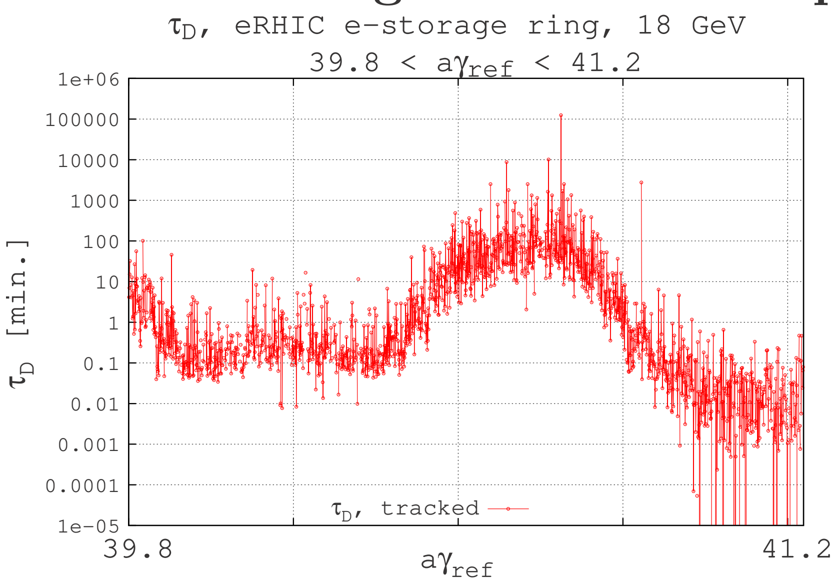
- Averaging over a small a$\delta\gamma_{\text{ref}}$ interval (a few particles / bins) greatly smooths the fluctuations.

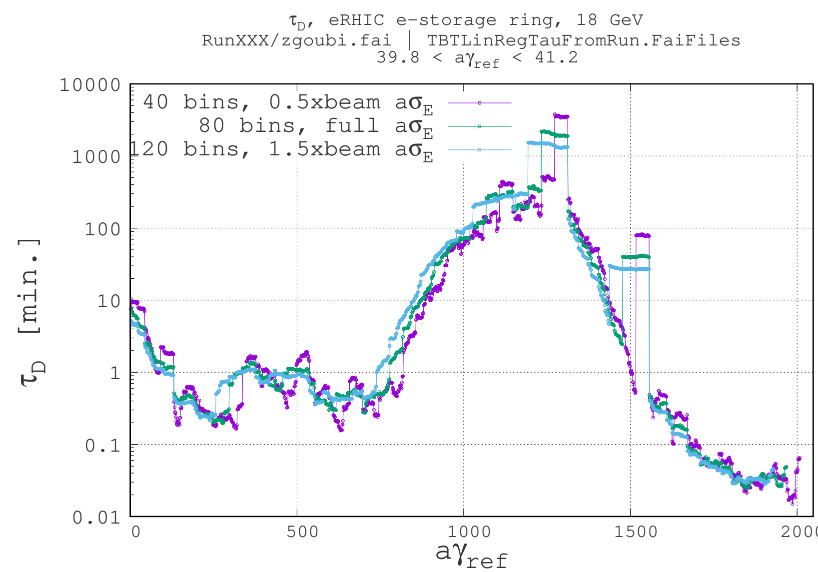
- Justification for a sliding average:

(i) with a$\Delta\gamma_{\text{ref}} = 1$ being covered in 1024 aγ_{ref} bins (or 2048),
a beam, which has $\sigma_\gamma/\gamma_{\text{ref}} \approx 10^{-3}$ (or a$\delta\gamma_{\text{ref}} = 0.04$),
extends over about 40 (or 80) bins,

(ii) so, a set of a few bins almost belong in the same ring,
thus averaging over a few bins is not very different from averaging over a few particles in the same bin

- In this sliding sampling, the distribution appears to evolve only weakly with increasing number of samples, N.





