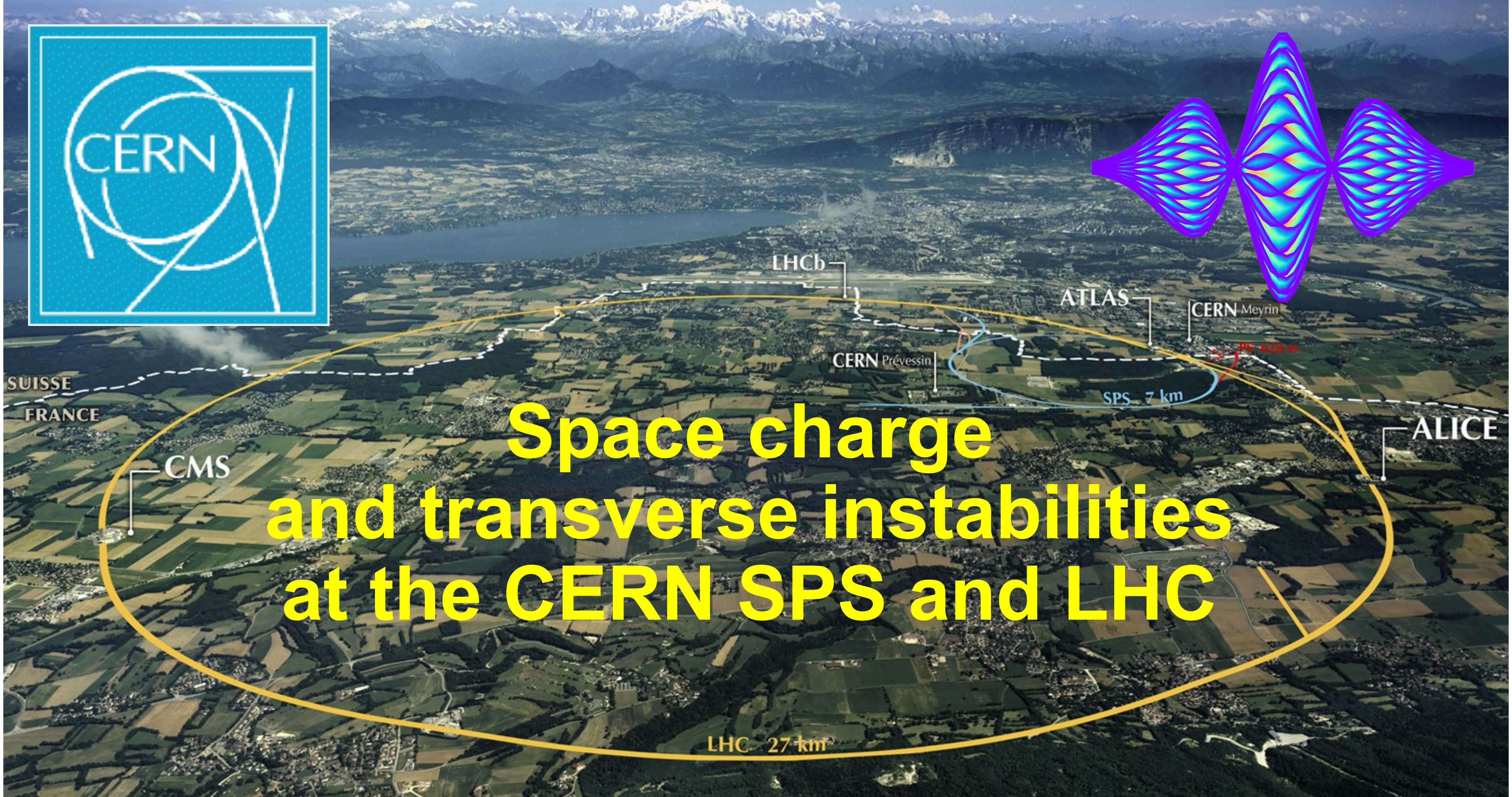
Work in collaboration with D. Amorim, G. Arduini, H. Bartosik, H. Burkhardt, E. Benedetto, K. Li, A. Oeftiger, D. Quatraro, G. Rumolo, B. Salvant, C. Zannini (CERN: now or <) and A. Burov (FNAL)



Space charge and transverse instabilities at the CERN SPS and LHC

Elias Métral

BE/ABP-HSC (Collective/Coherent Effects)

https://espace.cern.ch/be-dep-workspace/abp/HSC/SitePages/Home.aspx

Elias.Metral@cern.ch

Tel.: 00 41 75 411 4809

http://emetral.web.cern.ch/emetral/

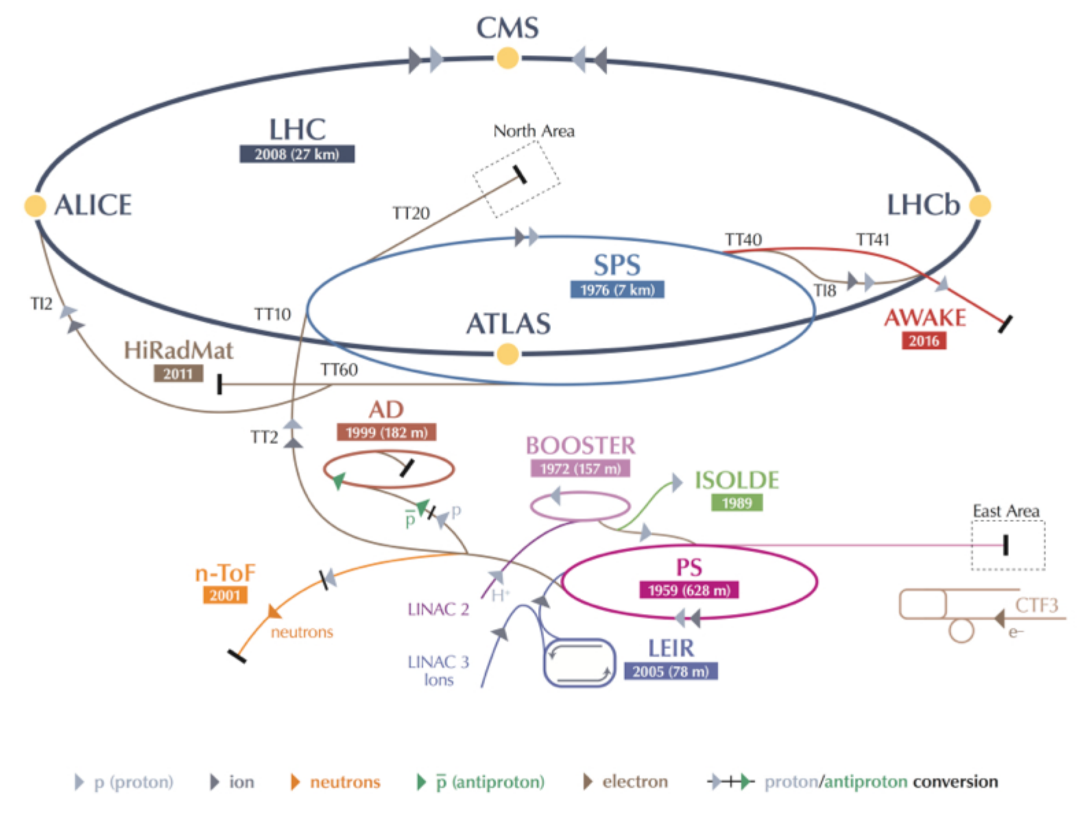
Abstract

- At the CERN accelerator complex, it seems that only the highest energy machine in the sequence, the LHC, with space charge (SC) parameter close to one, sees the predicted beneficial effect of SC on transverse coherent instabilities

- In the other circular machines of the LHC injector chain (PSB, PS and SPS), where the SC parameter is much bigger than one, SC does not seem to play a major (stabilising) role… maybe the opposite in the SPS…

- All the measurements and simulations performed so far in both the SPS and LHC will be reviewed and analyzed in detail

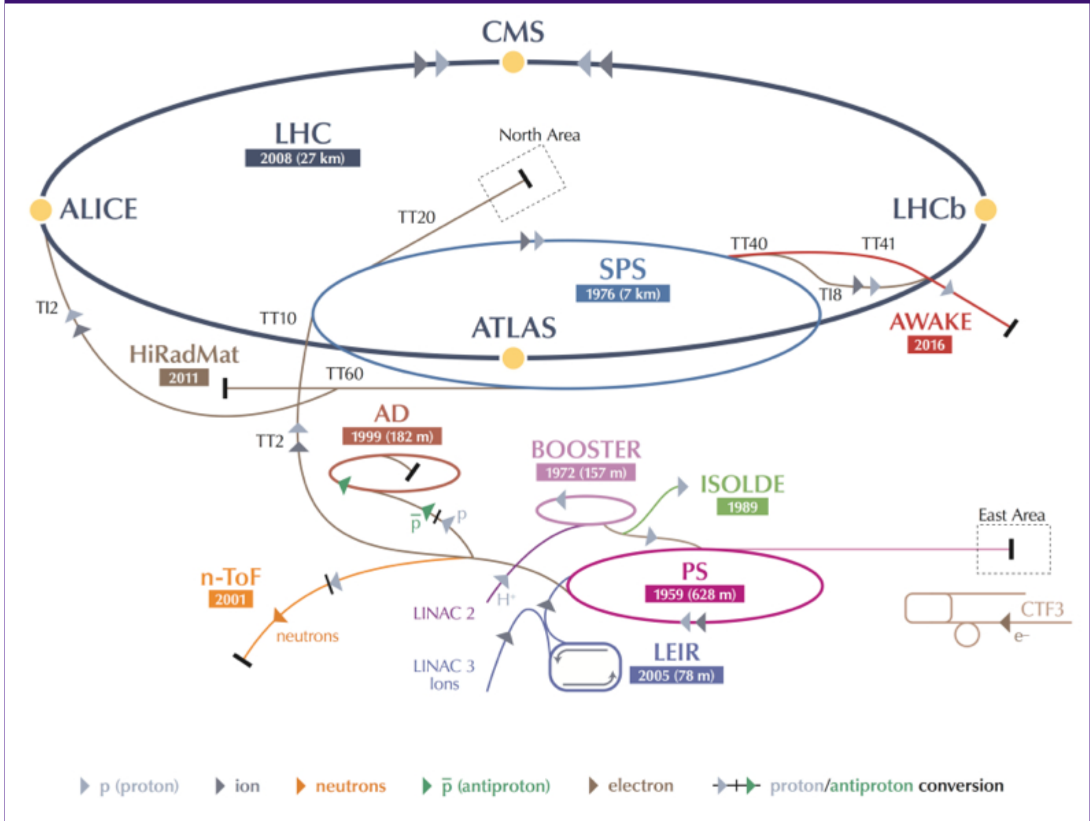
INTRODUCTION



	Operation with p^+
SPS	**25 GeV => 450 GeV**
LHC	**450 GeV => 7 (6.5) TeV**

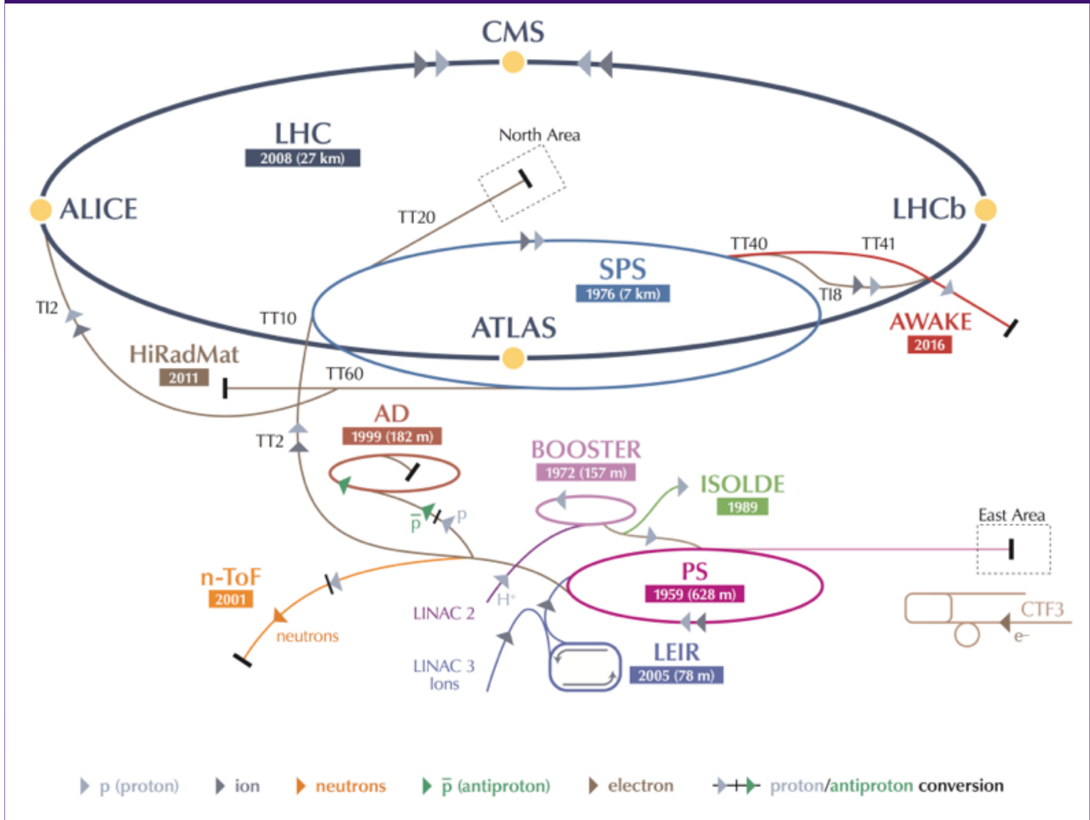
”Long-bunch” regime

- SPS ($\Delta Q_{sc} / Q_s >> 1$)
  - Observation of a fast vertical single-bunch instability (with a travelling-wave pattern along the bunch) at injection, above a certain threshold (depending on slip factor)
  - Several features are close to the ones from the predicted TMCI between modes - 2 and - 3 without SC (Q’ ~ 0)



	Operation with p^+
SPS	25 GeV => 450 GeV
LHC	450 GeV => 7 (6.5) TeV

"Long-bunch" regime

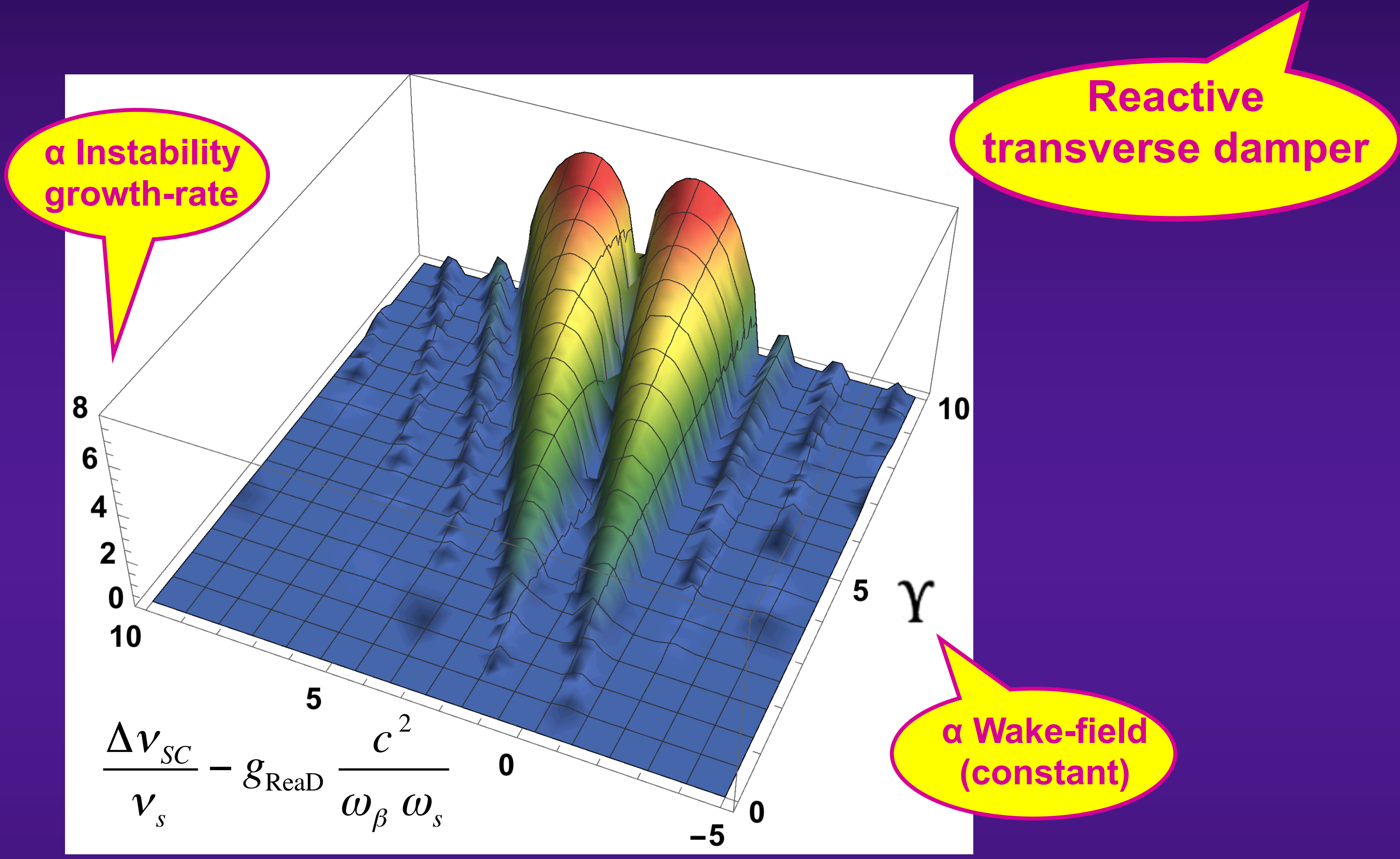


	Operation with p^+
SPS	25 GeV => 450 GeV
LHC	450 GeV => 7 (6.5) TeV

"Short-bunch" regime

- SPS (ΔQ_{sc} / Q_s >> 1)
  - Observation of a fast vertical single-bunch instability (with a travelling-wave pattern along the bunch) at injection, above a certain threshold (depending on slip factor)
  - Several features are close to the ones from the predicted TMCI between modes - 2 and - 3 without SC (Q' ~ 0)
- LHC (ΔQ_{sc} / Q_s ~ 1)
  - Predicted threshold for TMCI (modes - 1 and 0) at injection (Q' ~ 0) increased by SC
  - Head-Tail instability with 1 node (Q' ~ 5) => Stabilized by SC below a certain energy

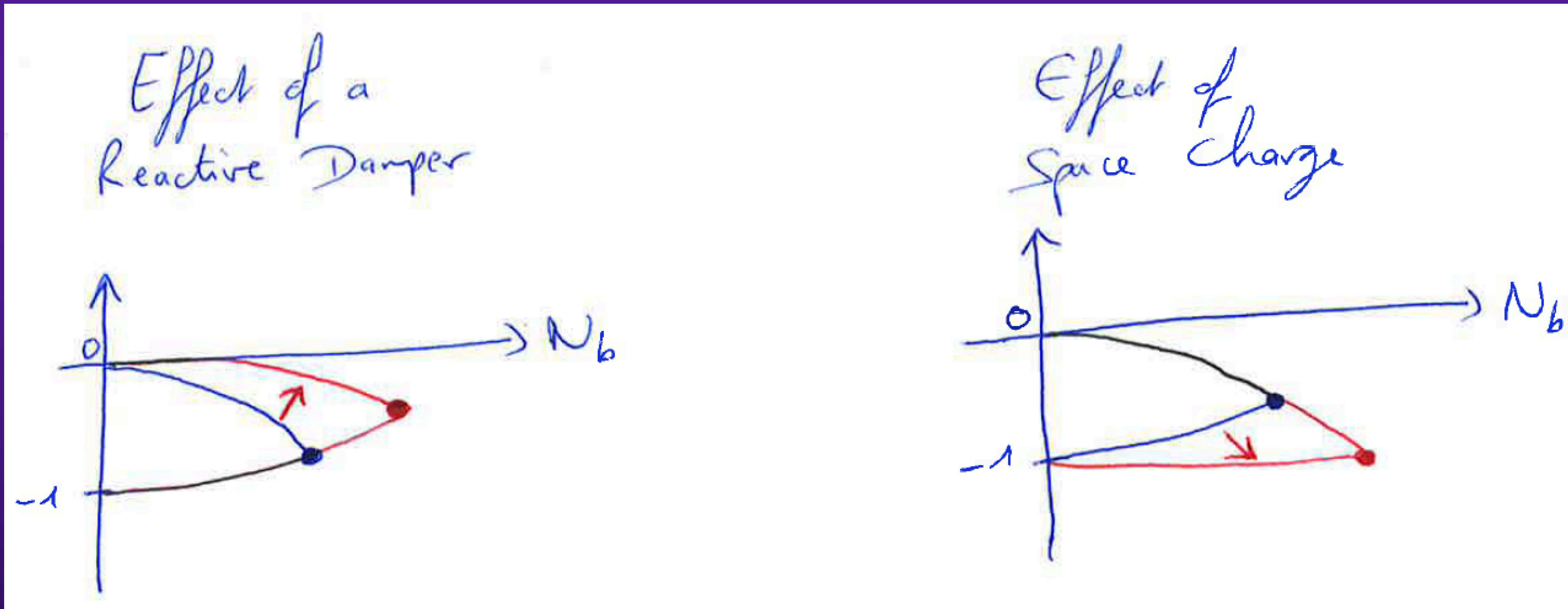
2 – PARTICLE MODEL FOR TMCI (Q' = 0) WITH SC AND/OR ReaD



- Results from Burov_2016 (using a ReaD only) and Chao-Chin-Blaskiewicz_2016 (using SC only) have been recovered and combined
- Both SC and ReaD affect TMCI in a similar way and can suppress it

2 – MODE MODEL FOR TMCI (Q' = 0) WITH SC AND/OR ReaD

- "Short-bunch" regime (TMCI between 0 and - 1) => LHC case
  - Both ReaD & SC are expected to be beneficial (as 2-part. model)
    - ReaD => Shifts mode 0 up
    - SC => Shifts mode – 1 down



2 – MODE MODEL FOR TMCI (Q' = 0) WITH SC AND/OR ReaD

- "Long-bunch" regime (TMCI of high-order modes) => SPS case
  - Situation is more involved due to higher-order mode-coupling
    - ReaD => Modifies only mode 0 and not the others (where the main mode-coupling occurs) => ReaD is expected to have no effect for main coupling
    - SC => Modifies all the modes (except 0) => ?: main subject of this presentation…

Still under discussion

CONTENTS

- **SPS**
  - 1) 1st observations with p^+ in 2003 and simulation studies
  - 2) 2nd simulation studies
  - 3) New measurement campaign
  - 4) Change of optics (Q26 => Q20) and new measurement and simulation studies
  - 5) Currently: closer look to Q26 with new results from
    - Theory by A. Burov => New detrimental effect of SC (see "Convective Instabilities of Bunched Beams with SC": https://arxiv.org/pdf/1807.04887.pdf)
    - Simulation with SC by A. Oeftiger
    - (Simple) 2-mode approach
  - 6) (Near) future: new measurement campaign planned

Linked to the interger part of the tune

- **LHC**
  - 1) Simulation studies of the TMCI (Q' = 0) at injection
  - 2) Measurement and simulation studies with Q' ~ 5

- **Conclusions**

- **Appendix**

SPS

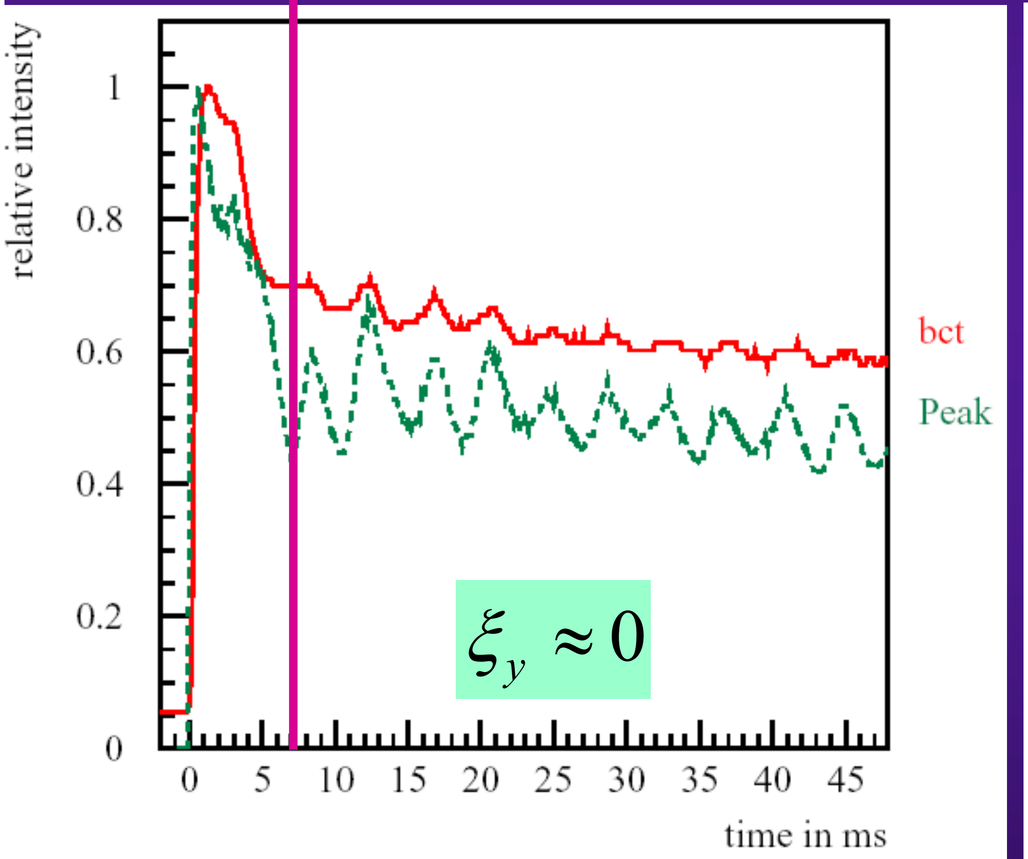
Fast vertical single-bunch instability with p+ at the SPS injection in 2003

