TMCI
& Higher-Harmonic RF Cavities
in electron-storage rings

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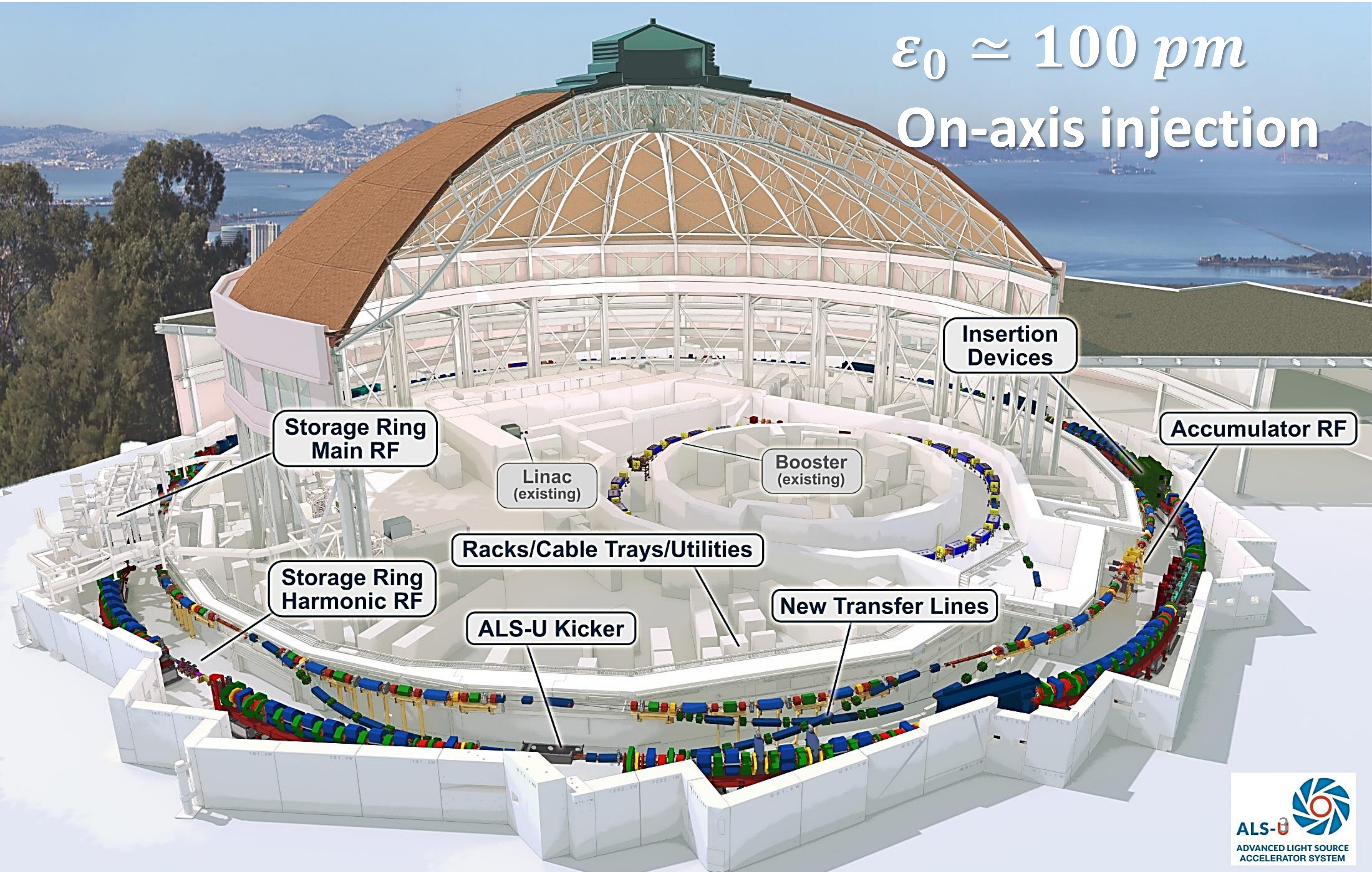
Goal: Revisit TMCI theory

- Conventional mode-analysis for TMCI applies to <u>linear</u> single-particle longitudinal motion

- HHCs ➔ motion in RF bucket is (highly) <u>non-linear</u>

- How should the theory be modified?

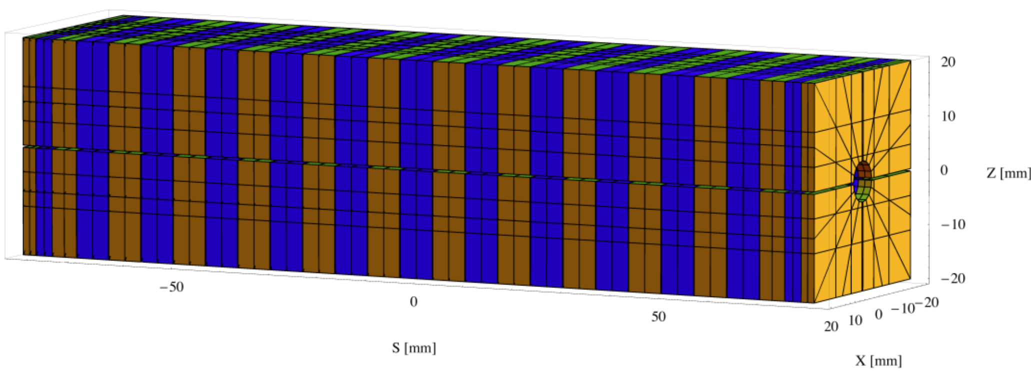
Motivation: ALS-U, the LBNL 4^{th} generation light source