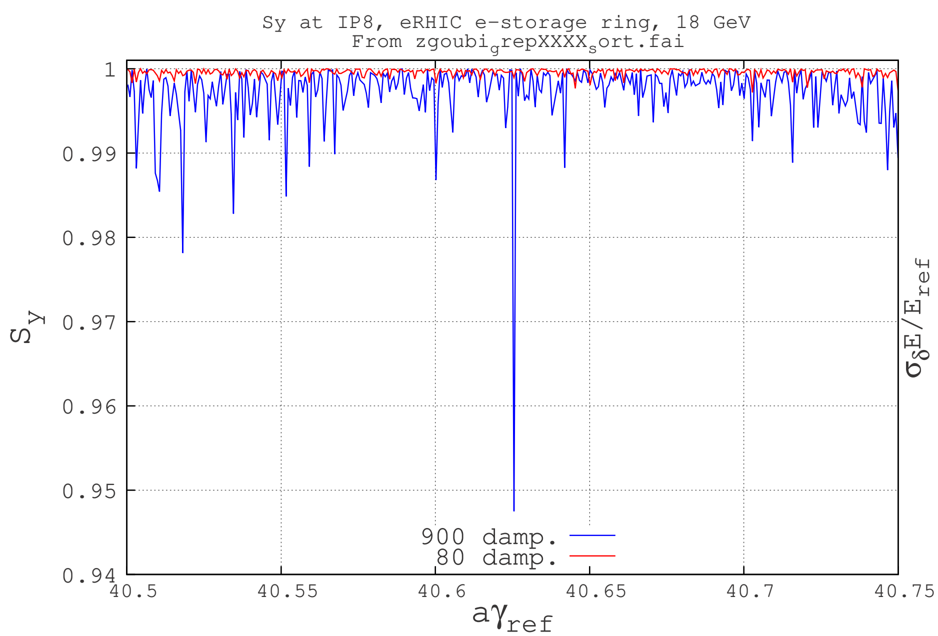
A metric (cont'd)

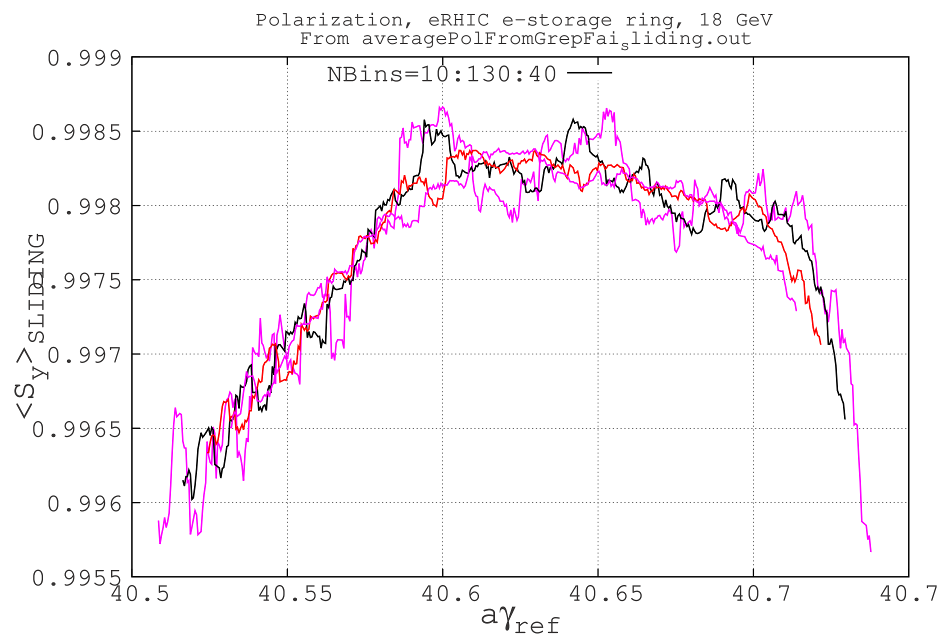
- Another possibility for comparing optics:

◇ A sliding average (right plot below) is applied on single particle spin values at a given time out of the multiturn tracking (left plot)

◇ The four curves below differ by the number of bins of the sliding sampling: N= 10, 50, 90 and 130 bins, respectively.

◇ Again, in this sliding sampling, the distribution appears to evolve only weakly with increasing number of samples, N.





COMMENTS

- **Assume similar resolution using both methods,**
  - "HPC-Hungry" and
  - "Ergodic",
  namely, the same number of reference rings, nRings (= number of bins), in a given interval $a\Delta\gamma_{ref}$.
  In the present hypotheses (eRHIC lattice, energy, etc.):
  - first method: the HPC volume is
  nRings$\times10^3$ [particles/ring]$\times$a few 10s of SR damping times
  - second method: the HPC volume is
  nRings$\times$1 to 10 times more SR damping times.
  This is a two to three orders of magnitude difference.

- **Larger HPC volume translates in one or the other of,**
  - longer queues, longer computing time, more processors, greater volume of I/Os, larger data analysis HPC volume,...

- Faster computation allows easier exploration of parameter space in design optimizations.

- **It remains to determine how close the single-particle method can get to the accuracy of the 1000-particle bunch method (an on- going work).**

CONCLUSION

- **The single-particle method seems an efficient first approach for qualifying an evolution of a lattice (optics variants, effects of errors, correction schemes, etc.).**
- **Plans: use it and improve it at eRHIC!**

THANK YOU FOR YOUR ATTENTION

BIBLIOGRAPHY

- BNL eRHIC collaboration and documents
- eRHIC p-CDR, BNL 205809-2018-FORE (2018)