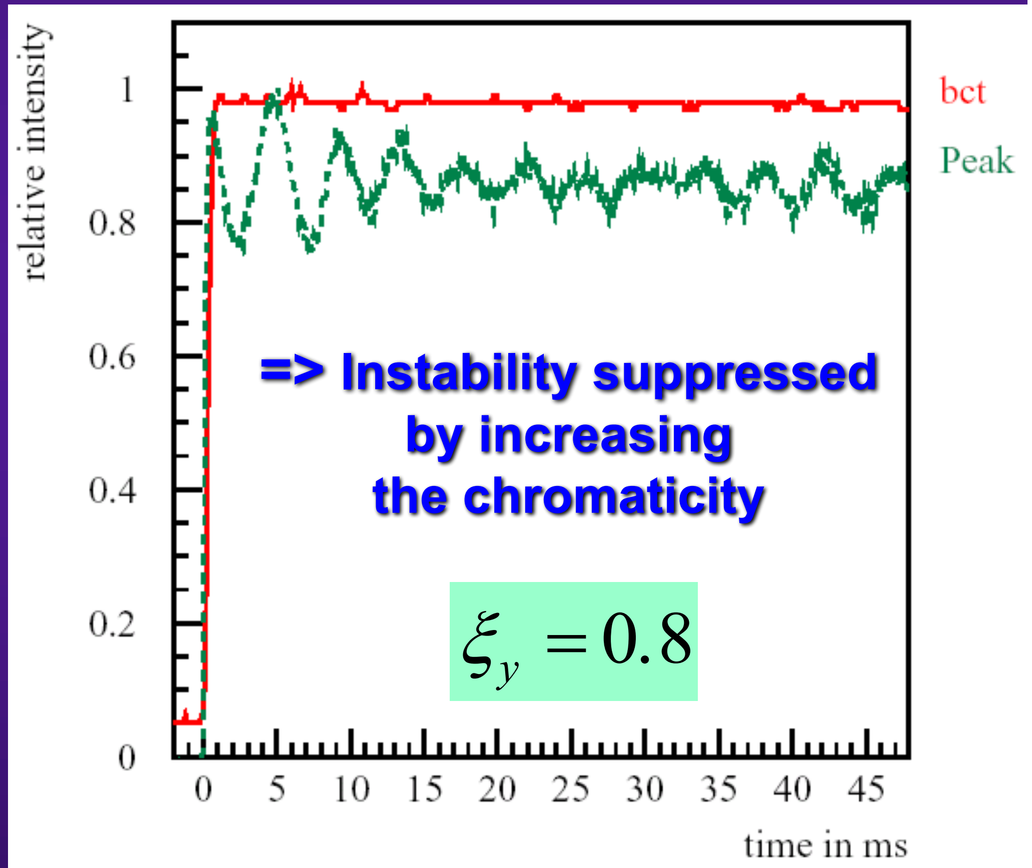
$p = 26$ GeV/c

$N_b \approx 1.2\,10^{11}$ p/b

$\varepsilon_l \approx 0.2$ eVs $< \varepsilon_l^{LHC} = 0.35$ eVs

Synchrotron period ≈ 7 ms

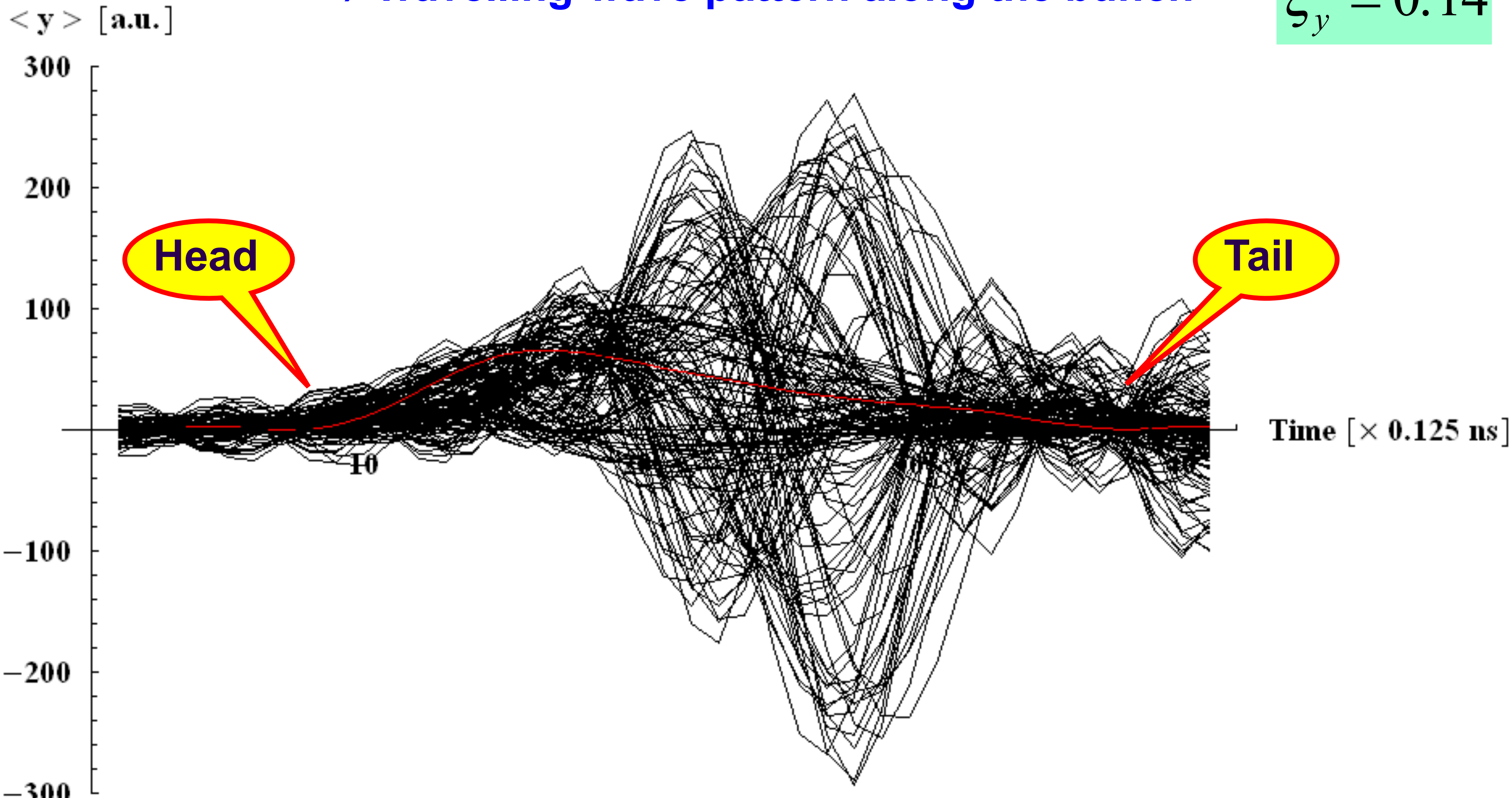




=> Instability suppressed by increasing the chromaticity

$\Rightarrow$ **Travelling-wave pattern along the bunch**

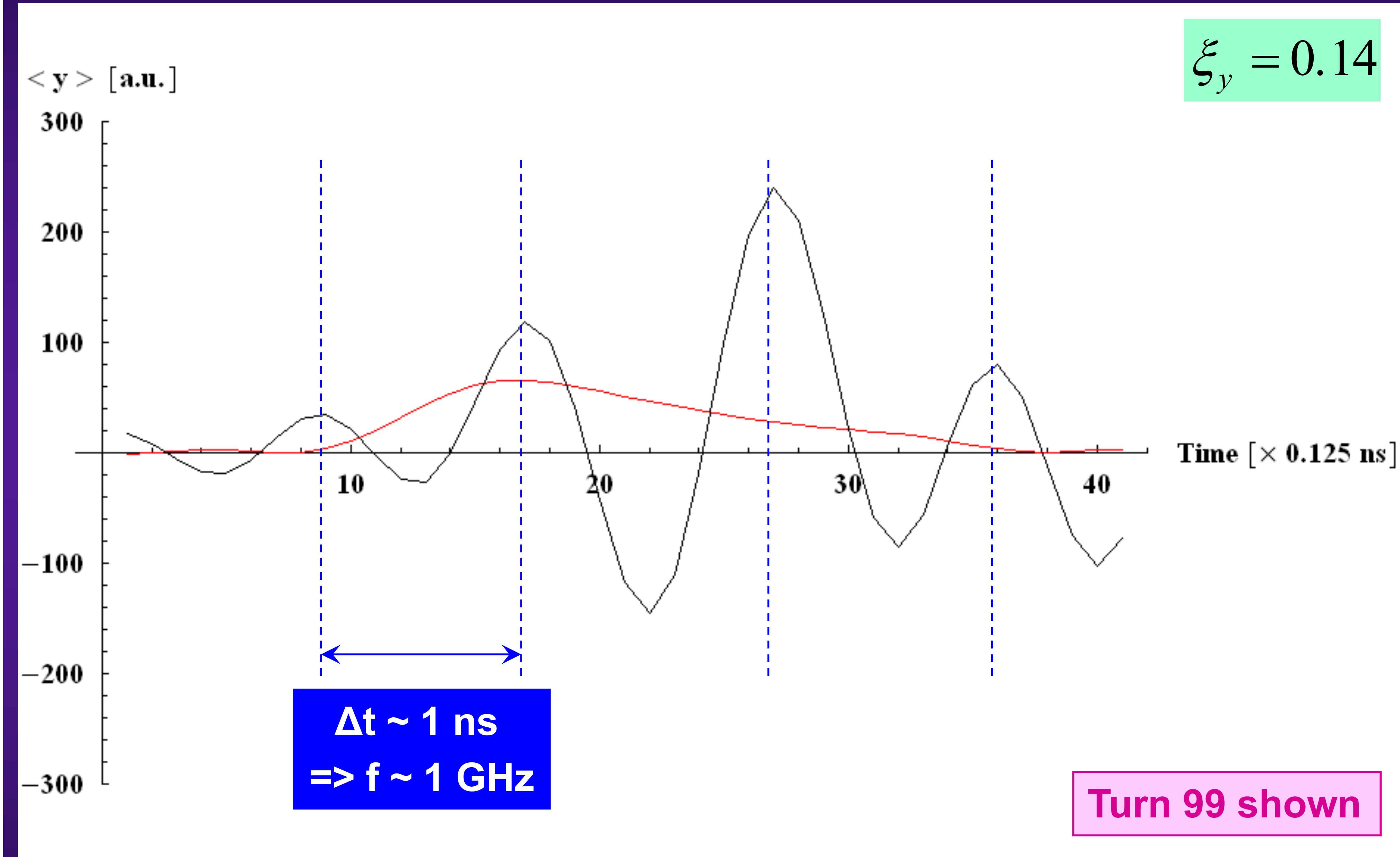
$\xi_y = 0.14$



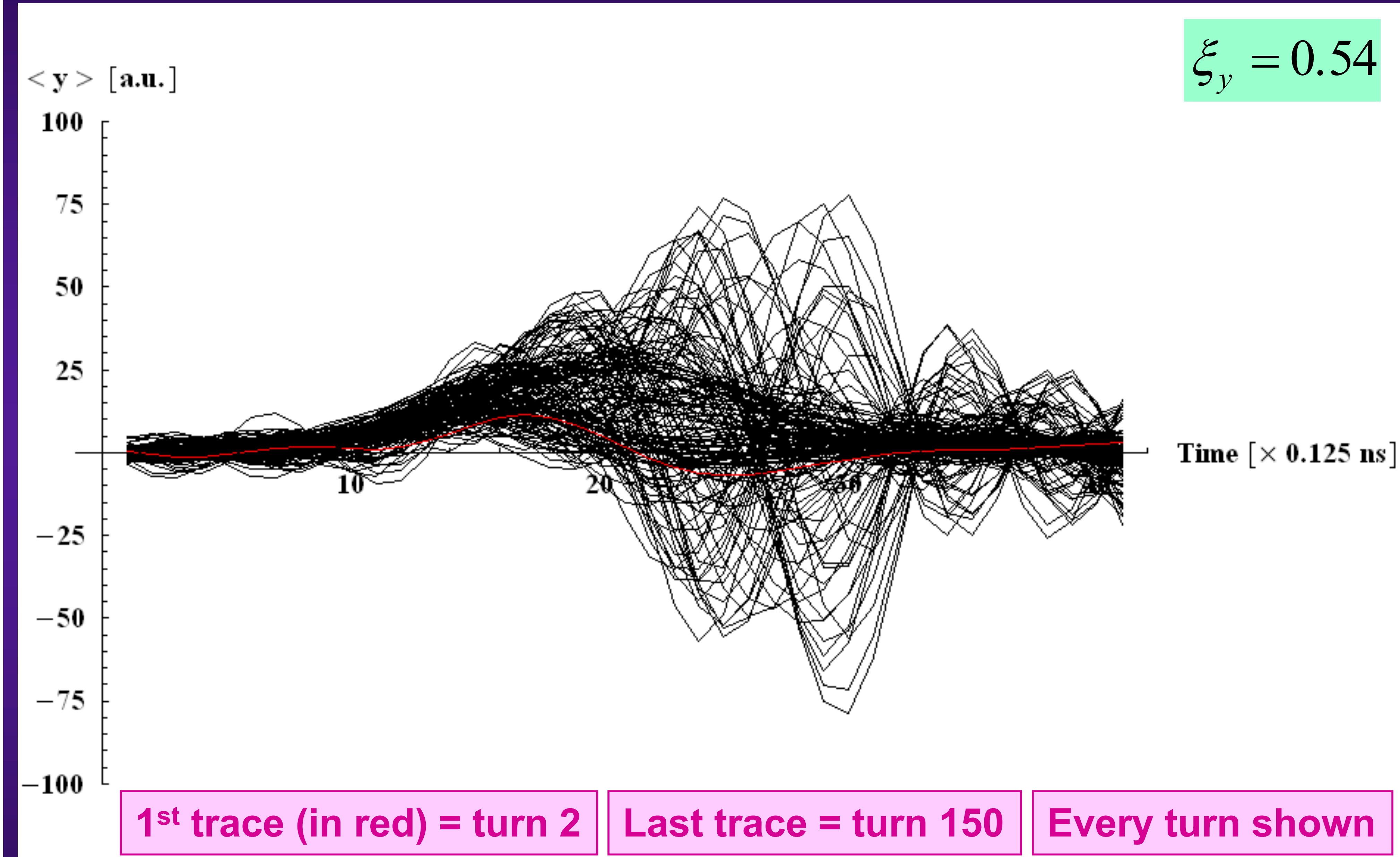
1st trace (in red) = turn 2

Last trace = turn 150

Every turn shown

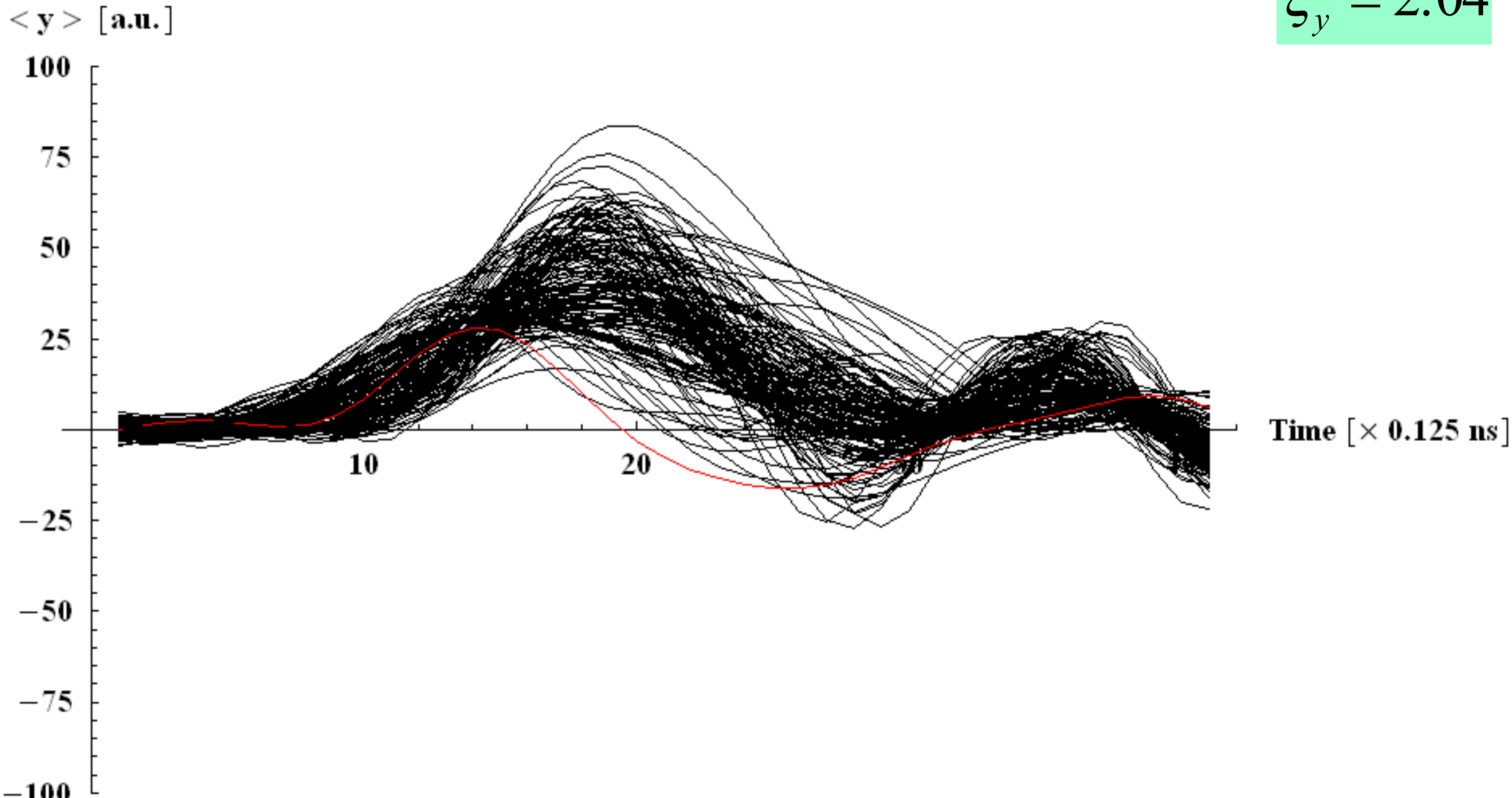


< y > [a.u.]
300
200
100
−100
−200
−300
10
20
30
40
Time [× 0.125 ns]
$\xi_y = 0.14$
Δt ~ 1 ns
=> f ~ 1 GHz
Turn 99 shown



$\xi_y = 0.54$
< y > [a.u.]
100
75
50
25
−25
−50
−75
−100
10
20
Time [× 0.125 ns]
1st trace (in red) = turn 2
Last trace = turn 150
Every turn shown