The new machine allows for narrow-aperture undulators

Under consideration:

Delta-undulator



- *Cu* vacuum chambers with NEG coating
- Aside: characterization of RW properties of NEG is an important topic in its own right

6mm ID

Vacuum chamber samples for NEG coating R&D

Resistive Wall as a major, if not the dominant, source of transverse impedance

Adopt familiar monolayer,
infinite thickness, round pipe,
DC conductivity RW impedance model:

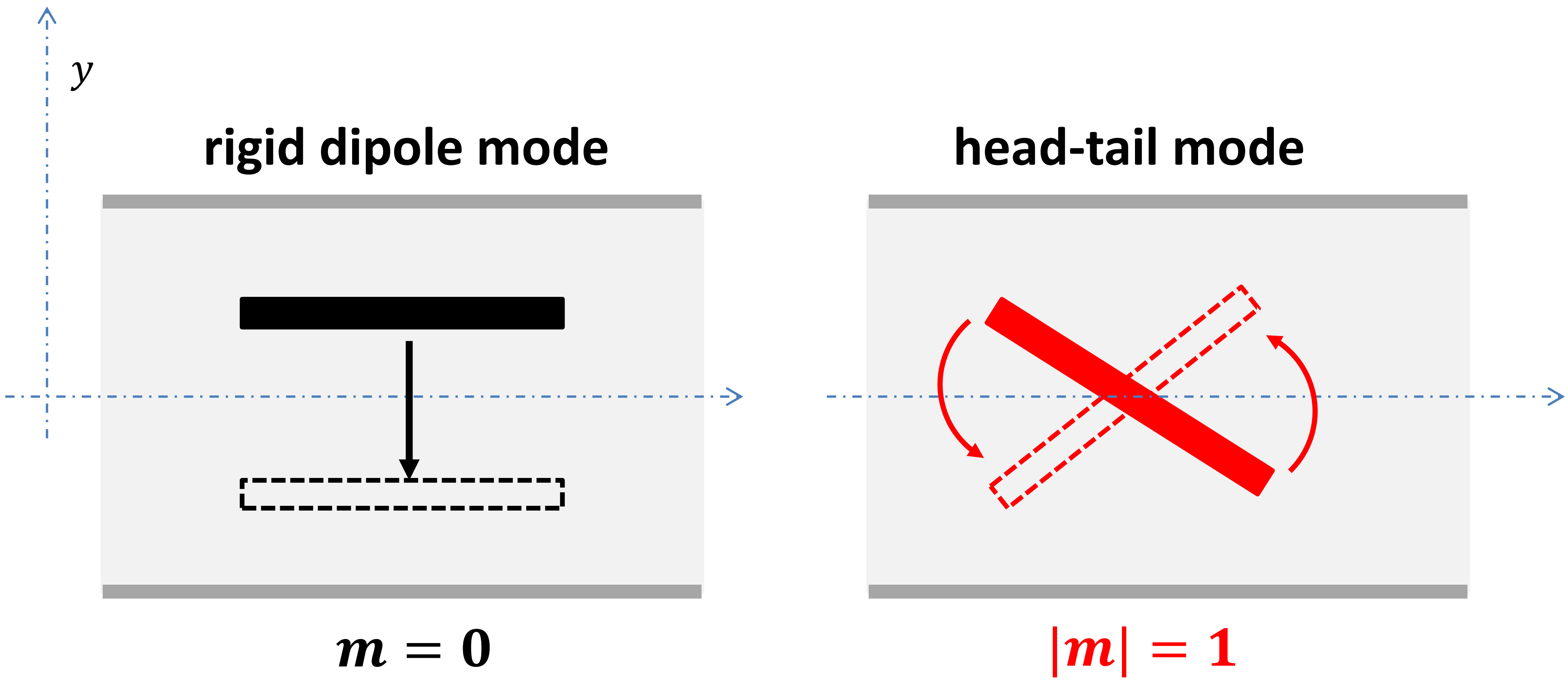
Strong dependence on chamber radius

$$Z_y(k) = \frac{\mathrm{sign}(k) - i}{\sqrt{|k|}} \frac{L}{b^3} \sqrt{\frac{2}{\pi c \sigma_c}},$$

Focus on:

- RW as the sole source of impedance
- Single bunch
- Vanishing chromaticities

The `classical' Transverse-Mode Coupling Instability (TMCI)