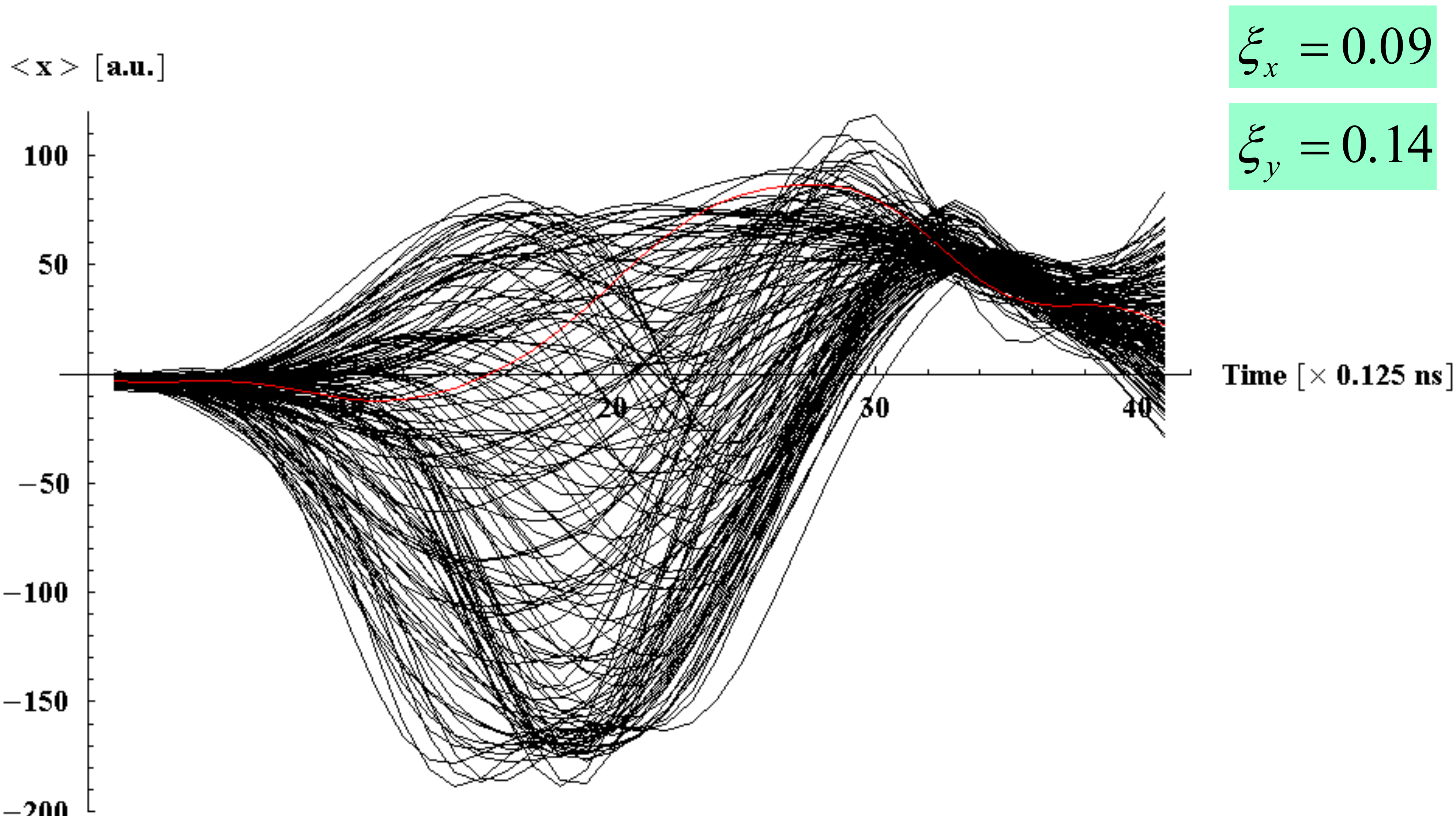
$$\xi_y = 2.04$$



1st trace (in red) = turn 2

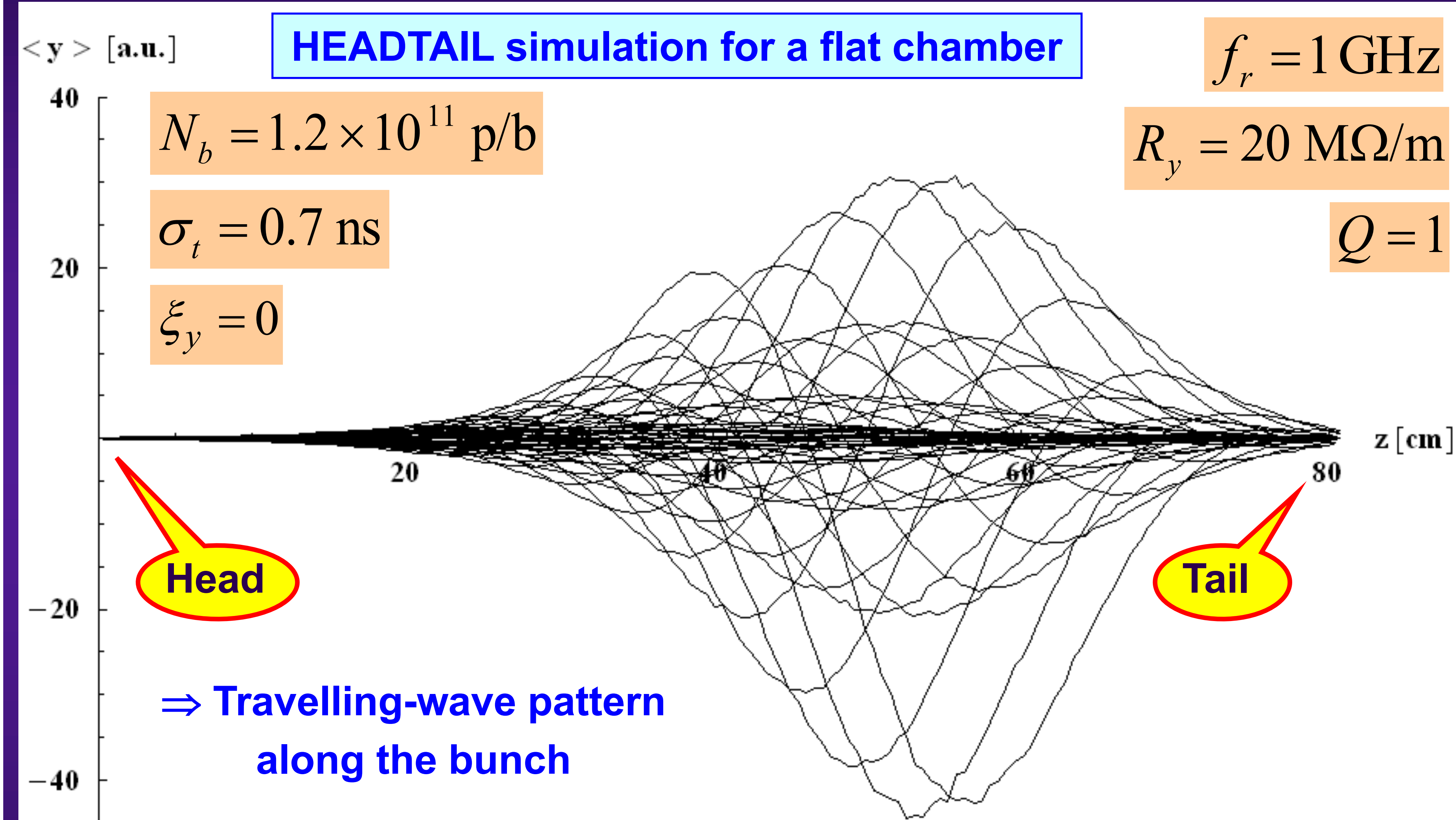
Last trace = turn 150

Every turn shown



1st trace (in red) = turn 2 | Last trace = turn 150 | Every turn shown

1^{st} simulation studies without SC (G. Rumolo)…



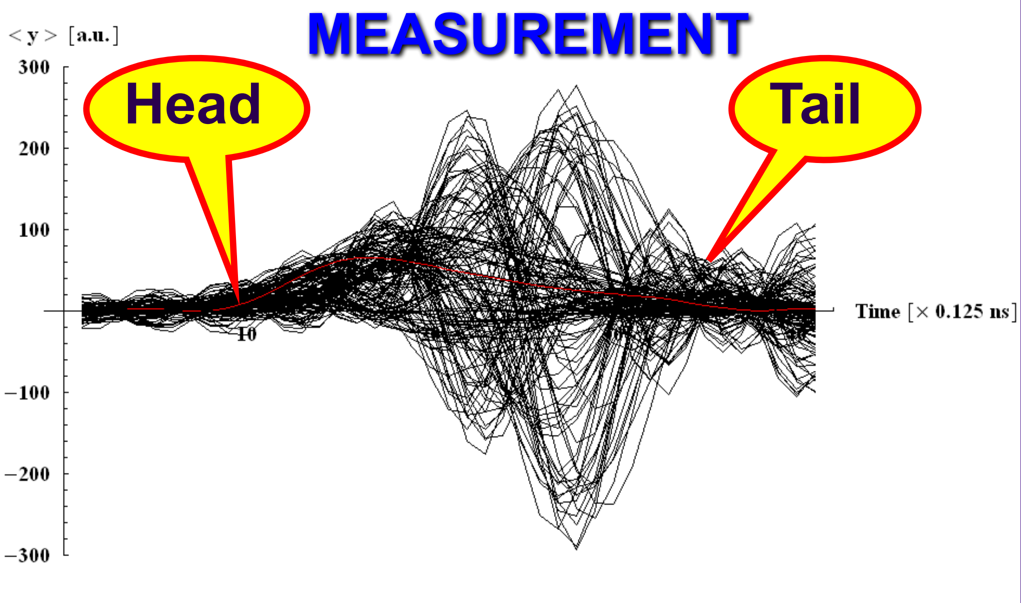
1^{st} trace = turn 1

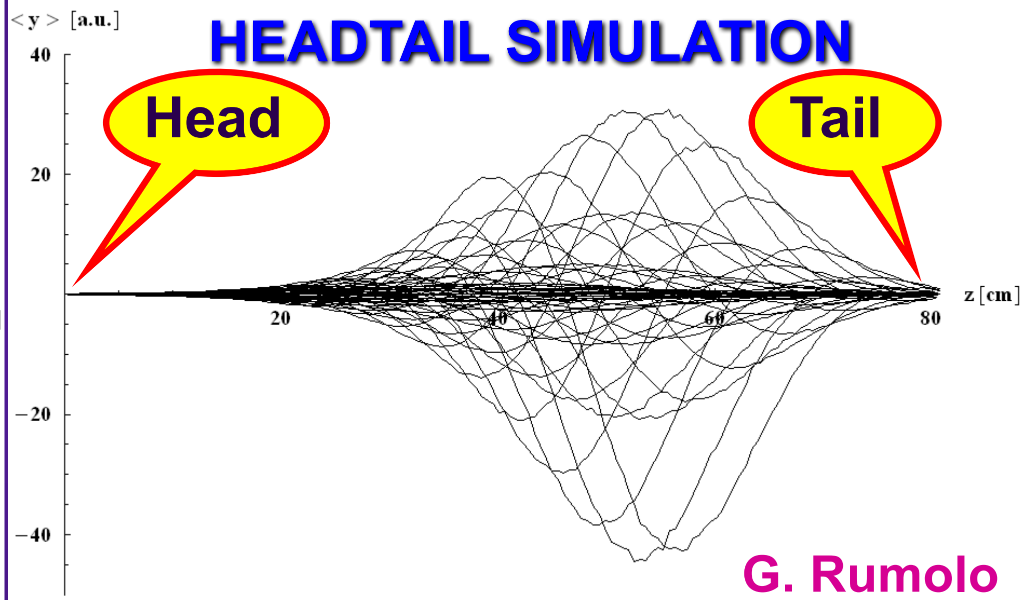
Last trace = turn 50

Every turn shown

… and 1st conclusions

=> Measured picture and movie close to simulated ones (using first a Broad-Band resonator only)

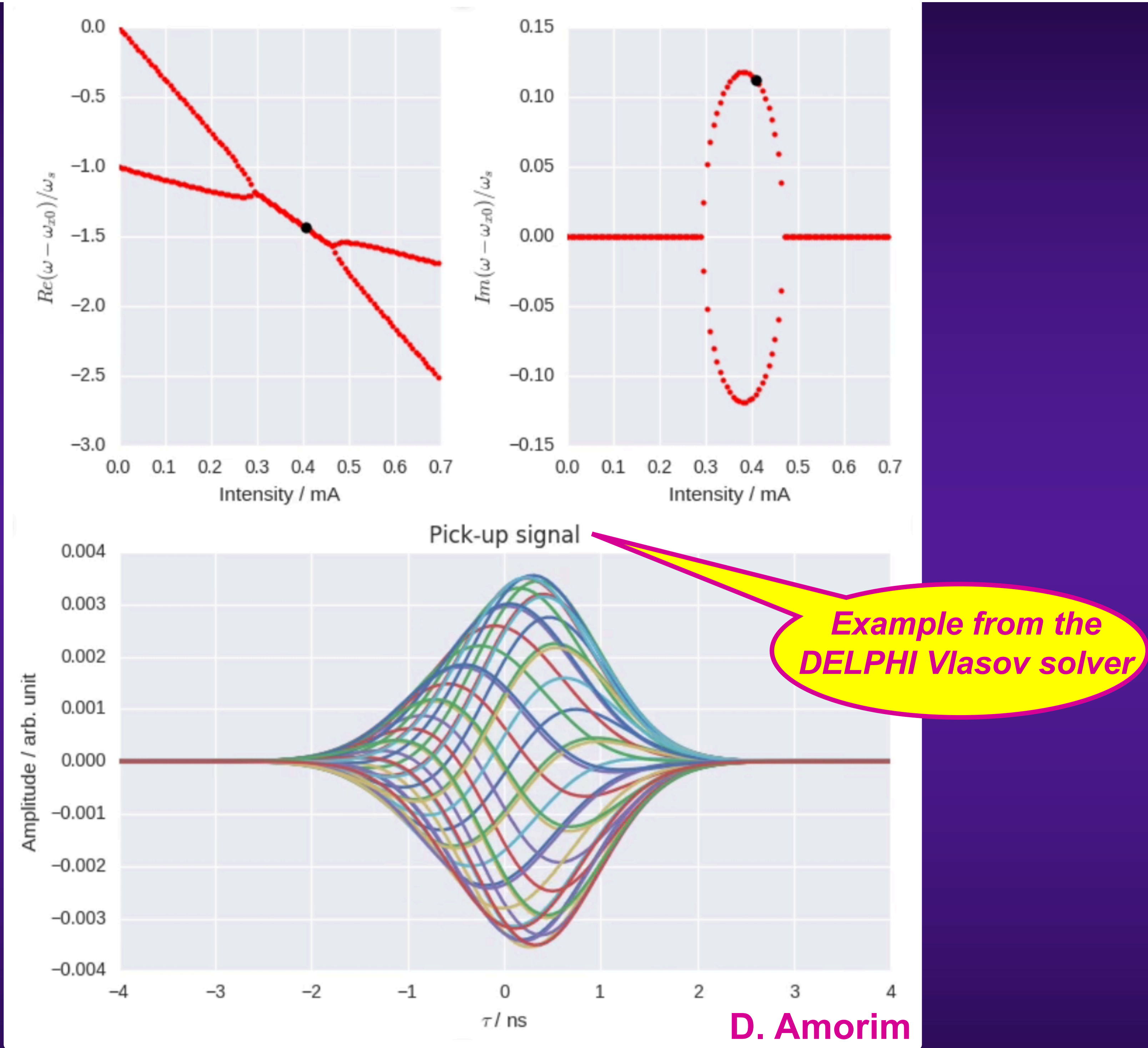


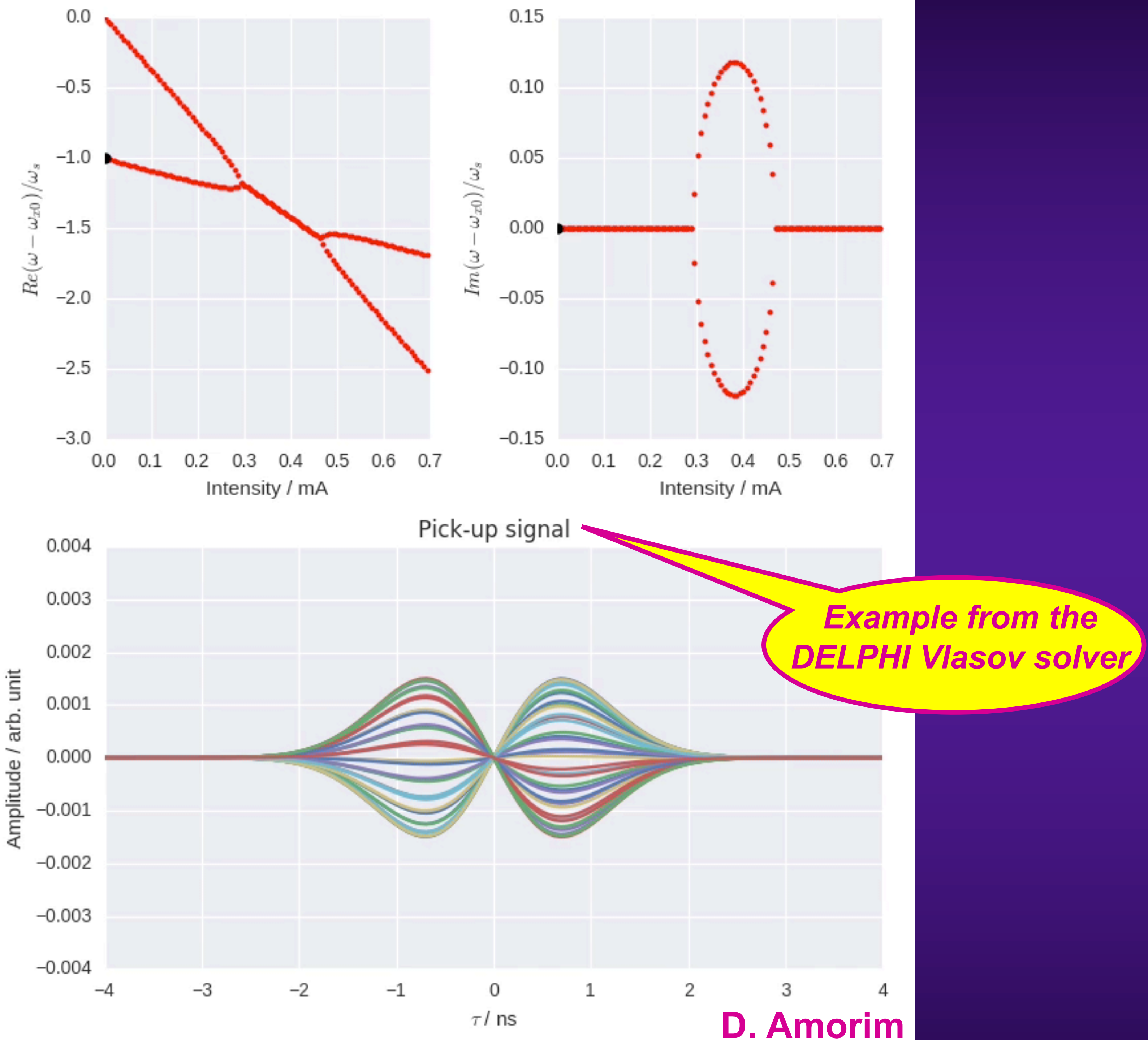


G. Rumolo

But can we state that it is a TMCI?

The coupling of 2 Head-Tail modes (standing-wave patterns) generates a travelling-wave pattern…

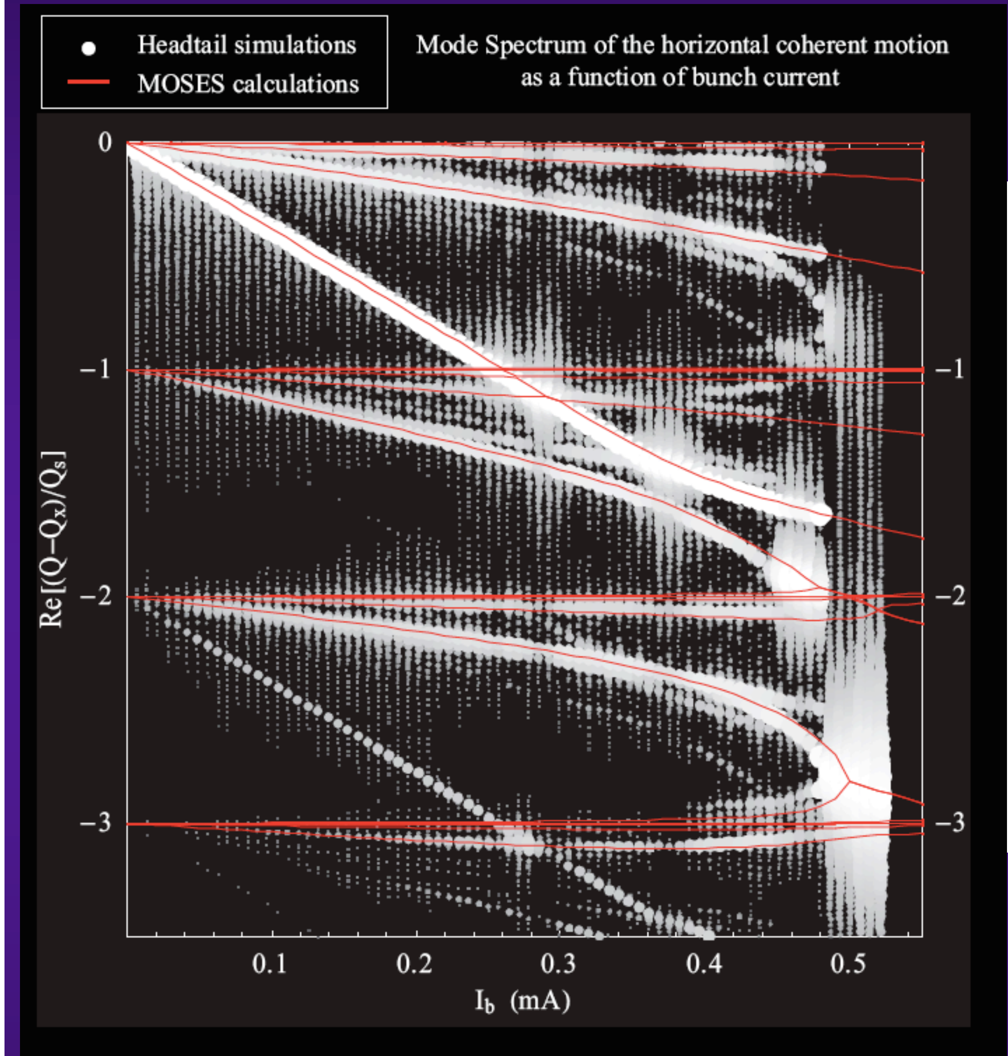

0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
$Re(\omega - \omega_{x0})/\omega_s$
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
Intensity / mA
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
$Im(\omega - \omega_{x0})/\omega_s$
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
Intensity / mA
Pick-up signal
0.004
0.003
0.002
0.001
0.000
−0.001
−0.002
−0.003
−0.004
Amplitude / arb. unit
−4 −3 −2 −1 0 1 2 3 4
τ / ns
Example from the DELPHI Vlasov solver
D. Amorim

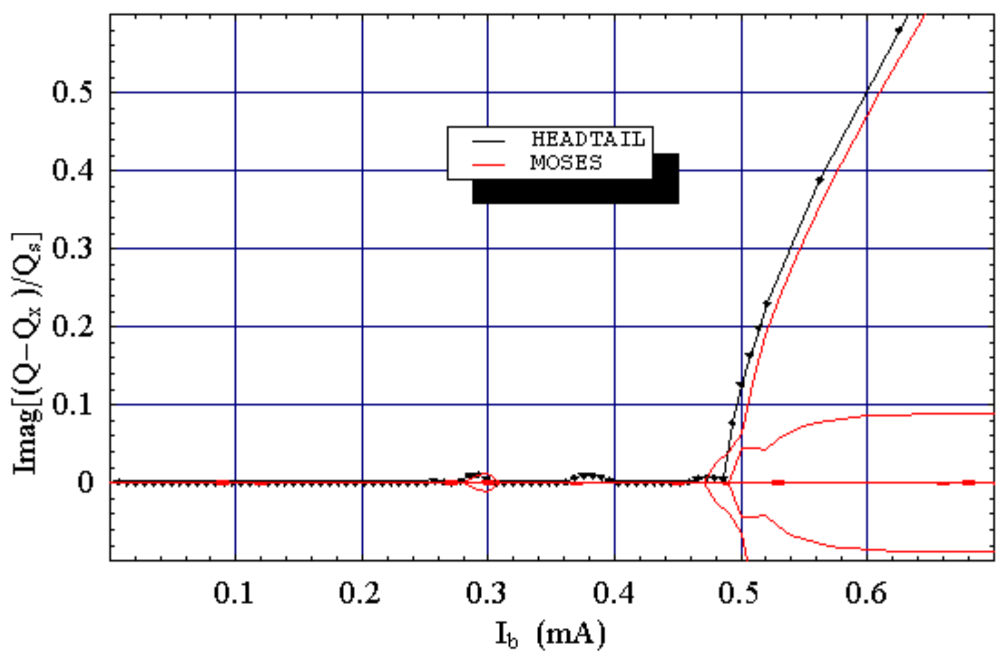


0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
$Re(\omega - \omega_{x0})/\omega_s$
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
Intensity / mA
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
$Im(\omega - \omega_{x0})/\omega_s$
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
Intensity / mA
Pick-up signal
0.004
0.003
0.002
0.001
0.000
−0.001
−0.002
−0.003
−0.004
Amplitude / arb. unit
−4 −3 −2 −1 0 1 2 3 4
τ / ns
Example from the DELPHI Vlasov solver

D. Amorim

2nd simulation studies (B. Salvant)…





=> TMCI between modes - 2 and - 3 is predicted (WITHOUT SC).

Also using the full impedance model which was developed in //

2^{nd} simulation studies (B. Salvant)…

Parameter Name	Symbol	Value	Unit
Beam momentum	p	26	GeV/c
Revolution frequency	f_{rev}	43375.9	Hz
Momentum compaction factor	α_{cp}	$1.92\ 10^{-3}$	
Betatron tune spread		0	
Synchrotron tune	Q_s	$3.24\ 10^{-3}$	
Average beta function	$\langle\beta_x\rangle = \langle\beta_y\rangle$	40	m
Linear chromaticity	$\xi_x = \xi_y$	0	
r.m.s. bunch length	σ_z	0.21	m
Resonator shunt impedance	R_s	10	MΩ/m
Resonator frequency	f_{res}	1	GHz
Resonator quality factor	Q	1	

Table B.3: SPS parameters for the LHC beam at injection used in *MOSES* calculations.

Parameter Name	Symbol	Value	Unit
Beam momentum	p	26	GeV/c
Revolution frequency	f_{rev}	43375.9	Hz
Momentum compaction factor	α_{cp}	$1.92\ 10^{-3}$	
Circumference length	L	6911	m
Lorentz factor	γ	27.7286	
Betatron tunes	Q_x / Q_y	26.185 / 26.13	
Synchrotron tune	Q_s	$3.24\ 10^{-3}$	
Average beta functions	$\langle\beta_x\rangle$ / $\langle\beta_y\rangle$	40 /40	m
Initial r.m.s. beam sizes	σ_x / σ_y	1.8 / 1.8	mm
Linear chromaticities	ξ_x / ξ_y	0 / 0	
Initial r.m.s. bunch length	σ_z	0.21	m
Initial r.m.s. longitudinal momentum spread	$\sigma_{\Delta p/p_0}$	$9.3\ 10^{-4}$	
Cavity harmonic number	h	4620	
Resonator shunt impedance	R_s	10	MΩ/m
Resonator frequency	f_{res}	1	GHz
Resonator quality factor	Q	1	
Initial kick amplitude		0.9	mm
Number of slices		500	
Number of macroparticles		10^6	
Longitudinal restoring force		linear	
Frozen wake field		yes	

Table B.4: SPS parameters for the LHC beam at injection used in *HEADTAIL* simulations.

3rd studies: new measurement campaign (B. Salvant)…

- **Why do we observe "what looks like a TMCI (with a travelling-wave along the bunch)" whereas SC should suppress it (according to some past theoretical analyses => Pioneer work of M. Blaskiewicz in 1998 followed by other analyses by A. Burov et al.)?**

3rd studies: new measurement campaign (B. Salvant)…

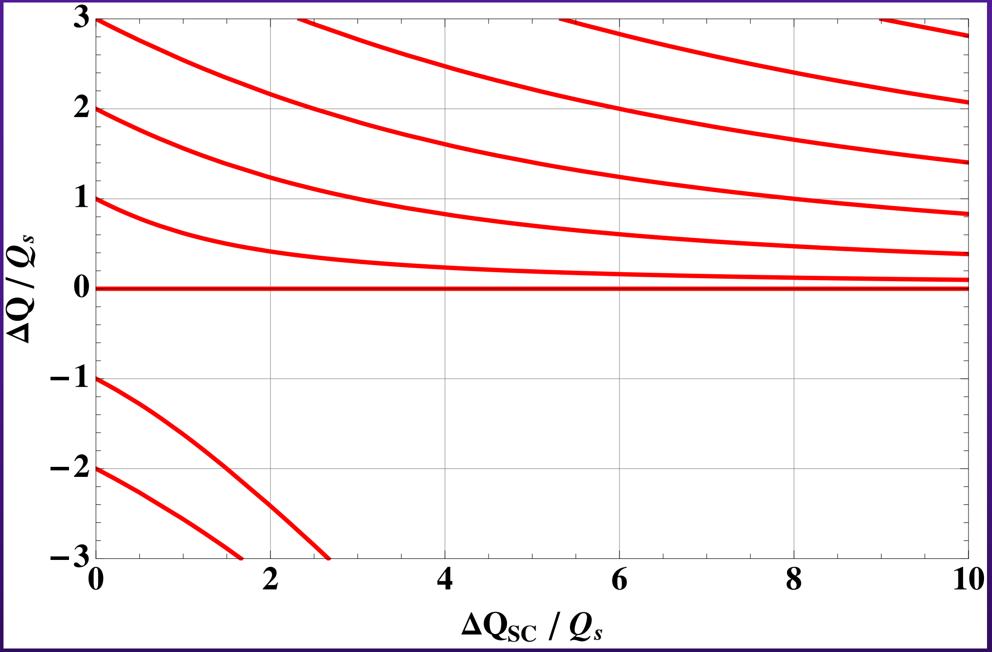
- **Why do we observe "what looks like a TMCI (with a travelling-wave along the bunch)" whereas SC should suppress it (according to some past theoretical analyses => Pioneer work of M. Blaskiewicz in 1998 followed by other analyses by A. Burov et al.)?**
  - **Can we observe the coupling of the (< 0 or > 0) modes?**
  - **How do measurements compare to HEADTAIL simulations?**

SC ONLY

(square-well air-bag, Blaskiewicz1998)

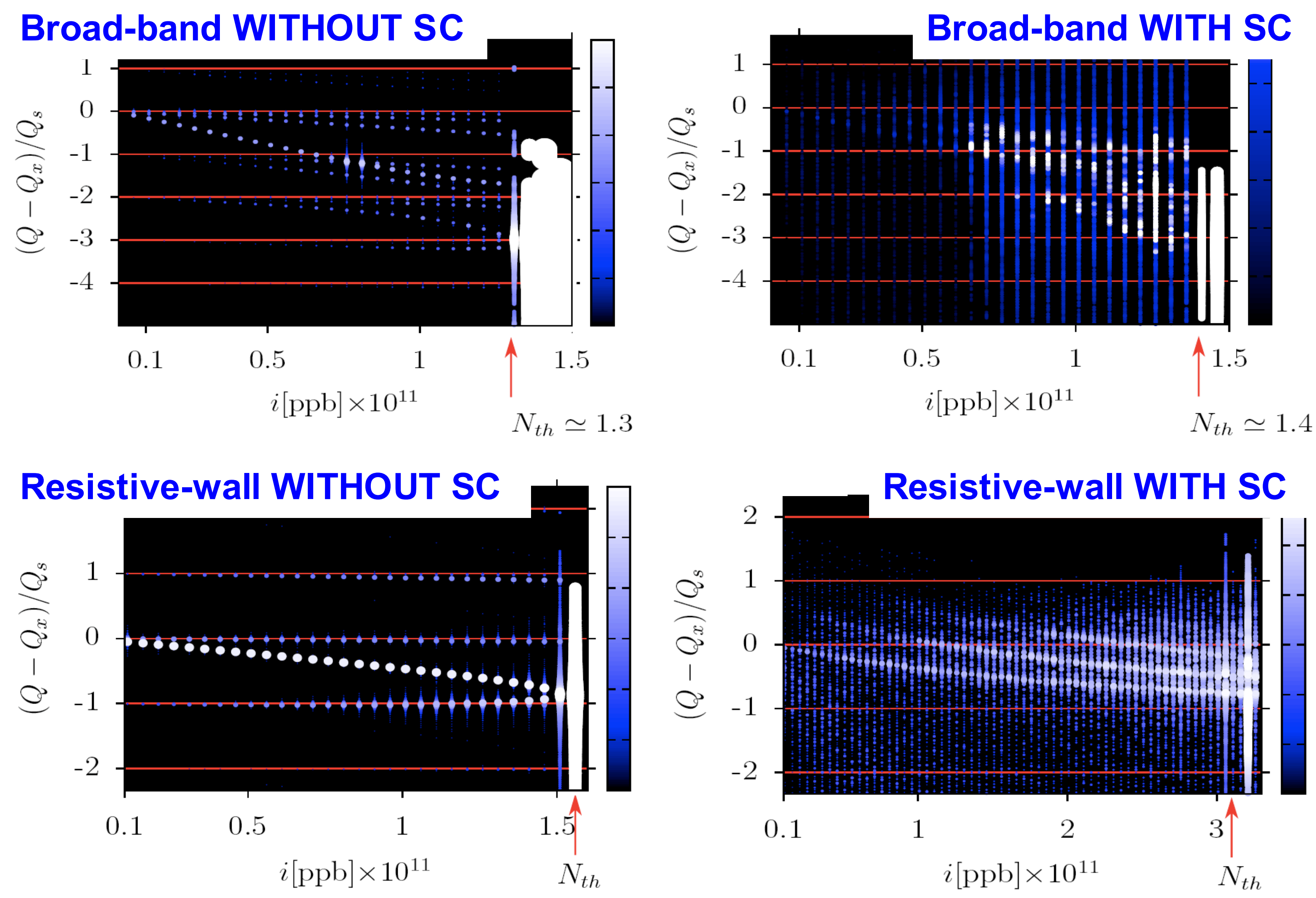
$$\Delta Q^{y}_{m \geq 0} = -\frac{\Delta Q_{SC}}{2} + \sqrt{\left(\frac{\Delta Q_{SC}}{2}\right)^2 + \left(m Q_s\right)^2}$$

$$\Delta Q^{y}_{m < 0} = -\frac{\Delta Q_{SC}}{2} - \sqrt{\left(\frac{\Delta Q_{SC}}{2}\right)^2 + \left(m Q_s\right)^2}$$



3rd studies: new measurement campaign (B. Salvant)…

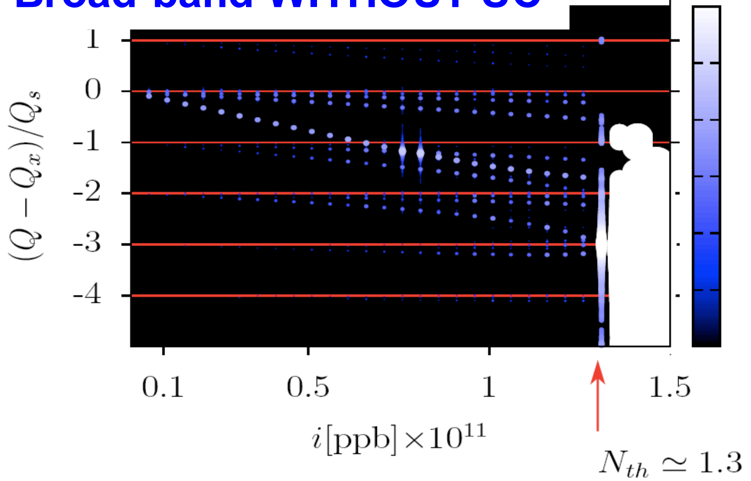
- **1st SC simulations from D. Quatraro and G. Rumolo in 2010 using a 3rd order symplectic integrator for the equation of motion, taking into account non linear space charge forces coming from a Gaussian shaped bunch** (http://accelconf.web.cern.ch/accelconf/IPAC10/papers/tupd046.pdf)



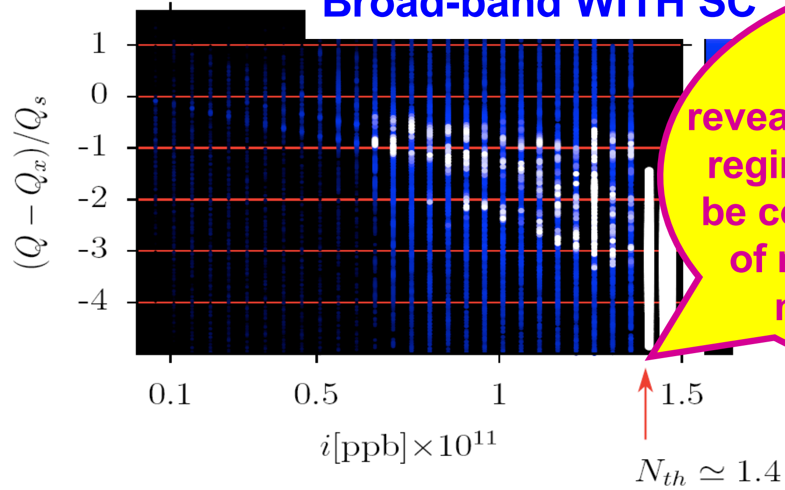
3rd studies: new measurement campaign (B. Salvant)…

- **1st SC simulations from D. Quatraro and G. Rumolo in 2010 using a 3rd order symplectic integrator for the equation of motion, taking into account non linear space charge forces coming from a Gaussian shaped bunch** (http://accelconf.web.cern.ch/accelconf/IPAC10/papers/tupd046.pdf)

Broad-band WITHOUT SC

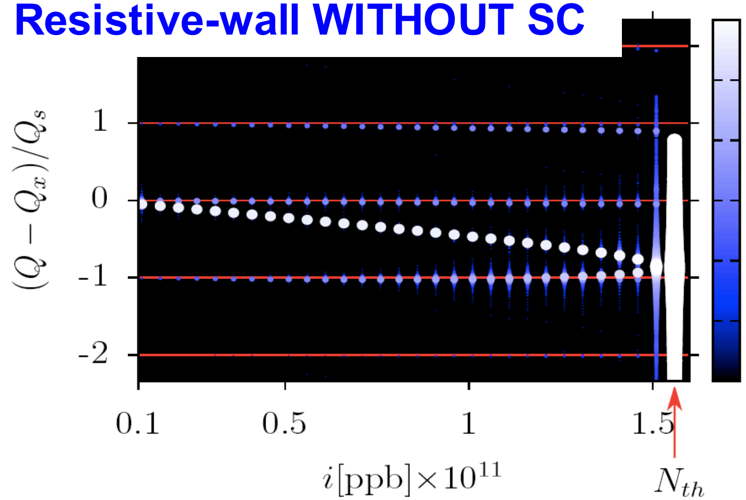


Broad-band WITH SC

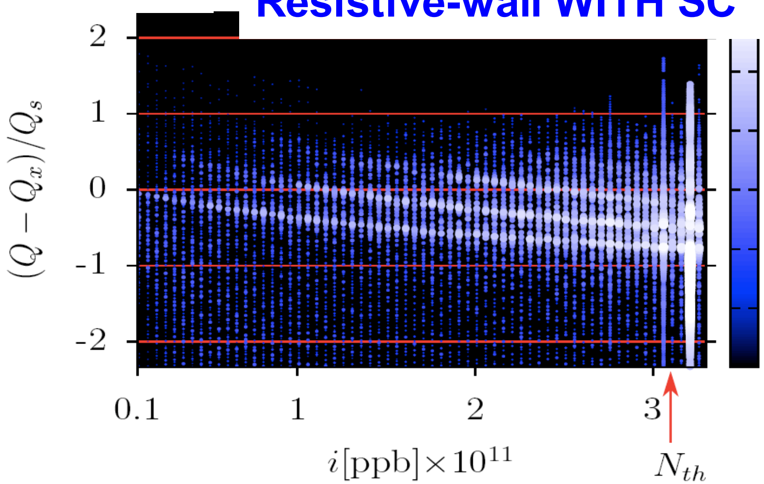


Only a minor (beneficial) effect revealed for the "long-bunch" regime of the SPS => Could be compatible with coupling of modes - 2 and - 3… (or maybe > 0 modes…)

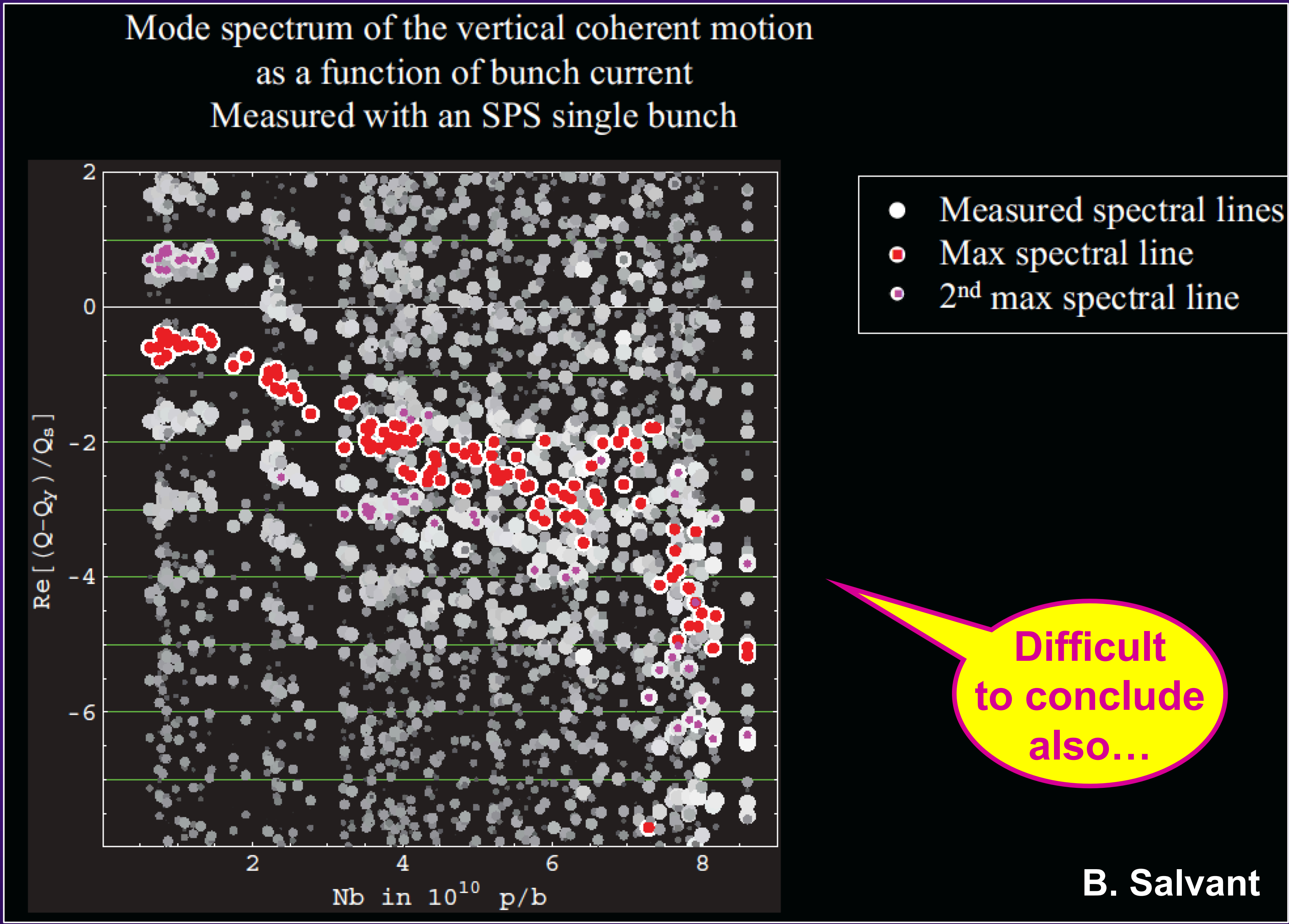
Resistive-wall WITHOUT SC



Resistive-wall WITH SC



3rd studies: new measurement campaign (B. Salvant)…



3rd studies: new measurement campaign (B. Salvant)…

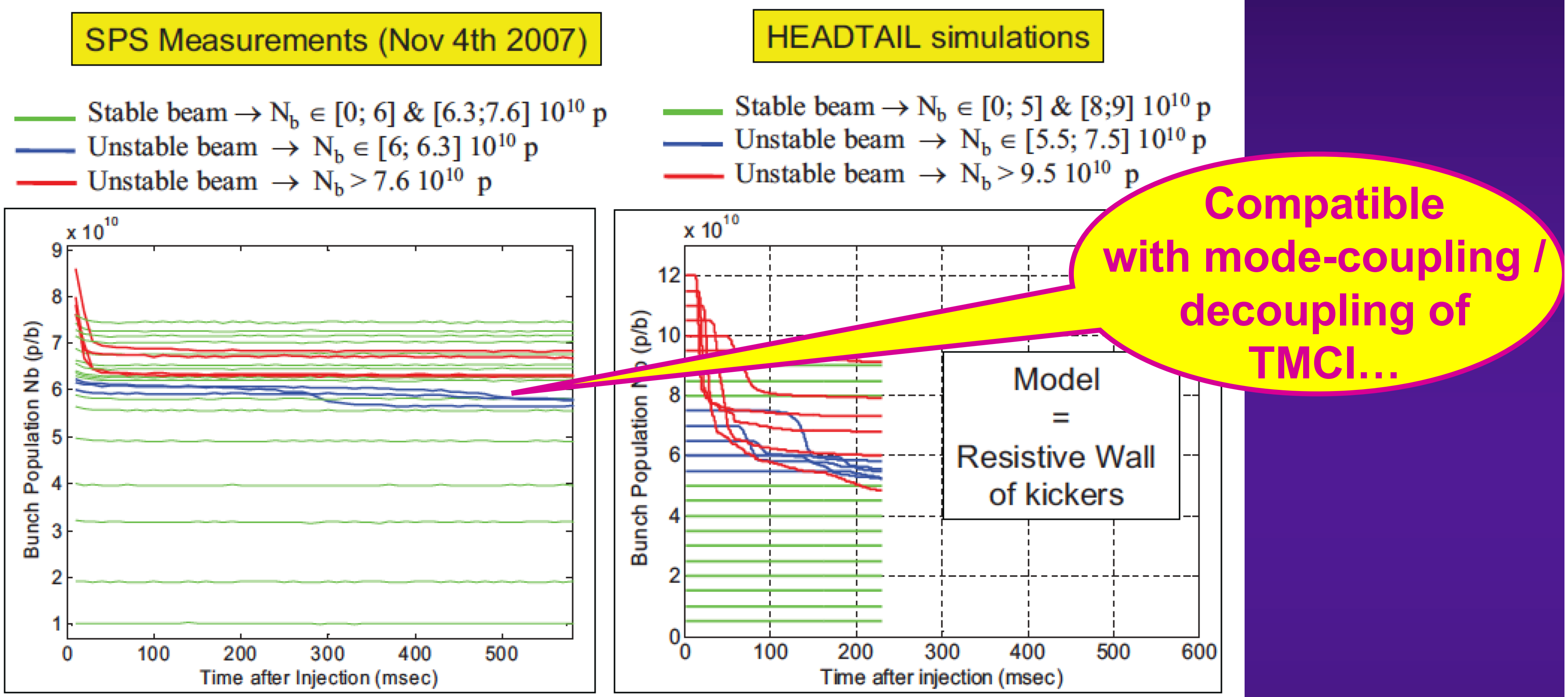


Figure 6.26: Bunch population measured by the SPS BCT for various cycles, SPS parameters $\varepsilon_l = 0.16$ eV.s, $\sigma_t = 0.7$ ns, and $\xi_y \approx 0$ (left), simulated with *HEADTAIL* for $\varepsilon_l = 0.16$ eV.s, $\sigma_t = 0.5$ ns, and $\xi_y = 0$ (right). Low bunch currents lead to stable bunch motion (in green). In both simulations and measurements, two distinct unstable ranges (slow instability in blue and fast instability in red) are separated by a stable range of bunch population (in green).

B. Salvant

4^{th} studies: Increasing the intensity threshold by increasing the slip factor (distance to transition)

- As
  - 1) SPS instability seemed to be relatively well described by TMCI using a Broad-Band resonator (without SC)
  - and 2) in this case ("long-bunch" regime) a simple formula exists (recently checked by A. Burov & T. Zolkin with NHT Vlasov solver => "TMCI with Resonator Wakes" (https://arxiv.org/pdf/1806.07521.pdf))

$$T_s = \pi \, \tau_{\mathrm{TMCI}}^{\mathrm{sm}}$$

$$N_{b,th} = \frac{4 \pi^3 f_s \, Q_{y0} \, E \, \tau_b^2}{e \, c} \times \frac{f_r}{\left| Z_y \right|}$$

$$N_{b,th} = \frac{8 \pi \boxed{Q_{y0} \left| \eta \right| \varepsilon_l}}{e \, \beta^2 \, c} \times \frac{f_r}{\left| Z_y \right|}$$

4th studies: Increasing the intensity threshold by increasing the slip factor (distance to transition)

- As
  - 1) SPS instability seemed to be relatively well described by TMCI using a Broad-Band resonator (without SC)
  - and 2) in this case ("long-bunch" regime) a simple formula exists (recently checked by A. Burov & T. Zolkin with NHT Vlasov solver => "TMCI with Resonator Wakes" (https://arxiv.org/pdf/1806.07521.pdf))

$$T_s = \pi\, \tau_{\mathrm{TMCI}}^{\mathrm{sm}}$$

$$N_{b,th} = \frac{4 \pi^3 f_s\, Q_{y0}\, E \tau_b^2}{e\, c} \times \frac{f_r}{\left| Z_y \right|}$$

$$N_{b,th} = \frac{8 \pi \boxed{Q_{y0} \left| \eta \right| \varepsilon_l}}{e\, \beta^2\, c} \times \frac{f_r}{\left| Z_y \right|}$$

it was proposed to modify the optics to increase the slip factor => "Q20 optics" by H. Bartosik (with Y. Papaphilippou)

$$\eta = - \frac{df_{rev} / f_{rev}}{dp / p} = \alpha_p - \frac{1}{\gamma^2} = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$$

4th studies: Increasing the intensity threshold by increasing the slip factor (distance to transition)

- **Simple rough estimate of γ_t for machines made of simple FODO cells**

  - **Approximating the machine radius by the bending radius, yields**

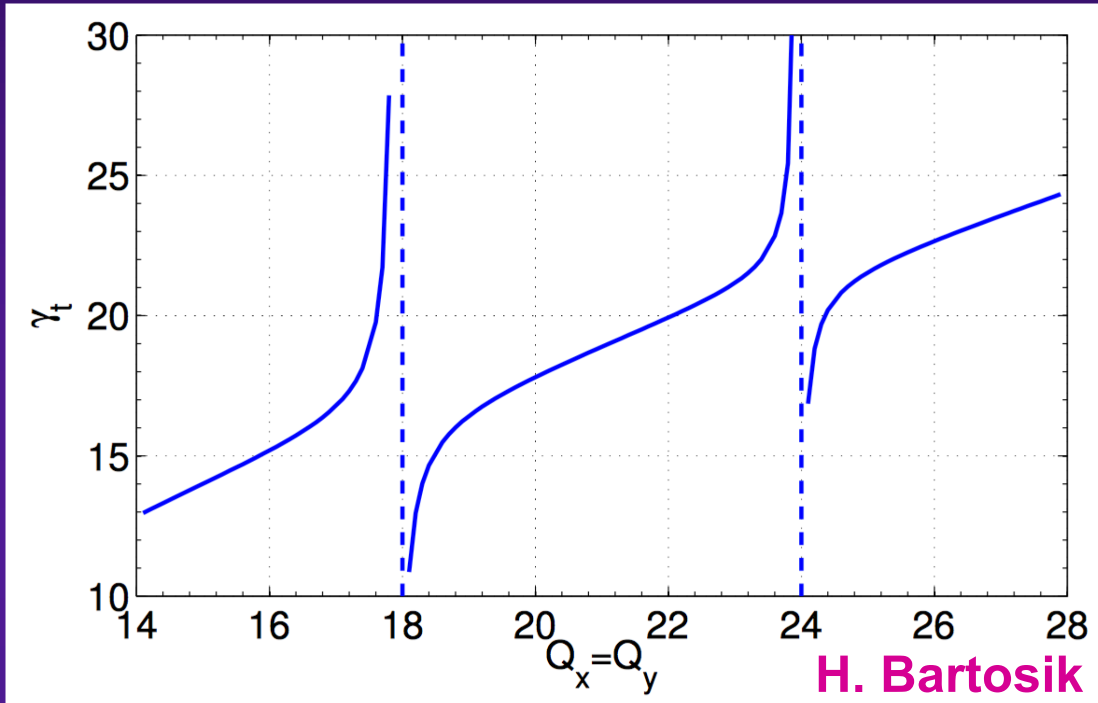
$$D_x \approx \frac{\rho}{Q_x^2}$$

  - **Inserting this in the definition of α_p (and then expressing γ_t) yields**

$$\gamma_t \approx Q_x$$

=> If one wants to modify γ_t, one should modify the horizontal tune

4^{th} studies: Increasing the intensity threshold by increasing the slip factor (distance to transition)



- **Q26:** $$|\eta|\, Q_y = 0.62\, 10^{-3} \times 26.13 \approx 0.0162$$ $\gamma_t = 22.8$