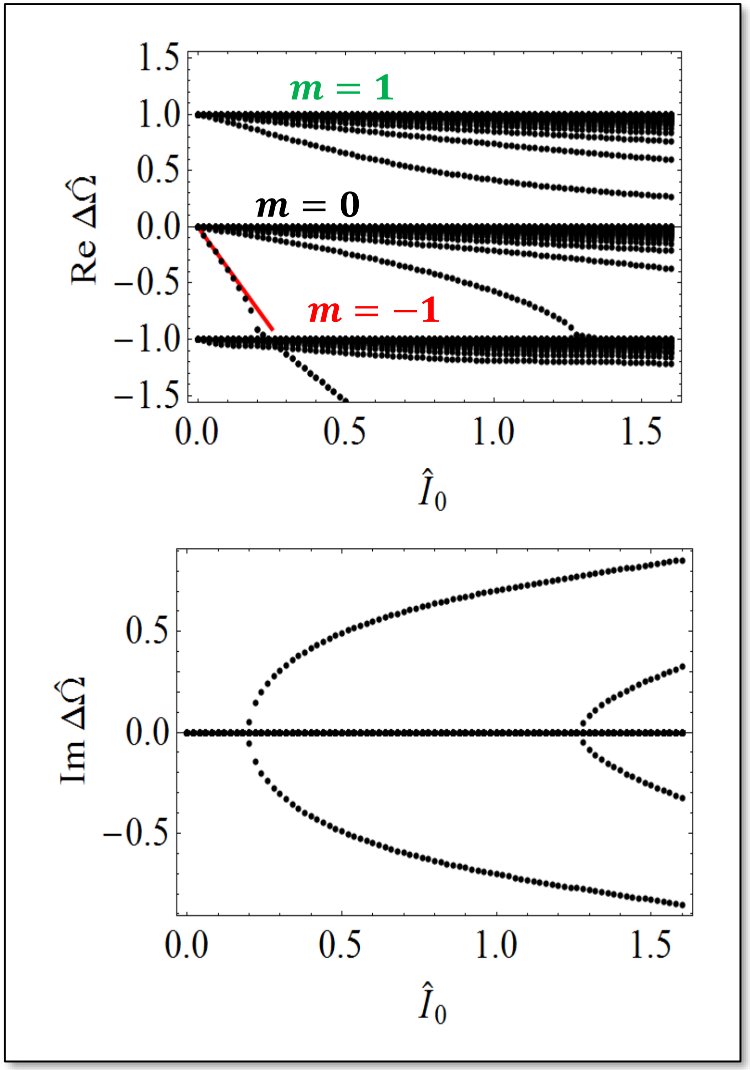
The instability happens

when the oscillation frequencies of the two modes merge

Mode-analysis accurately predicts the onset of the instability

Complex-number frequency shift:

$$\Delta\widehat{\Omega} = \frac{\Omega - \omega_y}{\omega_{s0}}$$



Dimensionless current parameter

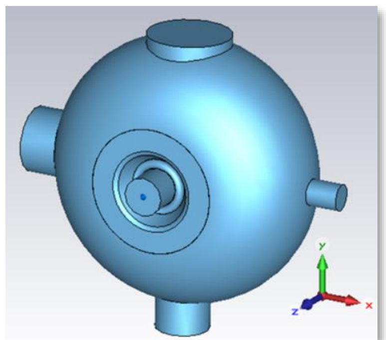
$$\hat{I}_0 = \frac{N r_c c}{(2\pi)^{3/2} \gamma \nu_{s0} b^3 \sqrt{c\sigma_c \sigma_{z0}}} \frac{\beta_y L_u}{2\pi}$$

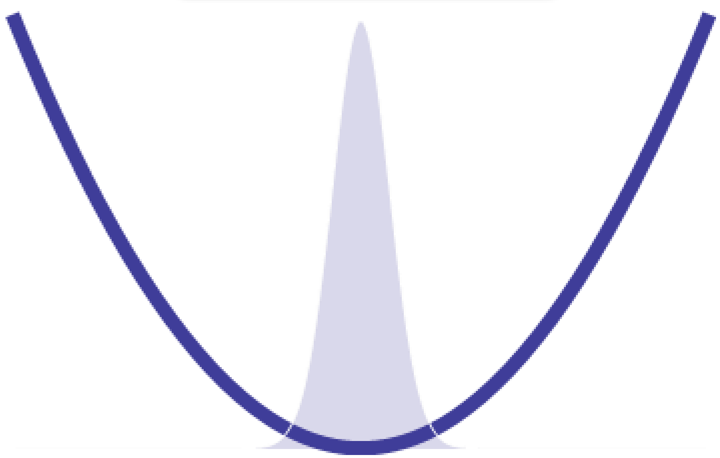
synchr. tune

Critical current: $\hat{I}_0 \simeq 0.197$

Enter the harmonic (aka Landau) RF cavities
(intended for bunch lengthening)