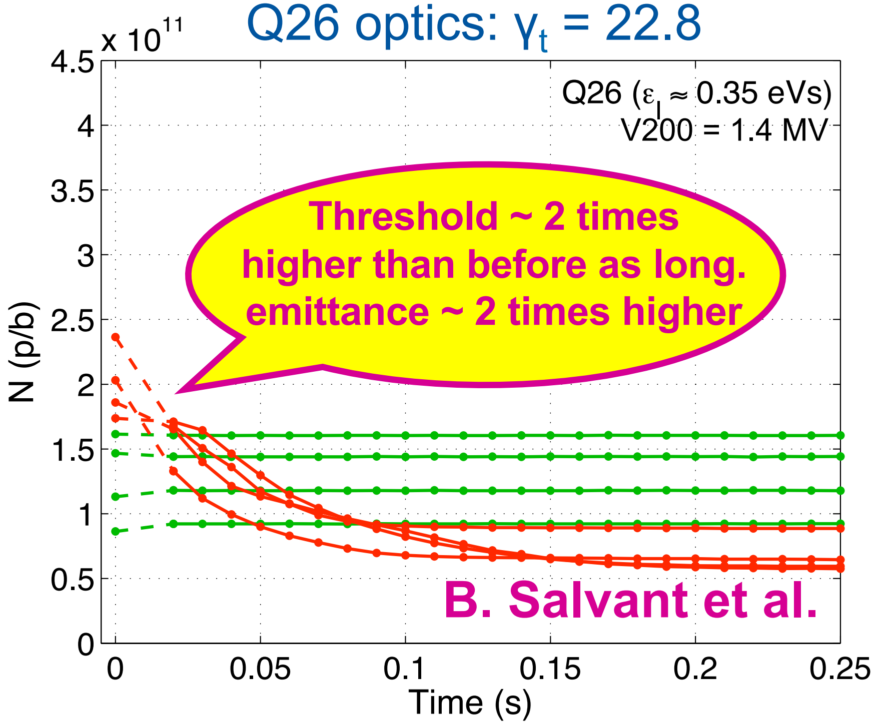
- **Q20:** $$|\eta|\, Q_y = 1.80\, 10^{-3} \times 20.13 \approx 0.0362$$ $\gamma_t = 18$

4th studies: Increasing the intensity threshold by increasing the slip factor (distance to transition)

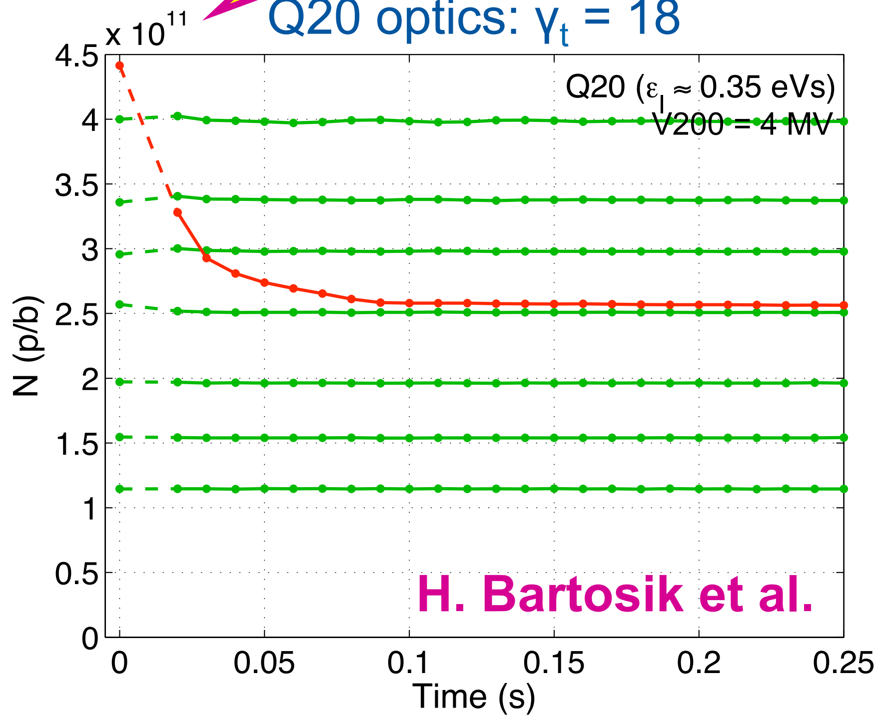
- Measurements in good agreement with simple formula

Gain of a factor 4.0 / 1.6 ~ 2.5 (compared to ~ 2.2 predicted)

Q26 optics: $\gamma_t = 22.8$

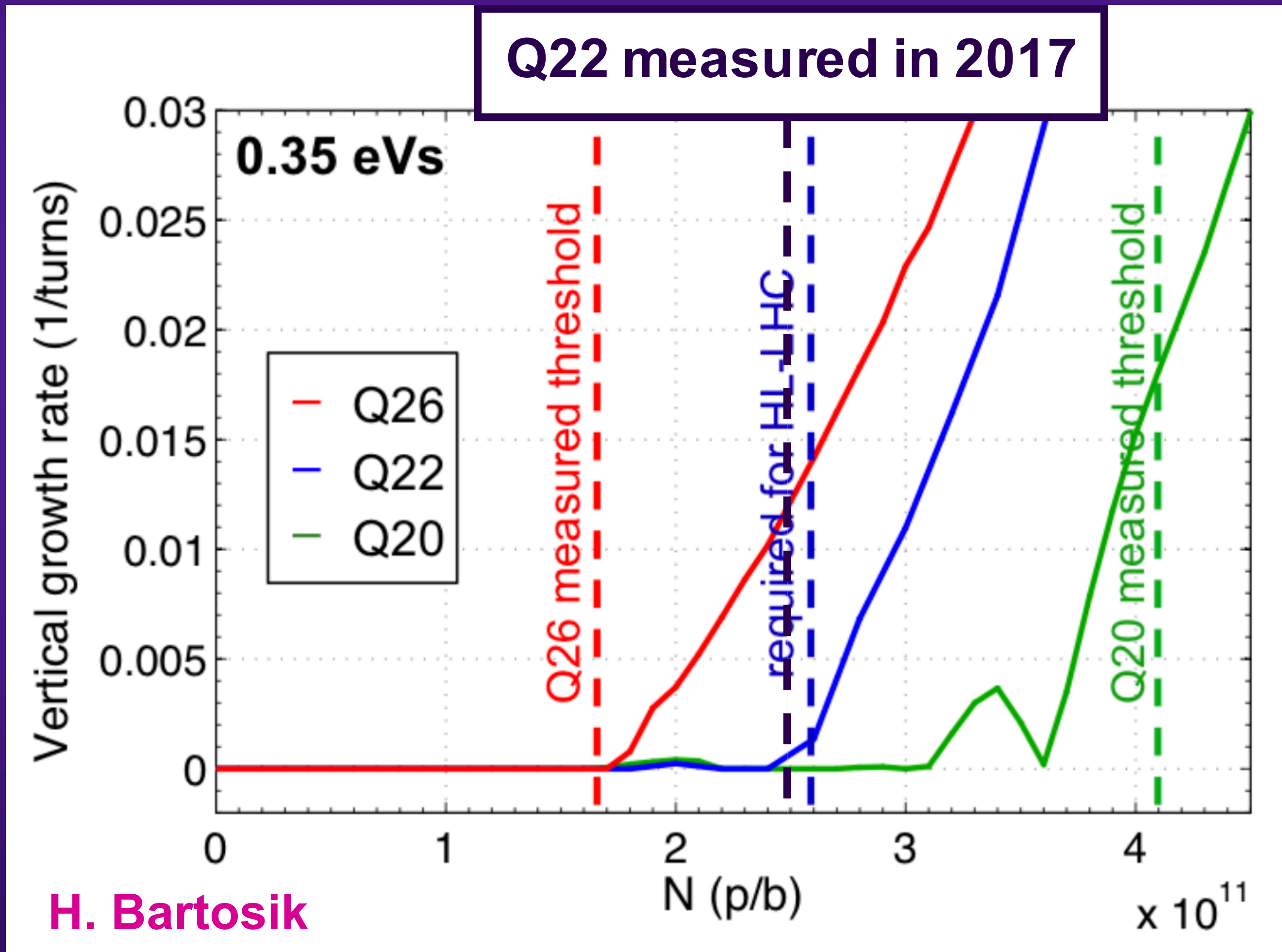


Q20 optics: $\gamma_t = 18$



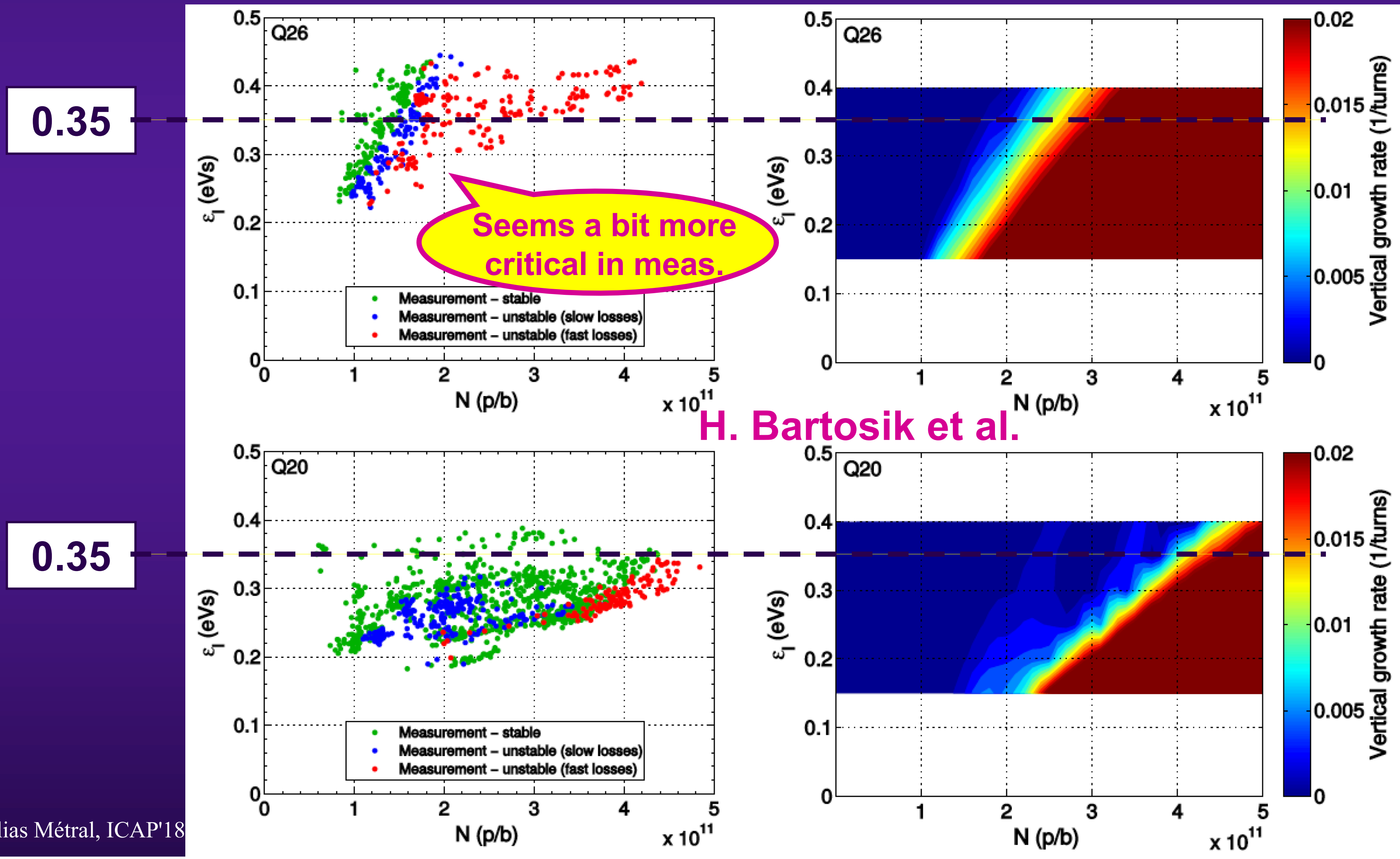
4^{th} studies: Increasing the intensity threshold by increasing the slip factor (distance to transition)

- Good agreement also with HEADTAIL simulations from 2014 for different optics, with full impedance model but WITHOUT SC



4th studies: Increasing the intensity threshold by increasing the slip factor (distance to transition)

- **Good agreement also between measurements (left) and HEADTAIL simulations (right) looking at different longitudinal emittances, with full impedance model but WITHOUT SC**



4th studies: Increasing the intensity threshold by increasing the slip factor (distance to transition)

Seems more towards the tail for Q26…

Without SC

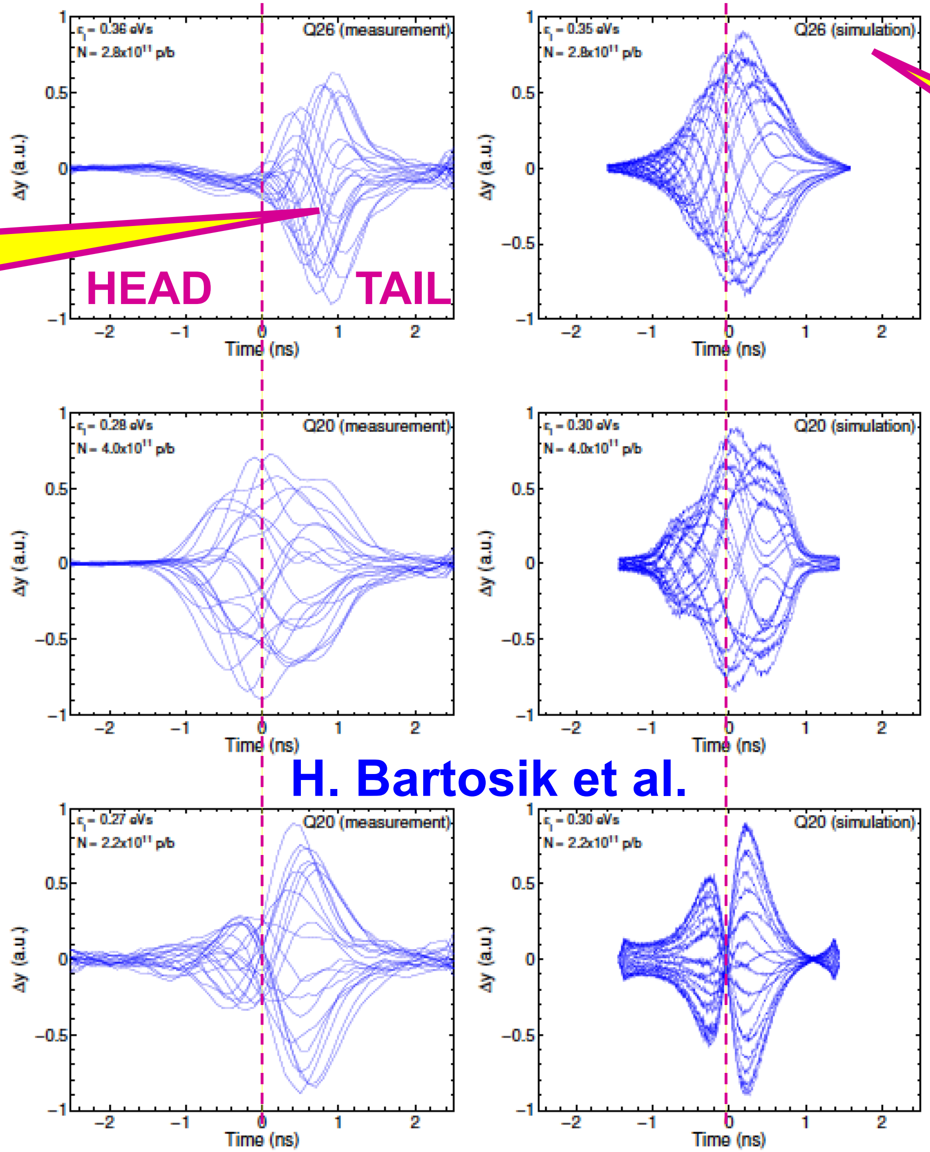
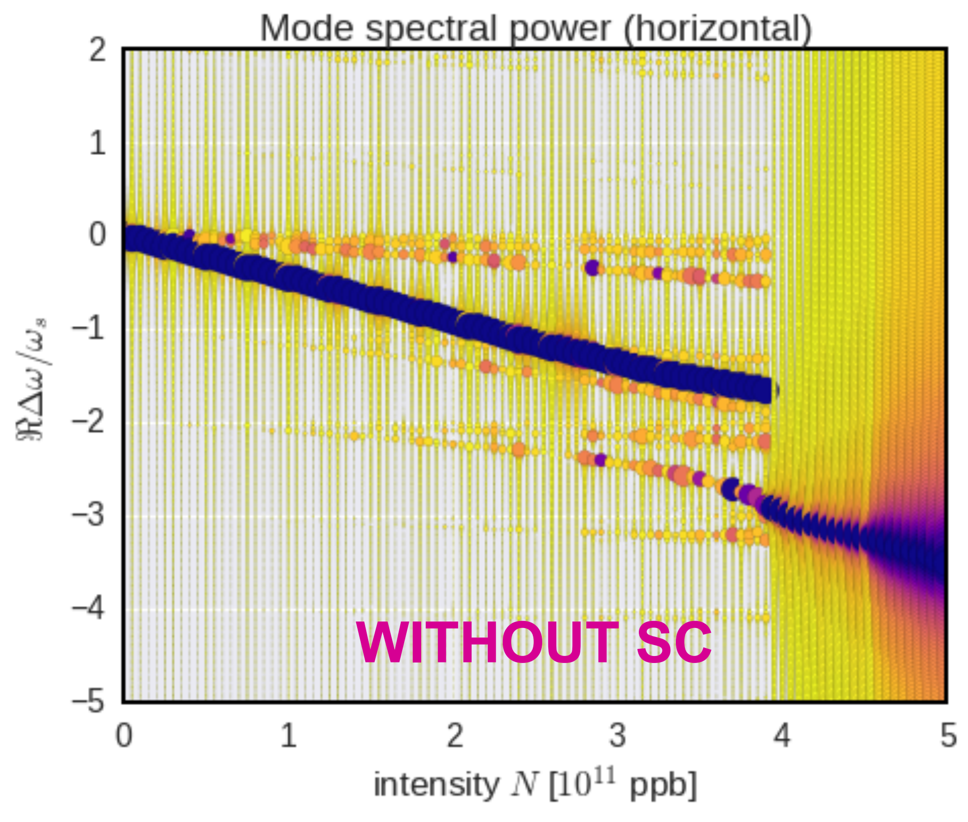
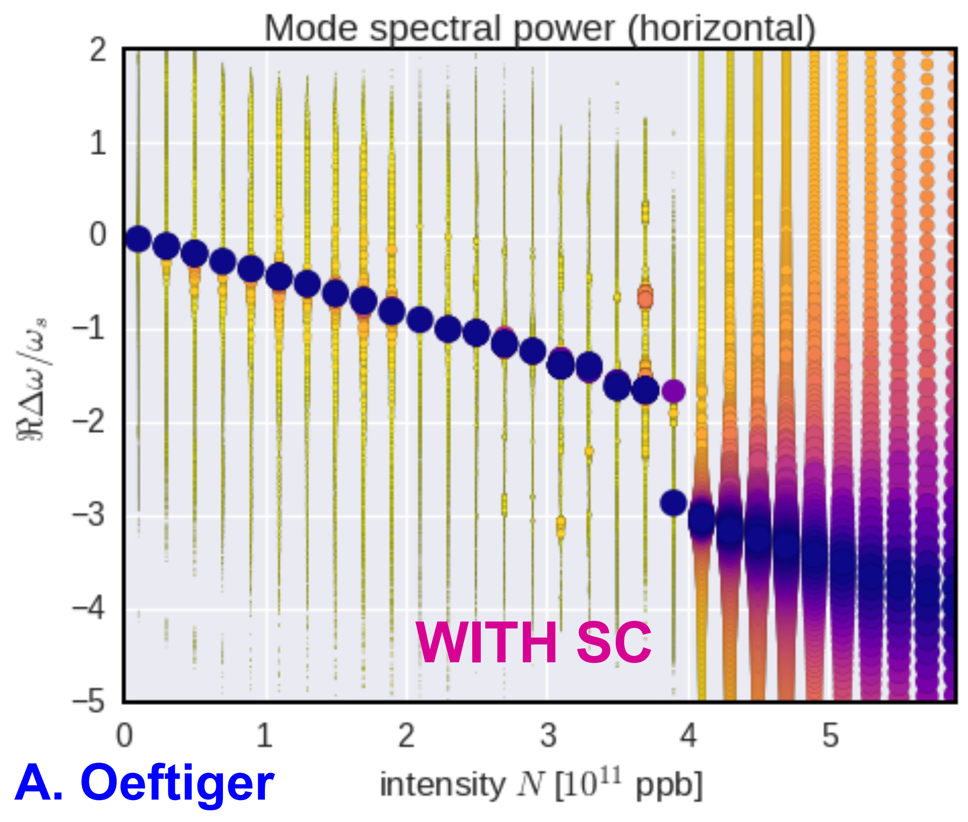


Figure 4.21: Comparison of the vertical intra bunch motion between the SPS Head-Tail monitor measurement (left) and the corresponding HEADTAIL simulations (right). One case is shown for the Q26 optics and two cases for the Q20 optics, as indicated together with the beam parameters in the graphs.