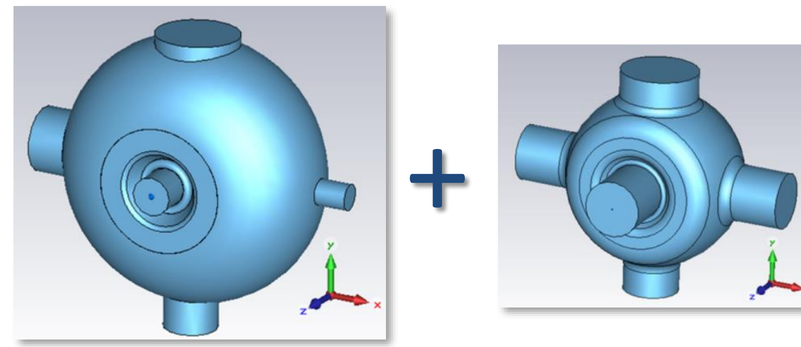
Main RF cavity only

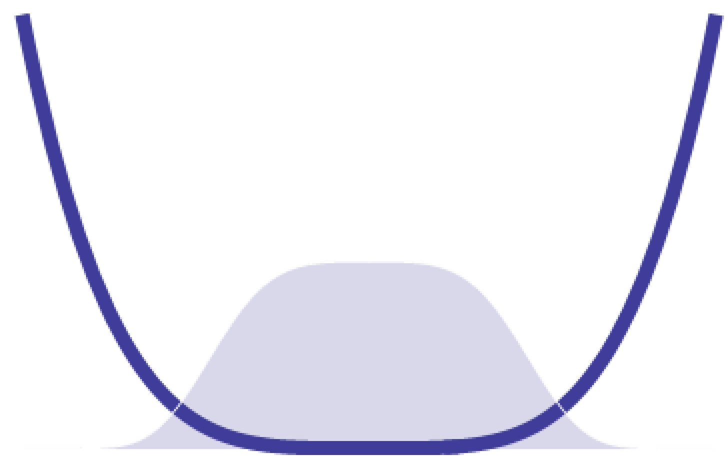




- Quadratic RF potential
- Linear long. motion
- Synch. oscill. freq ω_{s0}

Main + 3rd Harmonic Cavity



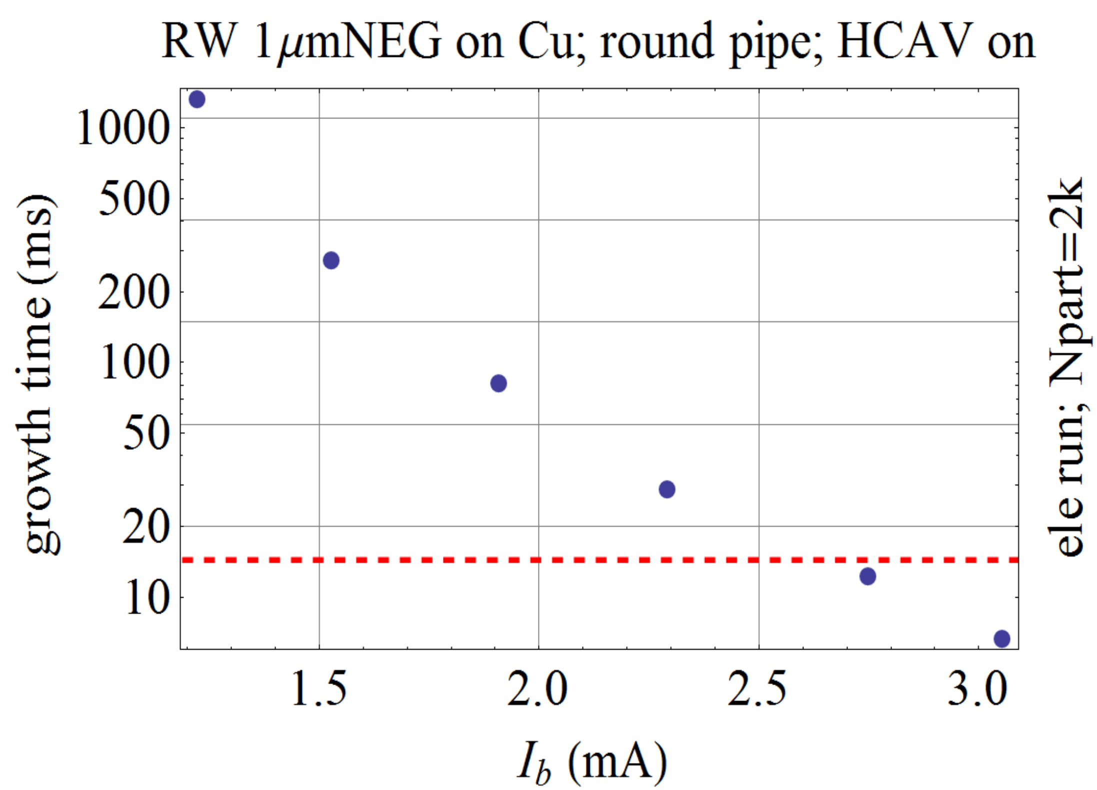


- Quartic RF potential
- Non-linear long. motion
- $\omega_s(r) \propto r$

Landau not doing his job … (?)