4th studies: Increasing the intensity threshold by increasing the slip factor (distance to transition)

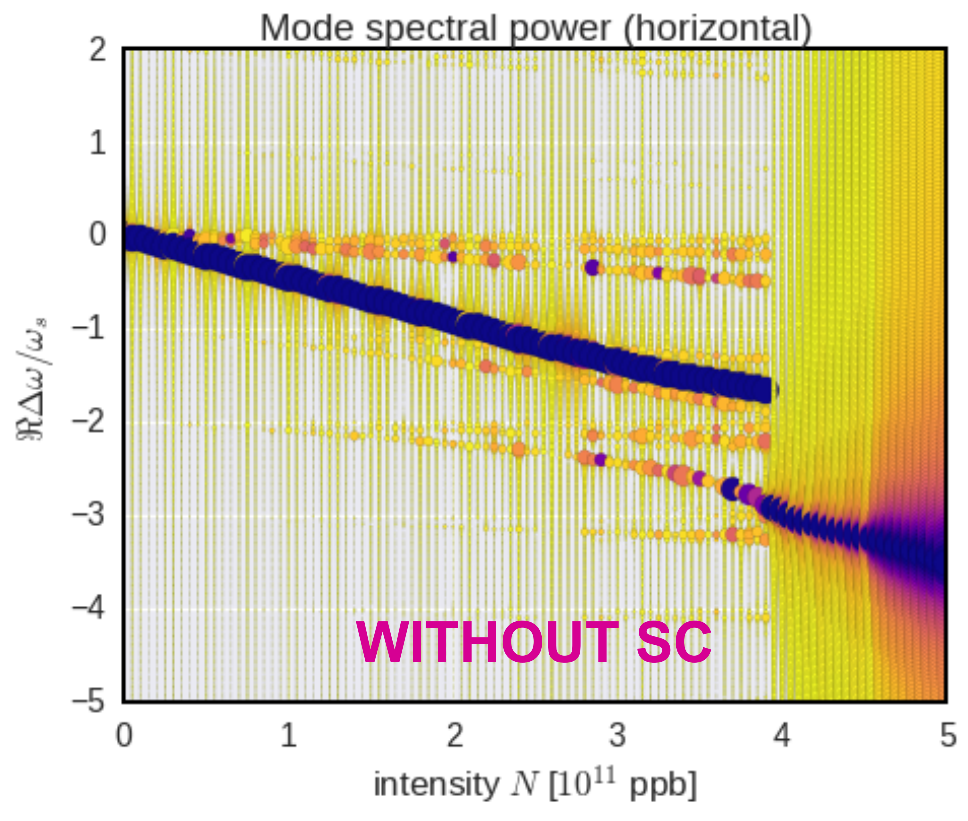
- Good agreement also between measurements and pyHEADTAIL simulations WITH SC for Q20 (considering the Broad-Band resonator model)

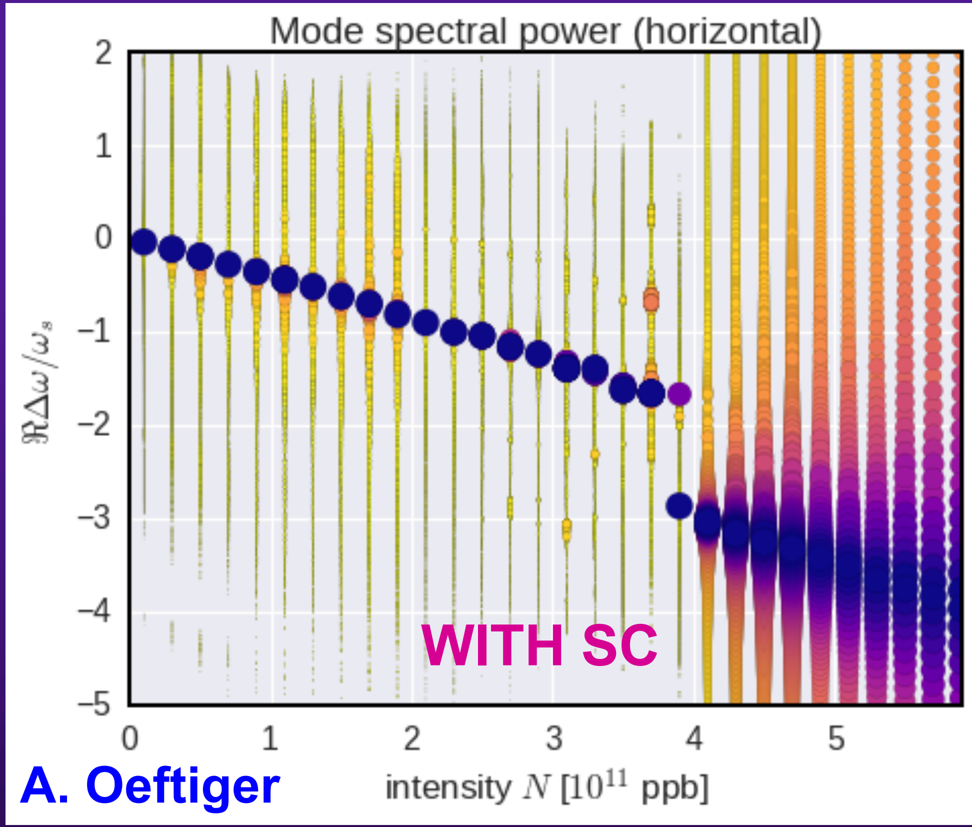




4th studies: Increasing the intensity threshold by increasing the slip factor (distance to transition)

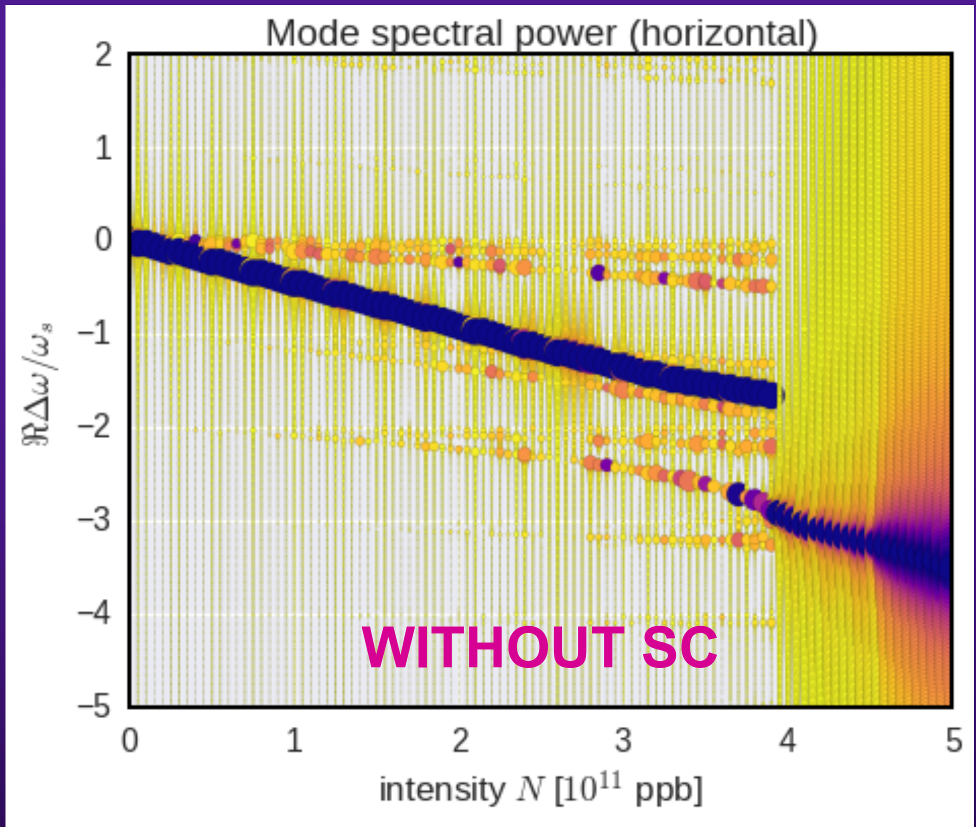
- **Good agreement also between measurements and pyHEADTAIL simulations WITH SC for Q20 (considering the Broad-Band resonator model) => Detailed analysis of the modes involved seems to reveal different modes at start of instability… on-going…**
  - **Without SC: azimuthal modes – 2 & – 3 with radial mode k = 0**
  - **With SC: azimuthal modes + 1 & + 2 with radial mode k = 1**

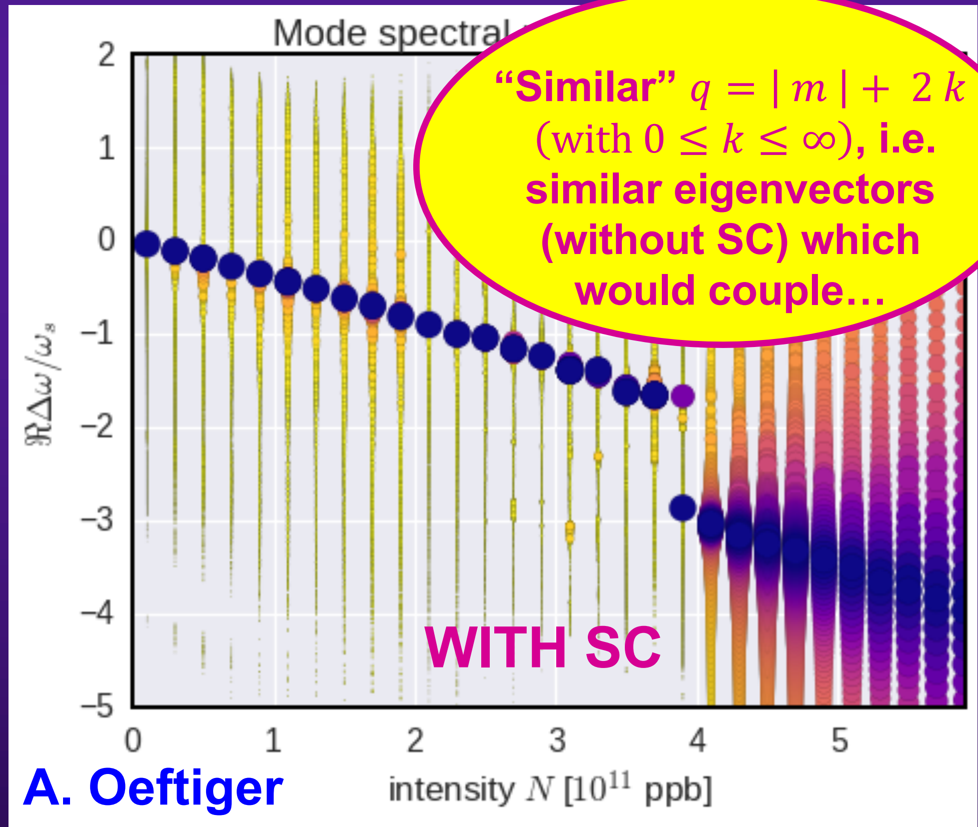




4th studies: Increasing the intensity threshold by increasing the slip factor (distance to transition)

- **Good agreement also between measurements and pyHEADTAIL simulations WITH SC for Q20 (considering the Broad-Band resonator model) => Detailed analysis of the modes involved seems to reveal different modes at start of instability… on-going…**
  - **Without SC: azimuthal modes – 2 & – 3 with radial mode k = 0**
  - **With SC: azimuthal modes + 1 & + 2 with radial mode k = 1**



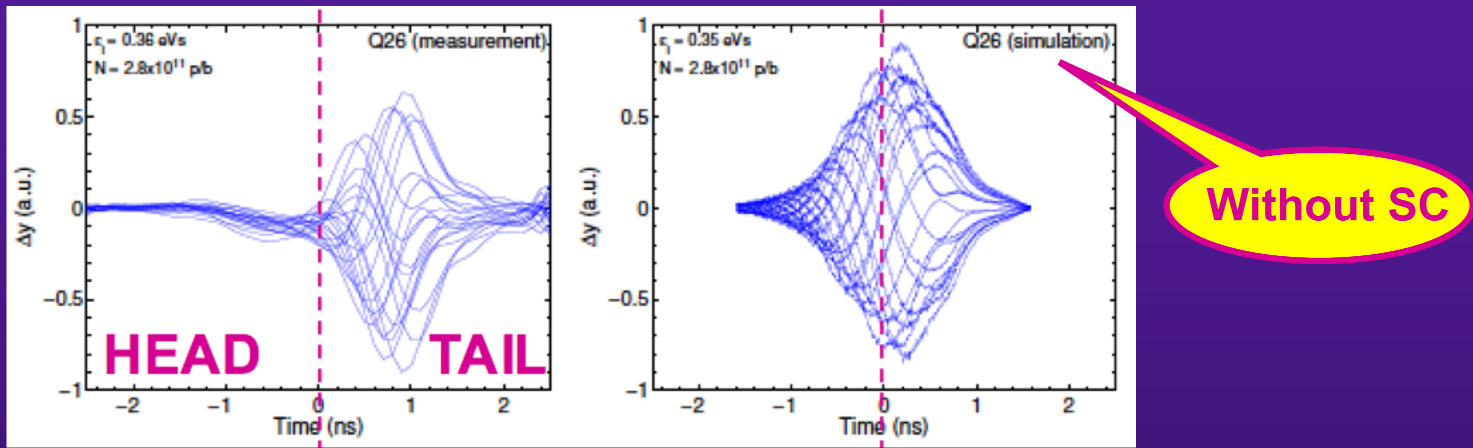


5^{th} studies: closer look to Q26 case

$$q_{SC} = \Delta Q_{x,y}^{SC,spread} / (2 Q_s)$$

$$q_{SC}\big|_{Q20} \approx 5 \ll 27 \approx q_{SC}\big|_{Q26}$$

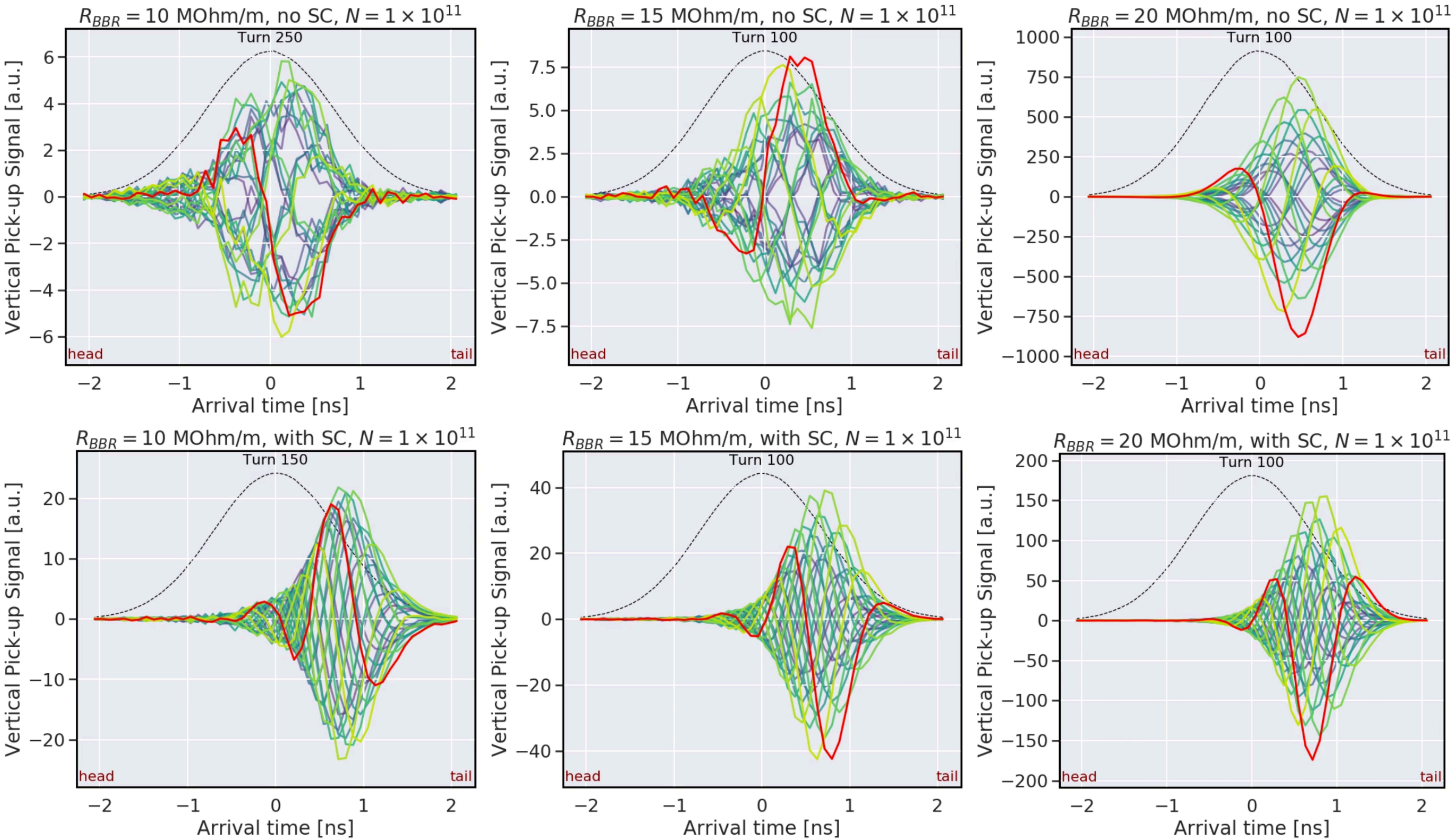
- **New theory from A. Burov: SC was recently found to be destabilising below TMCI without SC => "while the SC suppresses TMCI, it introduces saturating convective and absolute-convective instabilities, which could make the beam even less stable than without SC"**



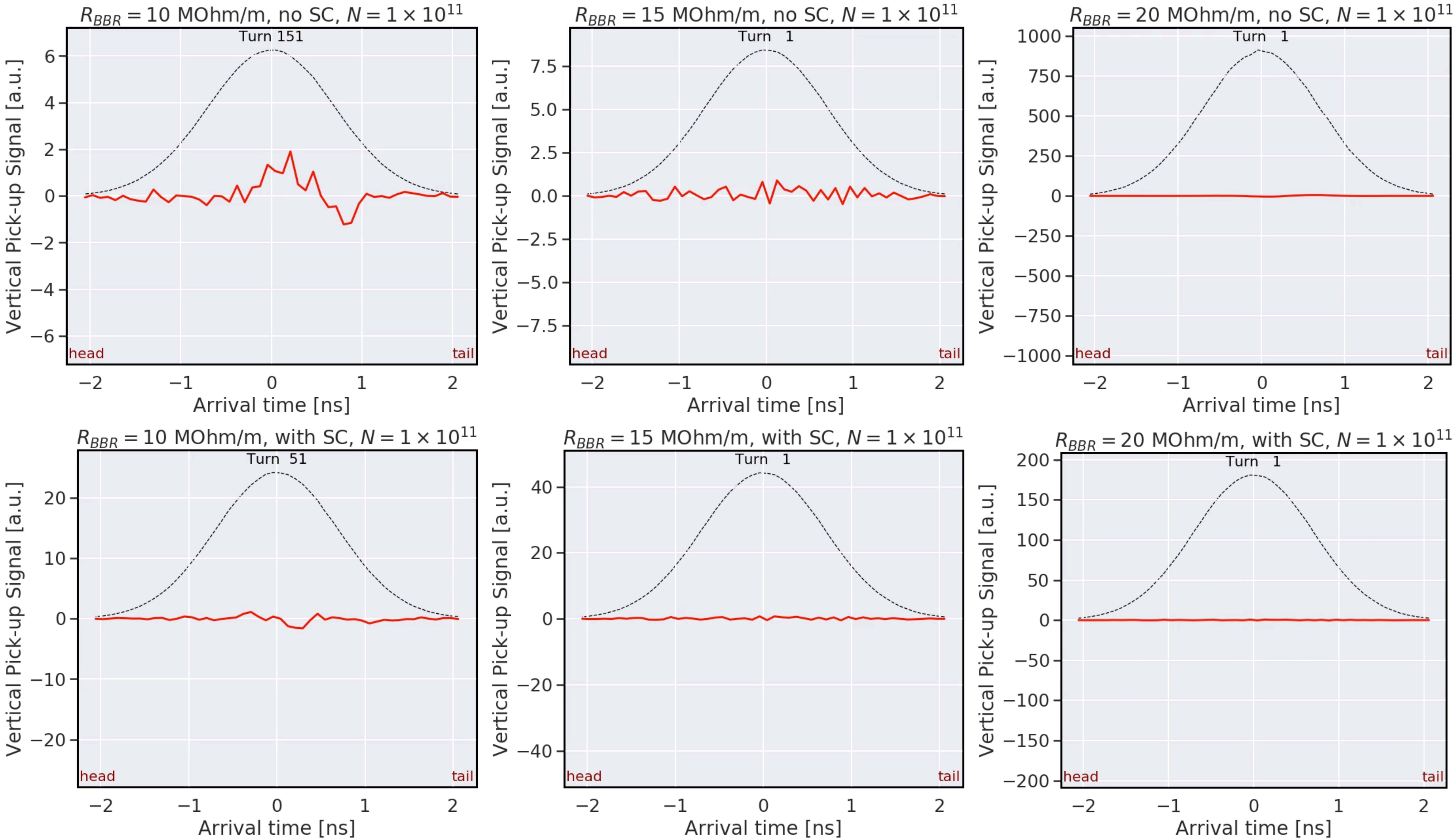
H. Bartosik et al.

Is SC responsible for this measured asymmetry between Head & Tail?

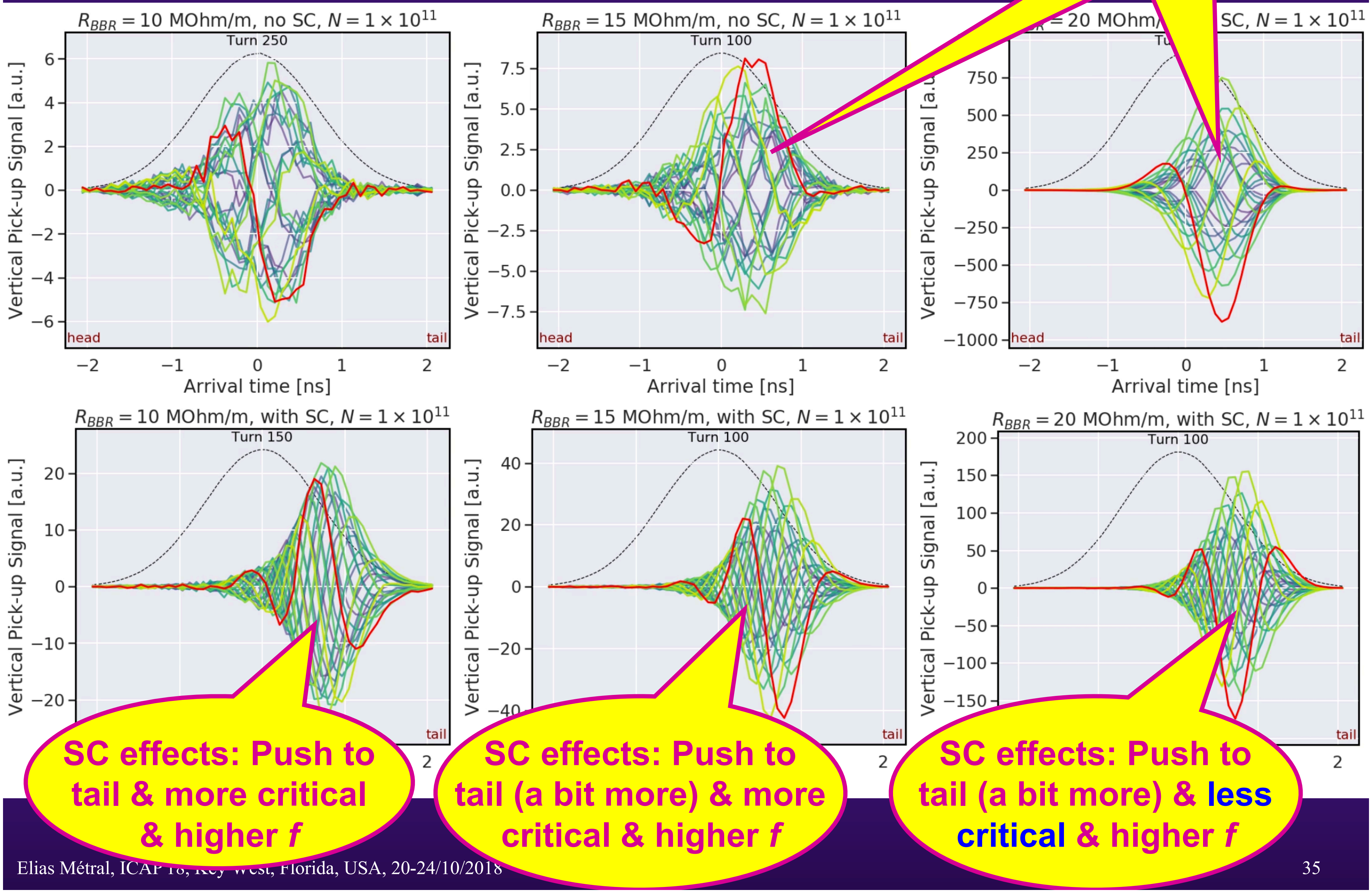
◆ New simulation results from A. Oeftiger for Q26



New simulation results from A. Oeftiger for Q26



◆ New simulation results from A. Oeftiger for Q26



- **Review of the 2-mode approach**
  - **Without SC**

Sacherer

$$\left| Q_s + \Delta Q_{m+1}^{S,y} - \Delta Q_m^{S,y} \right| = 2 \left| \Delta Q_{m,m+1}^{S,y} \right| \quad \Rightarrow \quad Q_s \approx 2 \left| \Delta Q_{m,m+1}^{S,y} \right|$$

~ 0

~ 0

Coasting-beam result with peak values (with Broad-Band)

Review of the 2-mode approach

- **Without SC**

Sacherer

$$\left| Q_s + \Delta Q_{m+1}^{S,y} - \Delta Q_m^{S,y} \right| = 2 \left| \Delta Q_{m,m+1}^{S,y} \right| \quad \Rightarrow \quad Q_s \approx 2 \left| \Delta Q_{m,m+1}^{S,y} \right|$$

~ 0 ~ 0

Coasting-beam result with peak values (with Broad-Band)

- **With SC (in the very "long-bunch" regime, as done in the past)**

$$\Delta Q^y_{m >> \frac{\Delta Q_{SC}}{2 Q_s}} \approx -\frac{\Delta Q_{SC}}{2} + m Q_s \quad \Rightarrow \quad \Delta Q_{m+1}^{S,y} - \Delta Q_m^{S,y} = \Delta Q_{m+1}^y - \Delta Q_m^y - Q_s \approx 0$$

Same result as before

- **Review of the 2-mode approach**

  - **Without SC**

Sacherer

$$\left| Q_s + \Delta Q_{m+1}^{S,y} - \Delta Q_m^{S,y} \right| = 2 \left| \Delta Q_{m,m+1}^{S,y} \right|$$

~ 0 ~ 0

=> $Q_s \approx 2 \left| \Delta Q_{m,m+1}^{S,y} \right|$

Coasting-beam result with peak values (with Broad-Band)

  - **With SC (in the very "long-bunch" regime, as done in the past)**

$$\Delta Q^y_{m >> \frac{\Delta Q_{SC}}{2 Q_s}} \approx - \frac{\Delta Q_{SC}}{2} + m Q_s$$

=> $\Delta Q_{m+1}^{S,y} - \Delta Q_m^{S,y} = \Delta Q_{m+1}^{y} - \Delta Q_m^{y} - Q_s \approx 0$

Same result as before

  - **With SC (in general case, new result)**

$q = | m | + 2 k$

$q_{sc} = \Delta Q_{SC} \; / \; (2 \, Q_s)$

$$Q_s + \Delta Q_{q+1}^{S,y} - \Delta Q_q^{S,y} = Q_s \left[\sqrt{q_{sc}^2 + (q+1)^2} - \sqrt{q_{sc}^2 + q^2} \right]$$

=> $\boxed{Q_s \left[\sqrt{q_{sc}^2 + (q+1)^2} - \sqrt{q_{sc}^2 + q^2} \right] = 2 \left| \Delta Q_{q,q+1}^{S,y} \right|}$

Same result as without SC => Only Q_s-term is reduced by SC

6^{th} studies: new measurement campaign planned…

- To try and disentangle between the impedance effect and the space charge effect => Varying the space charge tune spread (by varying the transverse emittances), etc.

6^{th} studies: new measurement campaign planned…

- To try and disentangle between the impedance effect and the space charge effect => Varying the space charge tune spread (by varying the transverse emittances), etc.

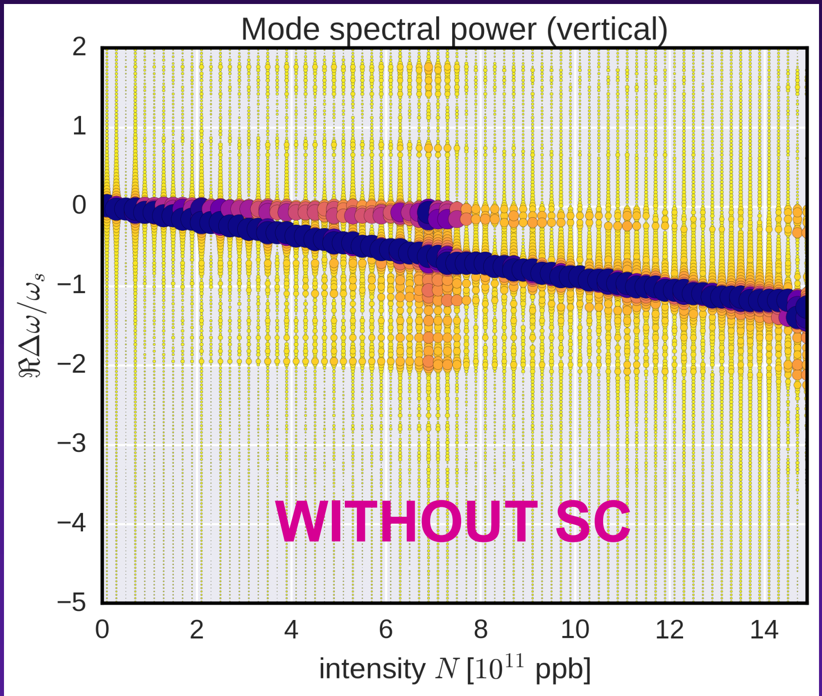
Conclusion for the SPS

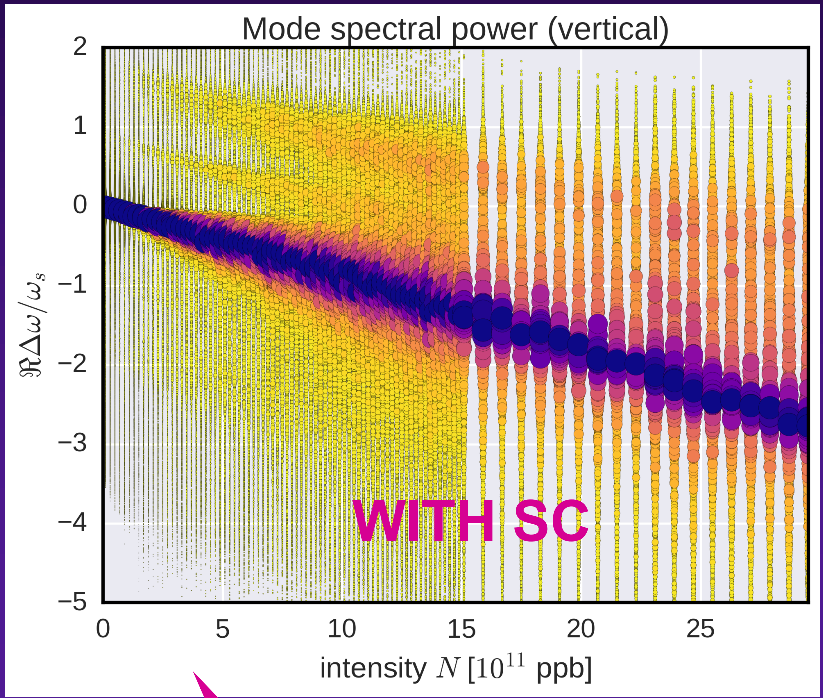
- We are not there yet for the “full understanding” => But we should be close now: new simulations with SC and full impedance model should be done soon and compared to the new measurements planned…
- A solution was found in practice for this instability in the SPS by increasing the slip factor (i.e. going farther away from transition)

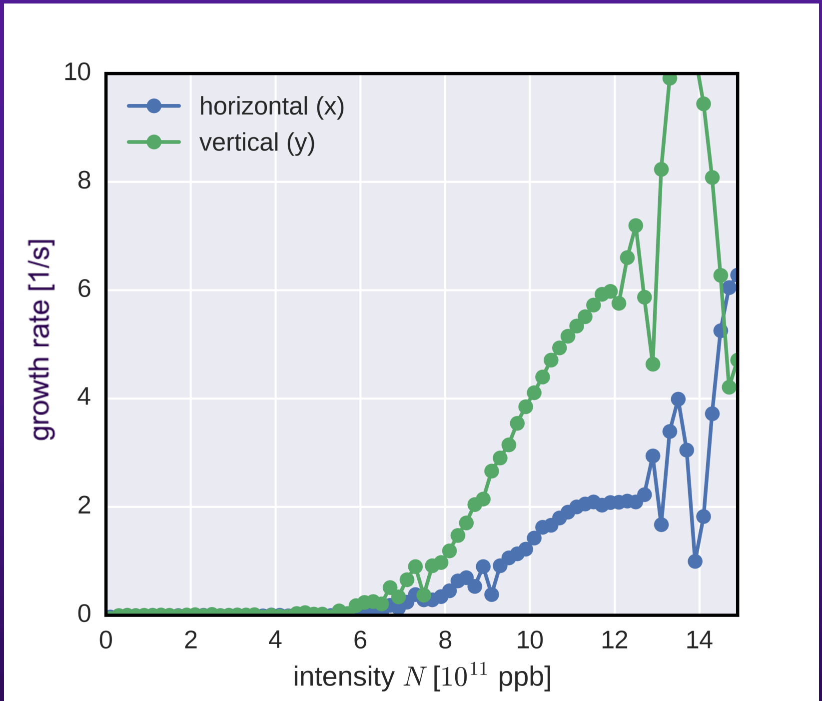
LHC

pyHEADTAIL SIMULATION WITH SC FOR (HL-) LHC TMCI (Q’ = 0)

=> Using the real impedance model







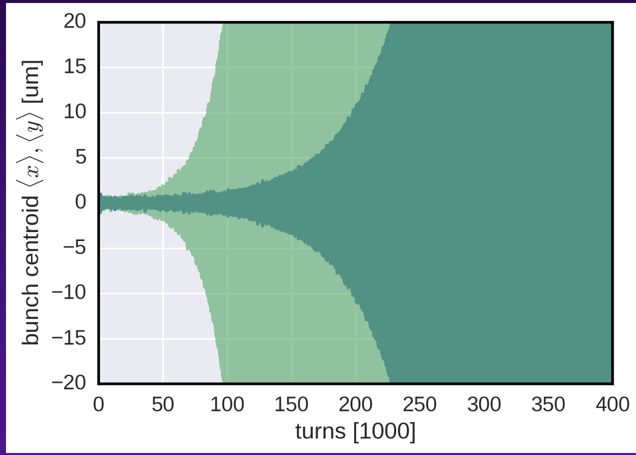
No instability anymore with SC

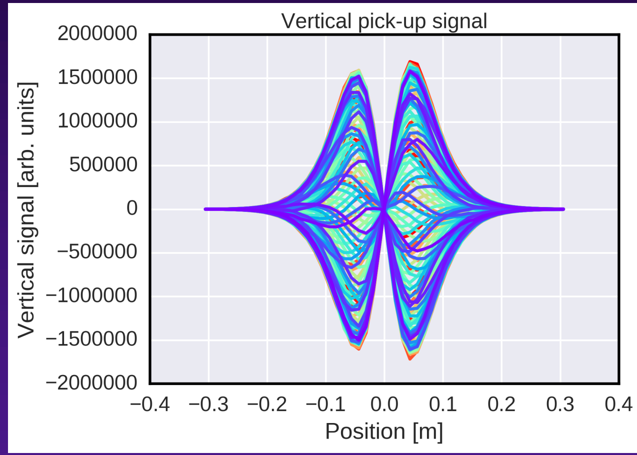
A. Oeftiger

pyHEADTAIL SIMULATION WITH SC FOR (HL-) LHC HEAD-TAIL INSTABILITY (Q' = 5)

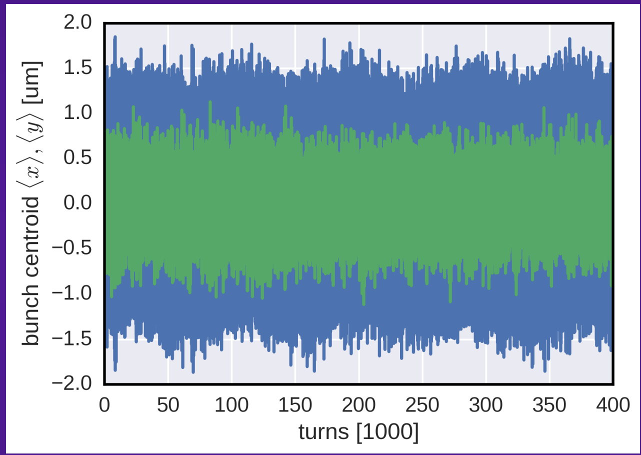
=> Using the real impedance model

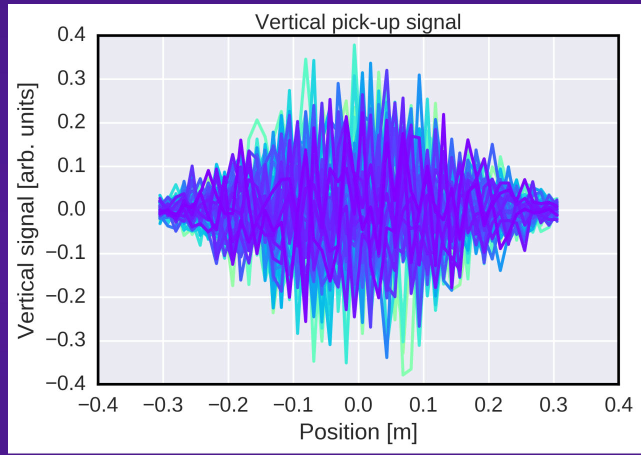
- **Impedance only**



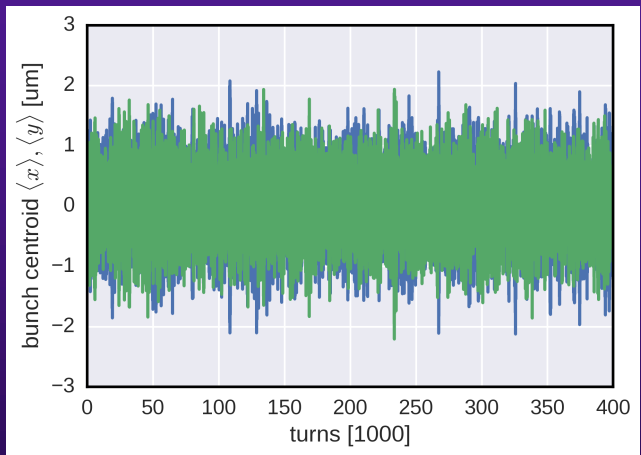


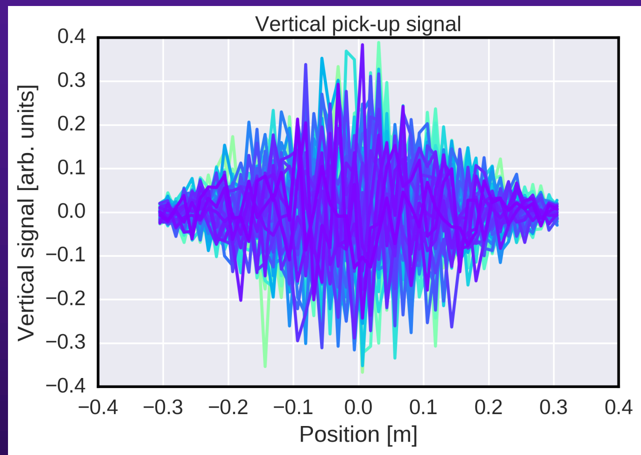
- **SC only**





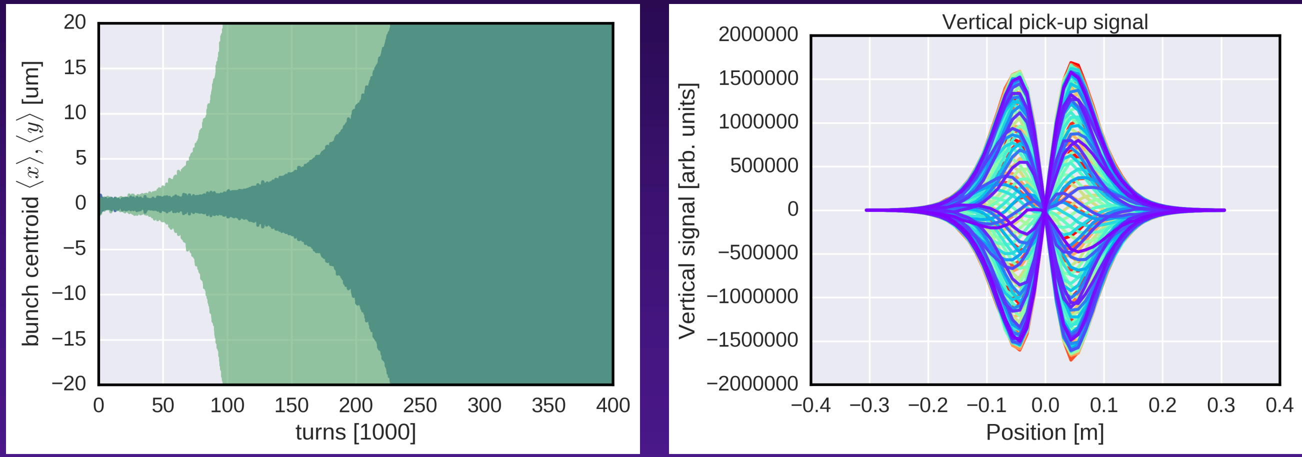
- **Impedance + SC**



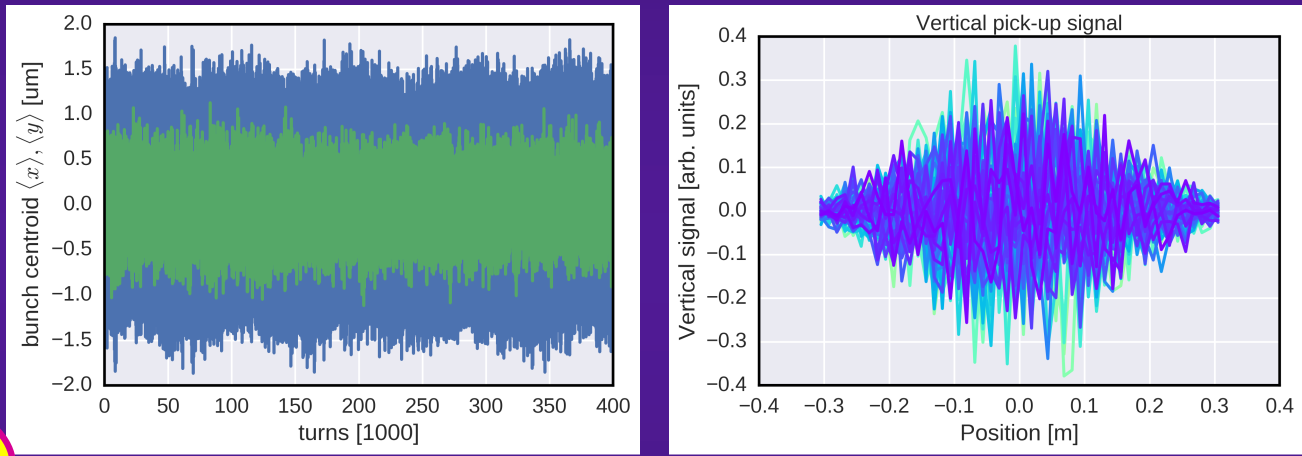


A. Oeftiger

- Impedance only

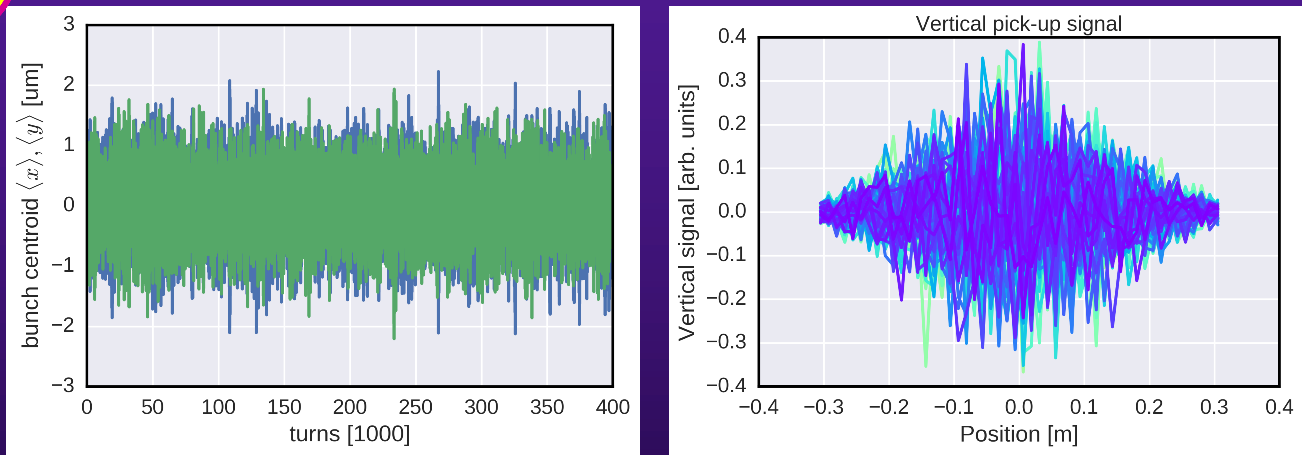


- SC only



SC stabilizes the Head-Tail instability

- Impedance + SC

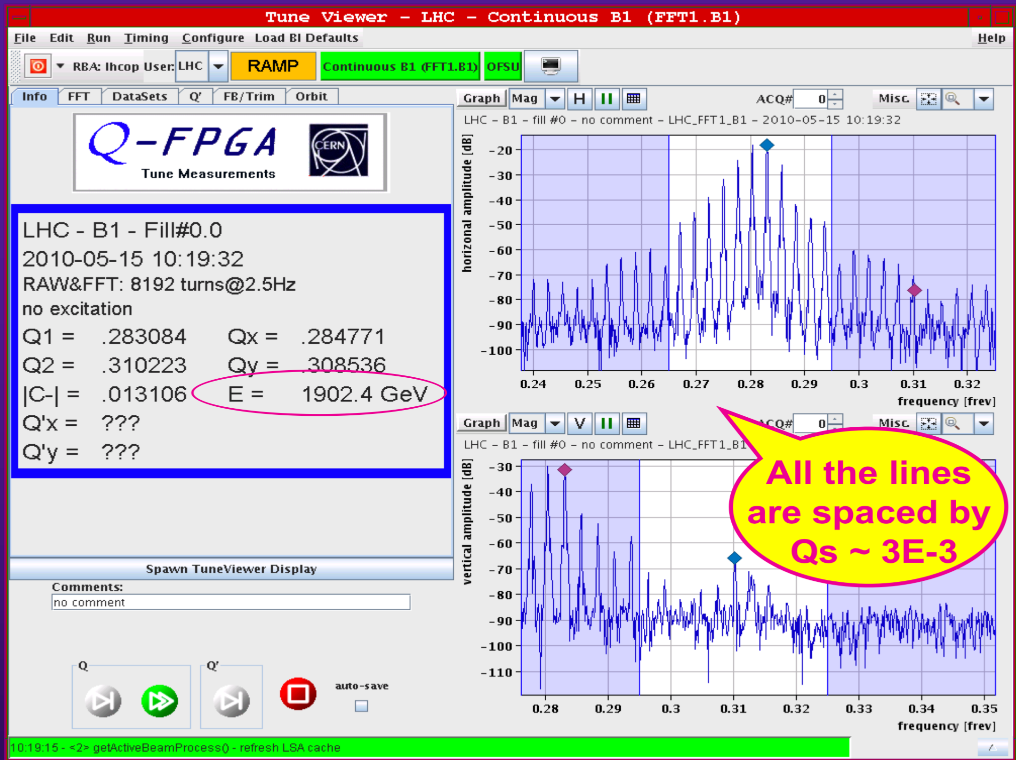


A. Oeftiger

- Studying the effect of energy during the ramp, which reduces the SC tune spread (by increasing the transverse emittances at injection energy), the instability re-appears at ~ 2 TeV…

- **Studying the effect of energy during the ramp, which reduces the SC tune spread (by increasing the transverse emittances at injection energy), the instability re-appears at ~ 2 TeV…**

- **… ~ 2 TeV is the energy at which the 1st transverse single-bunch instability was observed in the LHC during the 1st ramp performed in 2010 with neither Landau octupoles nor transverse damper (see https://accelconf.web.cern.ch/accelconf/IPAC2011/papers/mops074.pdf)**



CONCLUSIONS

- **Beneficial effect of SC in the LHC (“short-bunch” regime)**
  - SC simulation with pyHEADTAIL (2.5D PIC code from A. Oeftiger) gives an explanation of 1st single-bunch Head-Tail instability observed in LHC during 1st ramp in 2010 with neither Landau octupoles nor transverse damper => Might be good to re-do a controlled experiment to check / confirm…
  - SC simulation also predicts that SC increases significantly the TMCI intensity threshold (Q’ = 0) at (HL-) LHC injection => TMCI currently out of reach in LHC

CONCLUSIONS

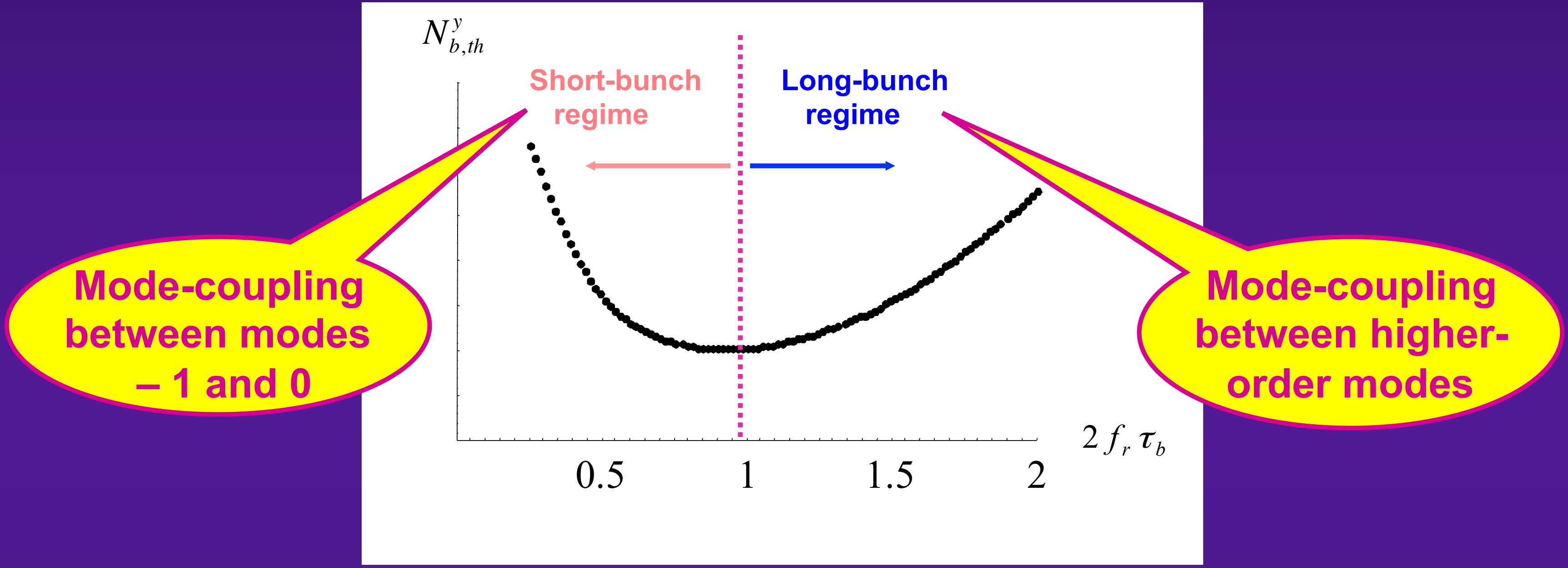
- **Beneficial effect of SC in the LHC ("short-bunch" regime)**
  - SC simulation with pyHEADTAIL (2.5D PIC code from A. Oeftiger) gives an explanation of 1st single-bunch Head-Tail instability observed in LHC during 1st ramp in 2010 with neither Landau octupoles nor transverse damper => Might be good to re-do a controlled experiment to check / confirm…
  - SC simulation also predicts that SC increases significantly the TMCI intensity threshold (Q' = 0) at (HL-) LHC injection => TMCI currently out of reach in LHC