Macroparticle simulations:

- beam always unstable (w/o radiation damping)
- growth rate higher compared to case w/o HHCs (everything else being equal)



Elegant simulations

Limited (and not all consistent) literature

- **Y. Chin, *et al*.: (**Part. Accel. 1985)
  - Mode analysis. BB Resonator Z model. Zero chromaticities
  - HHC non-linearity treated by perturbation theory
  - ➢ In fully nonlinear regime motion always unstable (conjecture)

- **S. Krinksy:** (Tech Note 2005)
  - Macroparticle simulations. RW Z. Zero chromaticities
  - ➢ Current threshold w/ HHCs is lower but there is a threshold

- **Cullinan, *et al*.: (**PRAB 2016)
  - Macro-particle simulations. RW Z. Finite chromaticities. Multi-bunches
  - ➢ HHCs stabilize motion

Toward a mode-analysis theory with HHCs

1. Avoid orthogonal polynomial basis to represent radial components of modes
   - Use step-wise representation on a grid

2. Be careful about the singular nature of the secular equation

Taking care of the singularity (remember Landau)

regular integral equation

w/o HHCs

- Eigen-functions are regular functions. Finite-dim approximation OK

$$(\Delta\hat{\Omega} - m)R_m(\rho) + i\hat{I}_0 e^{-\rho^2/2} \sum_{m'=-\infty}^{\infty} \int_0^{\infty} R_{m'}(\rho')\mathcal{G}_{m,m'}(\rho,\rho')\rho' d\rho' = 0$$

w/ HHCs

singular integral equation

- Eigen-functions may be generalized (distribution) functions. Finite-dim approx. not OK

$$(\Delta\hat{\Omega} - m\rho)R_m(\rho) + i\hat{I} e^{-h_1\rho^4} \sum_{m'=-\infty}^{\infty} \int_0^{\infty} R_{m'}(\rho')\mathcal{G}_{m,m'}(\rho,\rho')\rho'^2 d\rho' = 0,$$

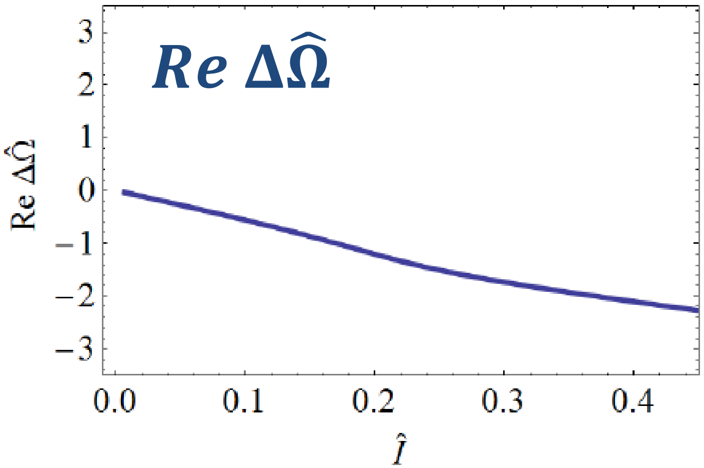
Change of unknown: $R_m(\rho) \to S_m(\rho) = (\Delta\hat{\Omega} - m\rho)R_m(\rho)e^{h_1\rho^4}$

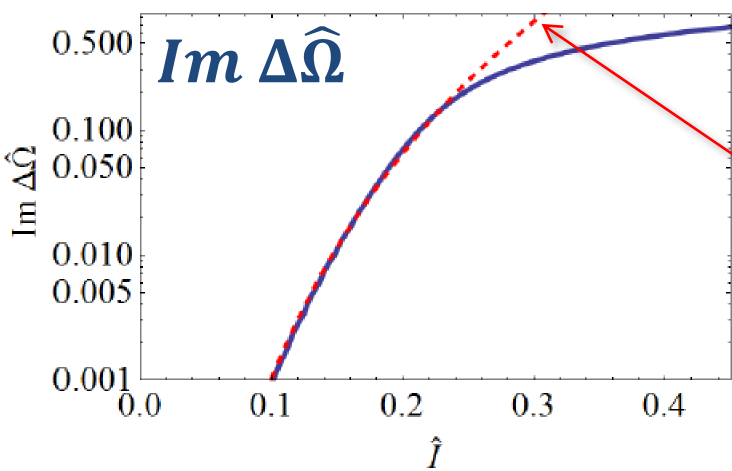
Discretize this!

$$S_m(\rho) + i\hat{I} \sum_{m'=-\infty}^{\infty} \int_0^{\infty} \frac{S_{m'}(\rho')e^{-h_1\rho'^4}}{\Delta\hat{\Omega} - m'\rho'} \mathcal{G}_{m,m'}(\rho,\rho')\rho'^2 d\rho' = 0.$$

$\mathrm{Im}\,\Delta\hat{\Omega} > 0$

Numerical solution of regularized Eq. suggests $\sim I_b^6$ scaling of instability threshold





$$\hat{I} = \frac{N r_c c}{\pi^{5/2}\gamma\langle\nu_s\rangle b^3\sqrt{c\sigma_c\sigma_z}}\frac{\beta_y L_u}{2\pi}$$

avg. synchr. tune over bunch

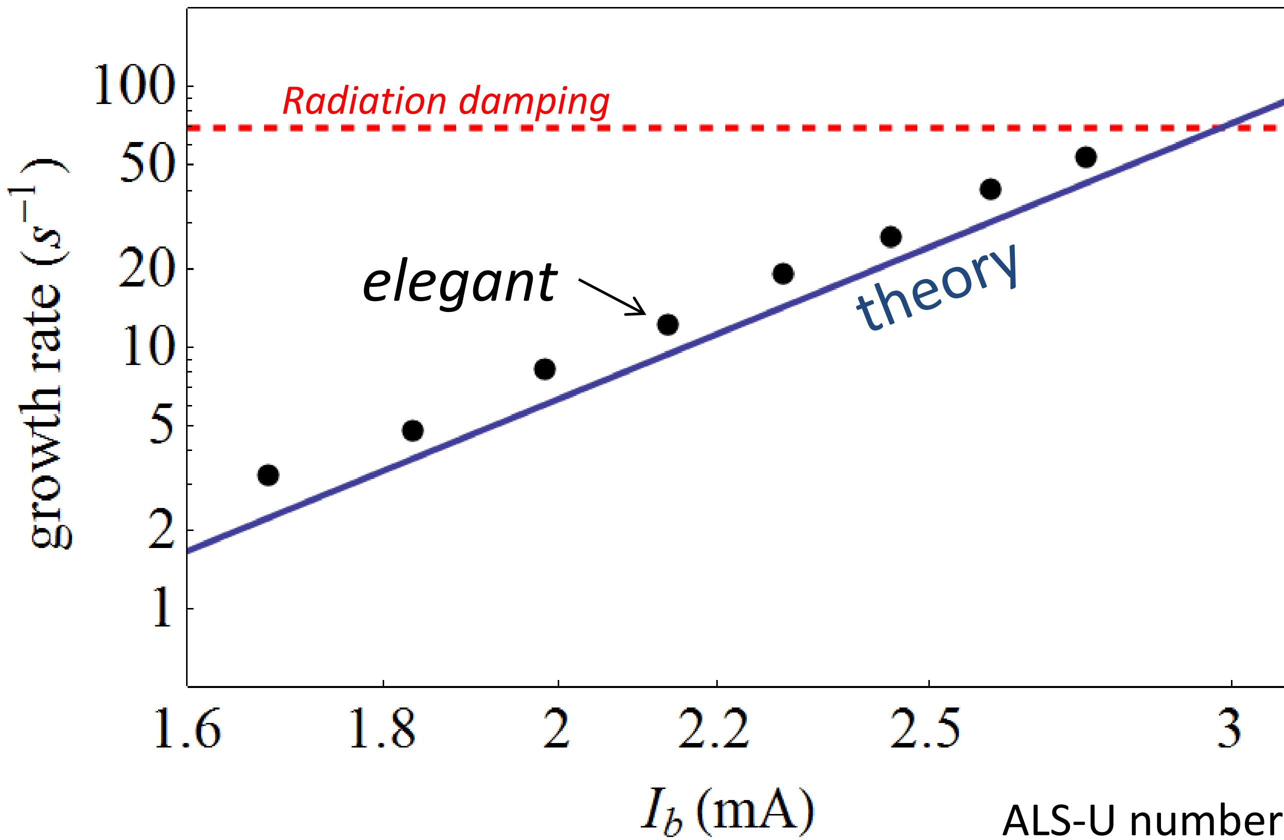
At any current there always exists a root with $Im\ \Delta\hat{\Omega} > 0$

➔ Motion always unstable

$$\boxed{\mathrm{Im}\ \Delta\hat{\Omega} = (2^{5/3}\hat{I})^6}$$

Conjecture based on numerical result. Uniqueness?

Reality check: macroparticle simulations confirm I_b^6 scaling



log-log scale

ALS-U numbers:
all straight sections
w/ 4m, $b = 3mm$,
ID Cu vacuum chamber

The take-home practical formula: stability with *vs.* without HHCs

Radiation damping accounted for

Threshold w/ HHCs   Threshold w/o HHCs

$$N_c \simeq 1.15 \times N_{c0} \left(\frac{T_0}{\tau_y \nu_{s0}}\right)^{1/6} \left(\frac{\sigma_{z0}}{\sigma_z}\right)^{1/3}$$

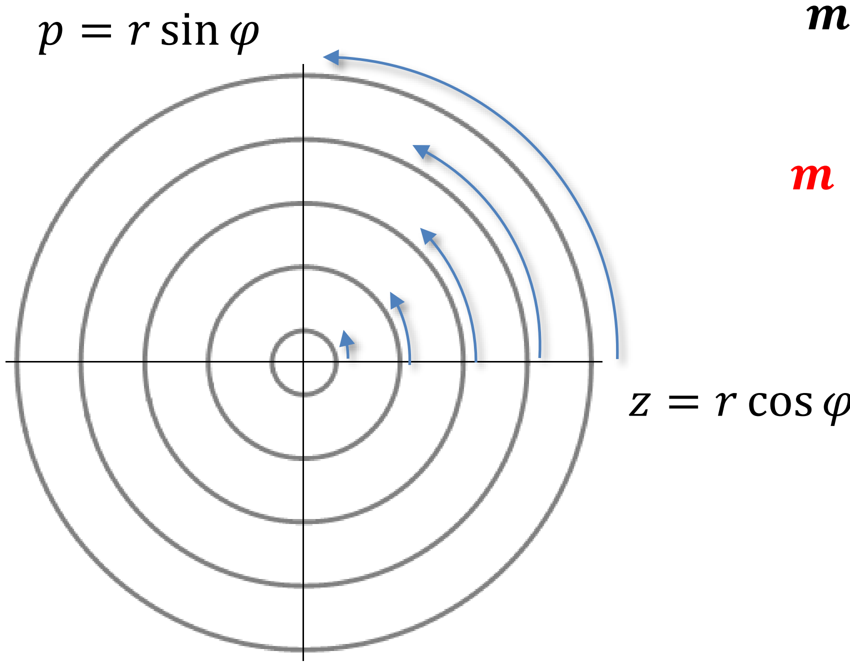
~ 0.52 $\sim \left(\frac{1}{4}\right)^{1/3} = 0.62$ ALS-U numbers

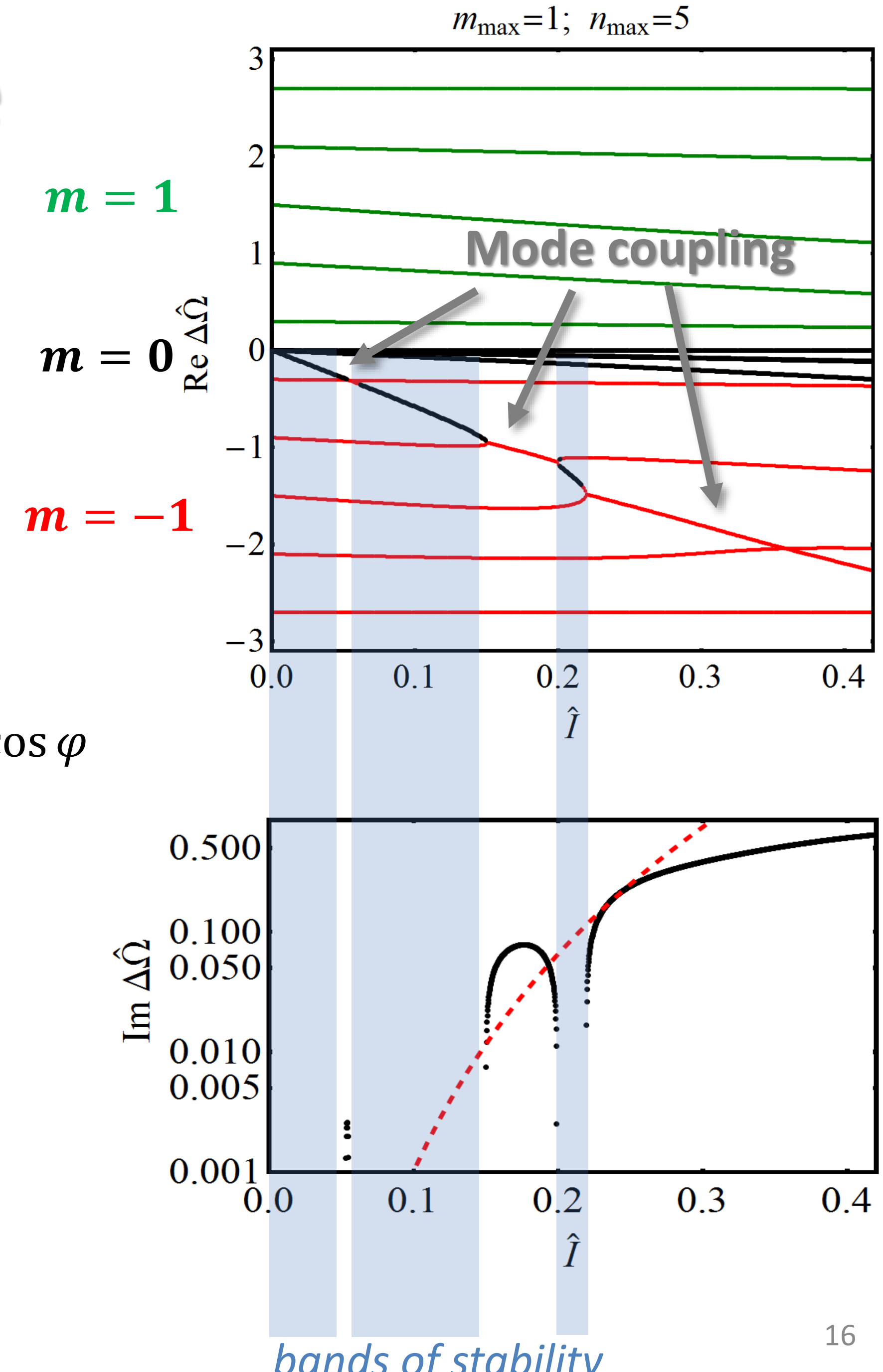
$N_c \simeq 0.4 \times N_{c0}$ HHCs cut instability threshold by half ☹

… but we should still be OK ☺

The intuitive picture

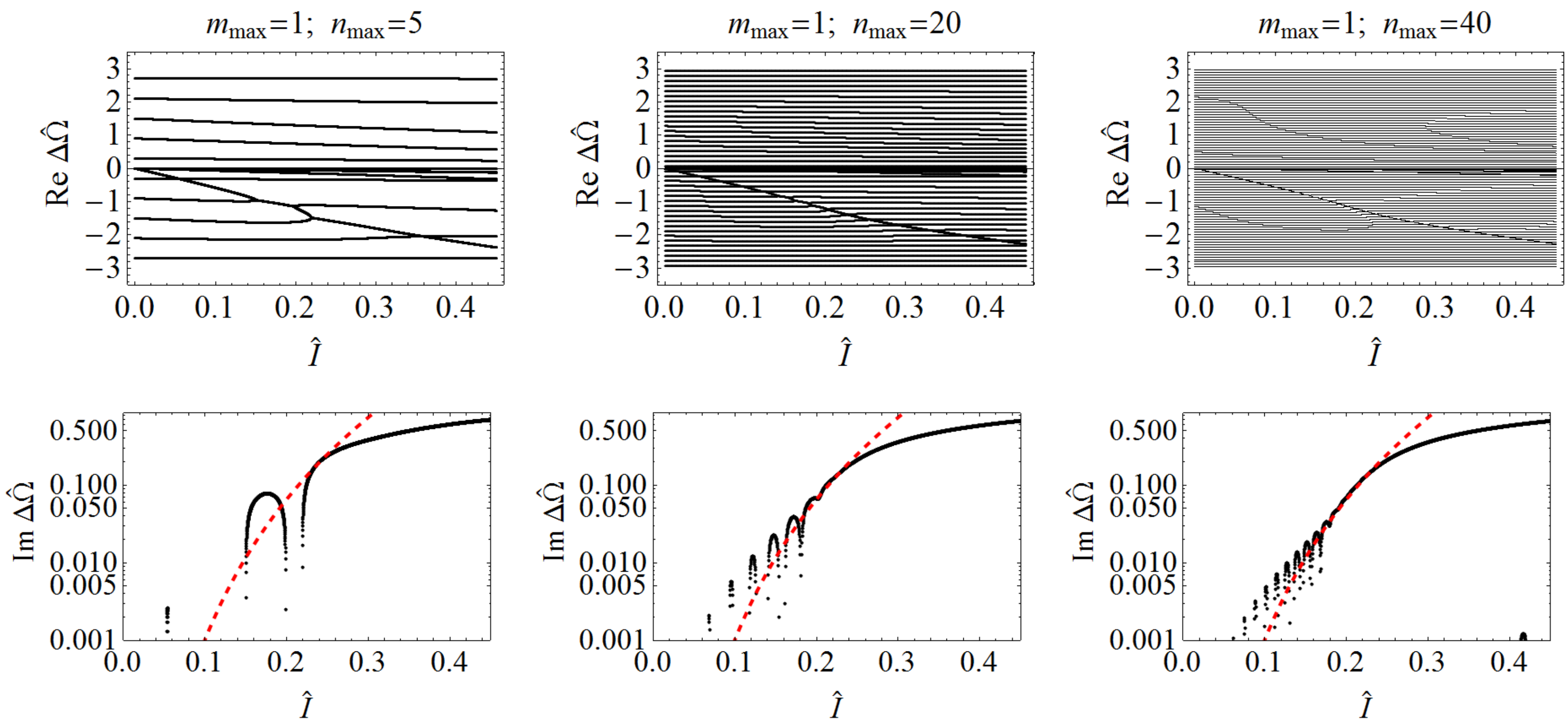
The beam as a set of nested shells in longitudinal phase space





The rigid-dipole mode **(m=0)** couples with head-tail mode **(m=-1)** at arbitrarily low current

Increasing no. of shells does not extrapolate well to the continuum limit …



…confirming the value of the regularizing transformation

Summary

- **New light-source storage rings ➔ narrow chambers**
  - Large RW transverse impedance

- **Higher-Harmonics Cavities (HHCs) are a fixture**
  - Bunch lengthening to reduce scattering effects

- **Developed theory for effect of HHCs on TMCI**
  - Regularize integral secular equation for accurate numerical work

- **Vanishing chromaticity + RW-dominated impedance ➔ HHCs degrade transverse beam stability**
  - But finite chromaticity will help (not discussed in this talk)