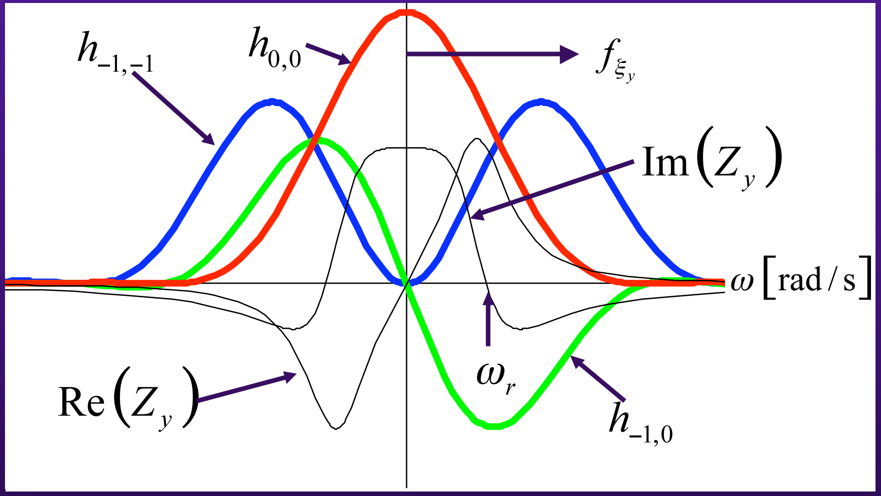
- **Small? / detrimental? effect of SC in the SPS ("long-bunch" regime)**
  - Several past measurements quite close to case without SC
  - The intensity threshold was increased considerably in practice by increasing the slip factor (based on theoretical analysis without SC) => Works very well: Q20 optics has replaced Q26 optics
  - However, a recent theoretical analysis (from A. Burov) predicts a detrimental effect of SC (even below the TMCI intensity threshold without SC)
    - Confirmed by SC simulations with Q26 (from A. Oeftiger) and simple 2-mode approach (same scaling as without SC, only Q_s-term reduced by SC)
    - To be looked at in more detail during a future measurement campaign…

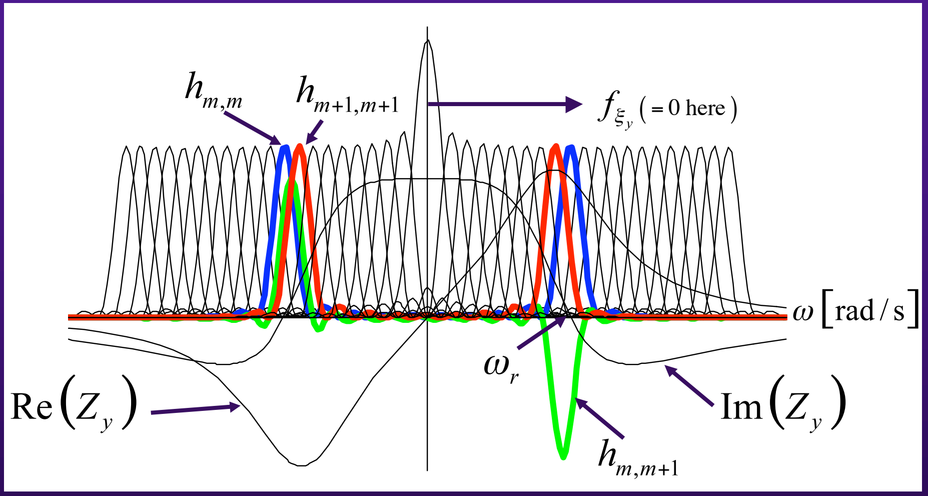
APPENDIX

2 – MODE MODEL FOR TMCI (Q' = 0) WITH SC AND/OR ReaD

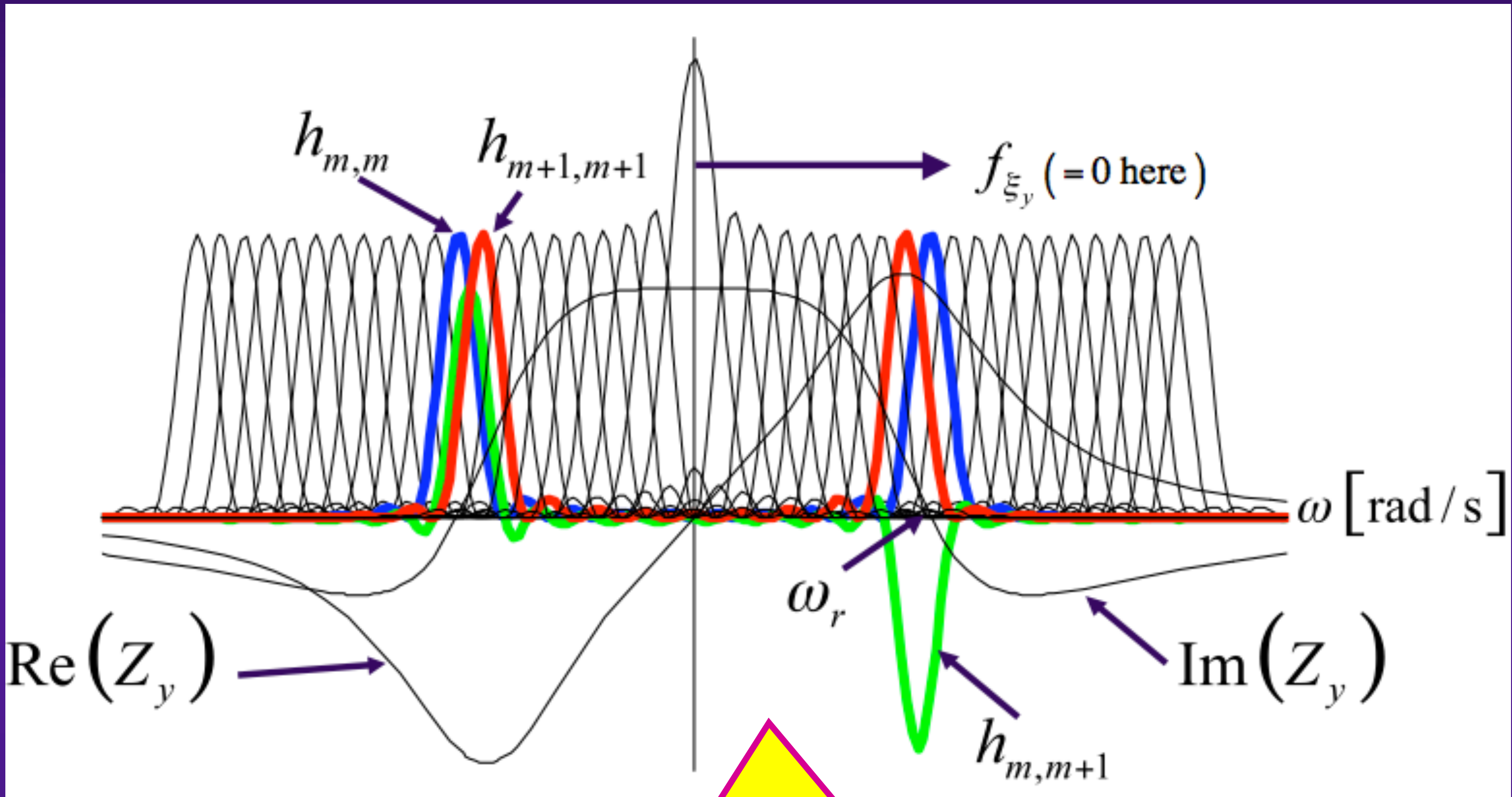
- **Broad-Band impedance with neither SC nor ReaD**







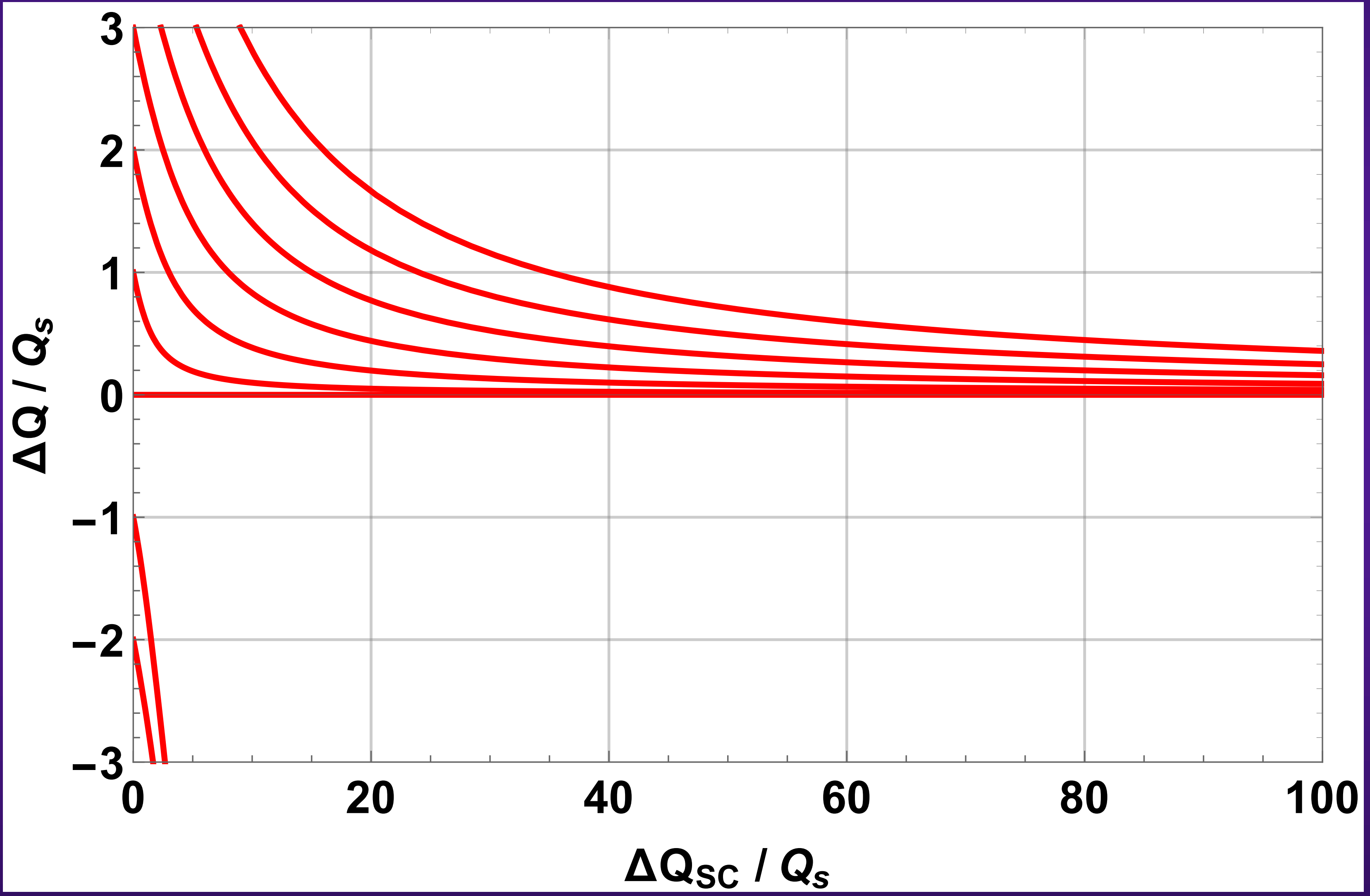
2 – MODE MODEL FOR TMCI (Q' = 0) WITH SC AND/OR ReaD



REMINDER: what defines the eigenvectors is $q = |\, m \,| + 2\, k$ (assumed in this plot to be equal to *m*) but not *m*! => In particular the simple 2-mode approach does not depend on the sign of *m*!

2 – MODE MODEL FOR TMCI (Q' = 0) WITH SC AND/OR ReaD

- **SC ONLY (square-well air-bag, Blaskiewicz1998)**



VLASOV SOLVER: GALACTIC WITH ReaD

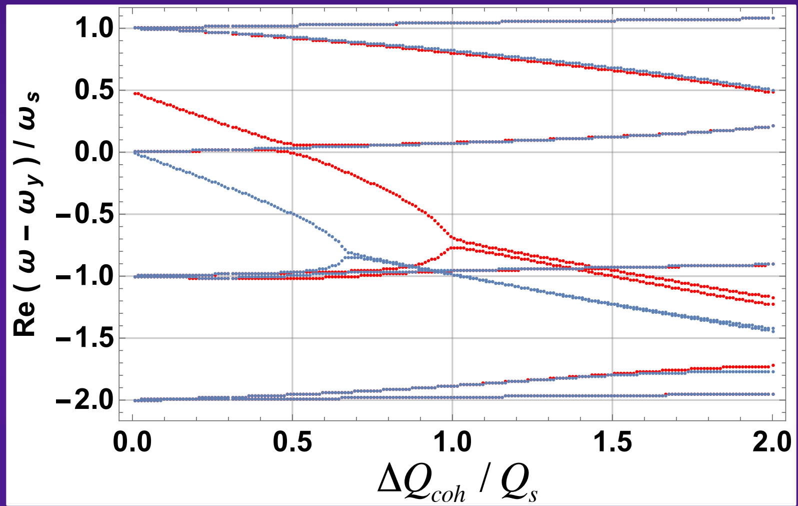
=> Application to LHC and SPS assuming a Broad-Band impedance

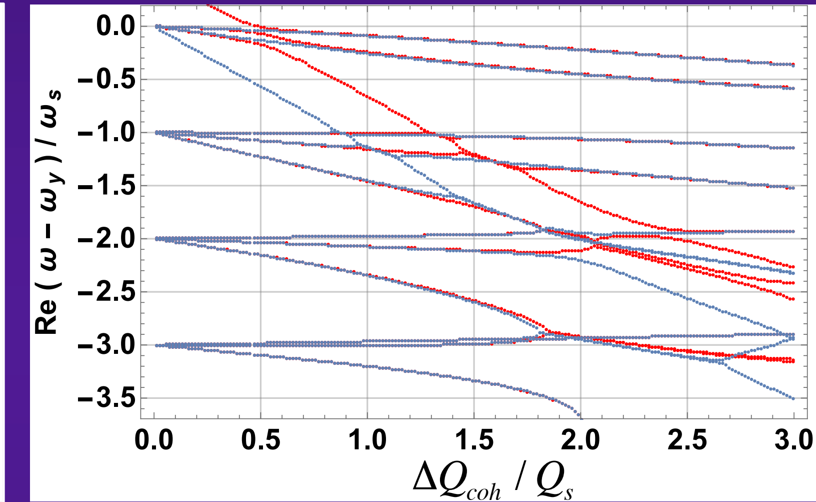
~ LHC case (Q' = 0) – No SC

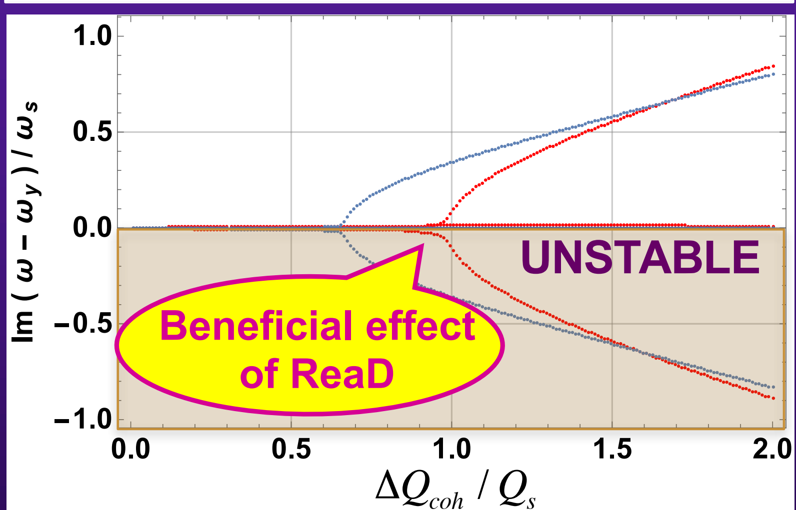
Without / with ReaD (50 turns) in blue / red

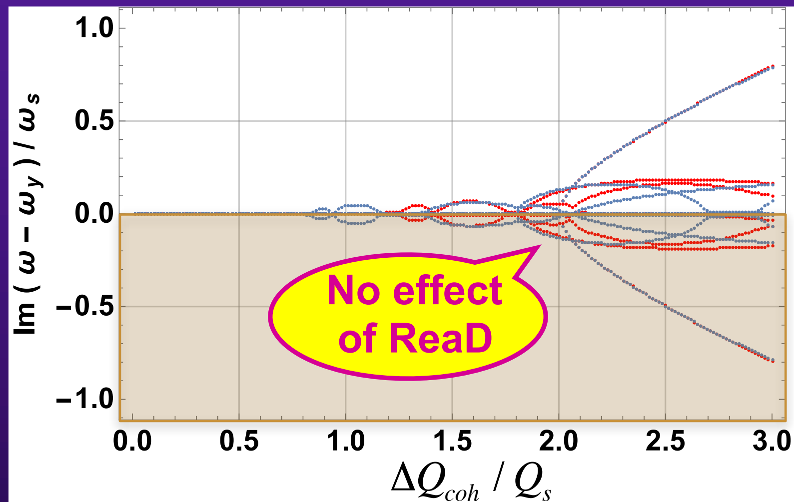
~ SPS case (Q' = 0) – No SC

Without / with ReaD (50 turns) in blue / red

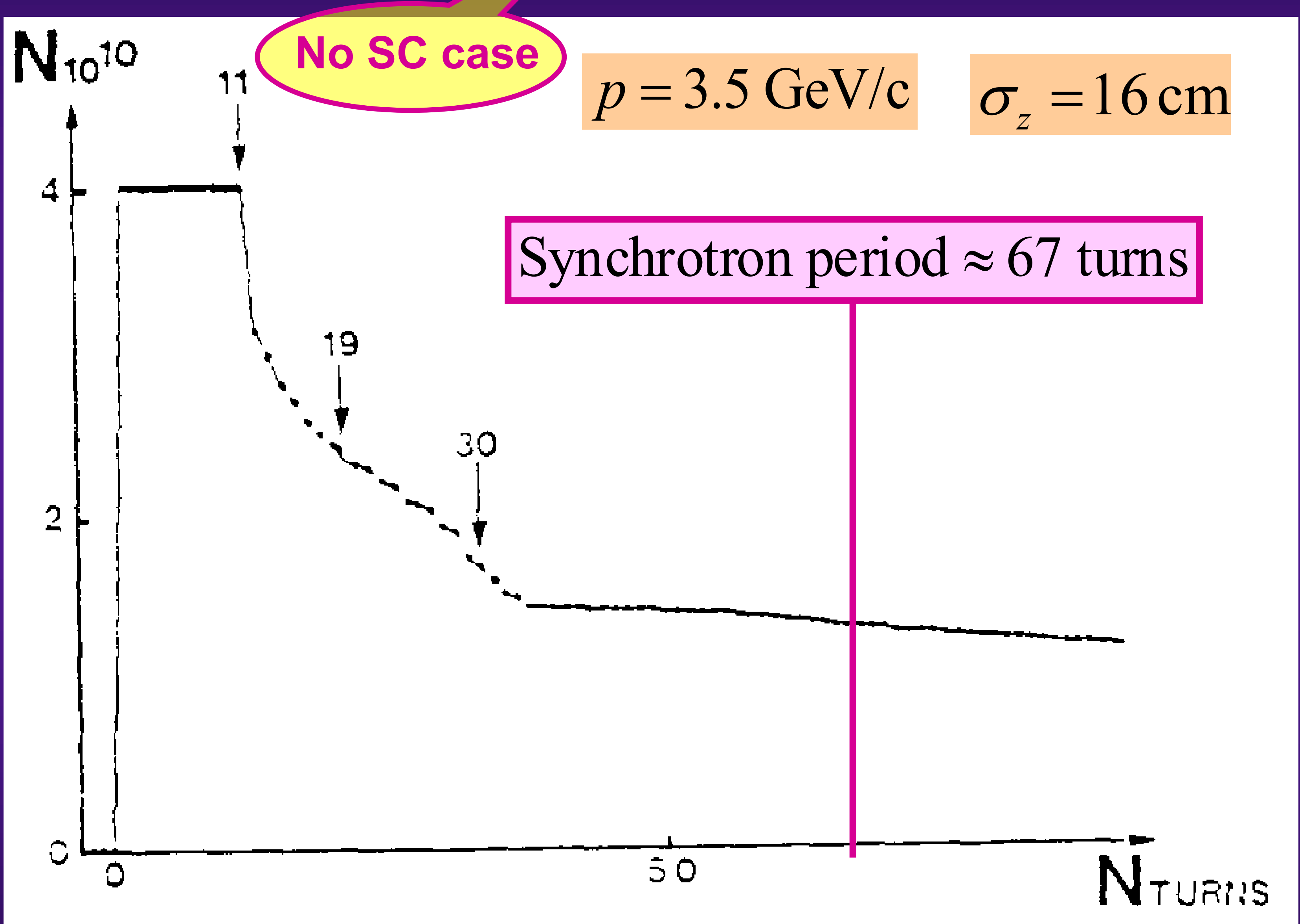








Fast instability of e⁺ bunches in the SPS
⇒ Gareyte & Brandt in 1988 (BBU analysis)



Fast instability of e+ bunches in the SPS ⇒ Gareyte & Brandt in 1988 (BBU analysis)

- **Yokoya's BBU theory for linacs (many bunches):** Cumulative Beam Breakup in Large-Scale Linacs, DESY 86-084, 1986 (https://lib-extopc.kek.jp/preprints/PDF/1986/8609/8609117.pdf)

$$x(s,t) = \frac{1}{2\sqrt{2\pi}} \sqrt{\frac{\gamma_0 k_0}{\gamma(s) k(s)}} \left(\frac{\Omega(s)}{t^3}\right)^{1/4} \Re\tilde{y}(0, -\varepsilon - i|\omega_0|) e^{-\varepsilon t - i|\omega_0|t + i\psi(s) + \sqrt{\Omega(s)t}}. \tag{3.22}$$

For the initial condition (2.16) and (2.17), we get

$$X(s,t) = \frac{X_0}{2\sqrt{2\pi}} \sqrt{\frac{\gamma_0 k_0}{\gamma k}} (\Omega(s)t)^{1/4} \frac{t_b}{t} e^{-\varepsilon t + \sqrt{\Omega(s)t}} \left\{ \begin{array}{c} \left[(e^{\varepsilon t_b} - 1)^2 + 4e^{\varepsilon t_b} (\sin \frac{\omega_0 t_b}{2})^2 \right]^{-1/2} \\ 1 \end{array} \right\} \tag{3.24}$$

where the upper and lower part in the curly bracket correspond to (2.16) and (2.17), respec-

For instance, when every bunch is injected with the same offset X_0 without slope, (2.15) gives

$$y(0,p) = \frac{X_0 t_b}{1 - e^{-p t_b}} \qquad \text{and} \qquad y'(0,p) = 0. \tag{2.16}$$

If only the first bunch is displaced by X_0 and the others are on the axis, we have

$$y(0,p) = X_0 t_b \qquad \text{and} \qquad y'(0,p) = 0. \tag{2.17}$$

Fast instability of e+ bunches in the SPS ⇒ Gareyte & Brandt in 1988 (BBU analysis)

- **Extension of Yokoya's BBU theory to synchrotrons (1 bunch):** D. Brandt, J. Gareyte, Fast Instability of Positron Bunches in the CERN SPS, CERN SPS/88-17 (AMS) (http://accelconf.web.cern.ch/accelconf/e88/PDF/EPAC1988_0690.PDF)
  - The long bunch is assumed to be made of many bunchlets so that the time between the bunchlets (t_b) is small compared to the decay time of the impedance and the oscillation period

= 1 <=> No acceleration

= 1 / $(\omega_0 t_b)$

For the initial condition (2.16) and (2.17), we get

$$X(s,t) = \frac{X_0}{2\sqrt{2\pi}}\sqrt{\frac{\gamma_0 k_0}{\gamma k}}\left(\Omega(s)t\right)^{1/4}\frac{t_b}{t}e^{-\varepsilon t+\sqrt{\Omega(s)t}}\left\{\begin{array}{c}\left[\left(e^{\varepsilon t_b}-1\right)^2+4e^{\varepsilon t_b}\left(\sin\frac{\omega_0 t_b}{2}\right)^2\right]^{-1/2} \\ 1\end{array}\right\} \tag{3.24}$$

where the upper and lower part in the curly bracket correspond to (2.16) and (2.17), respec-

Fast instability of e^+ bunches in the SPS $\Rightarrow$ Gareyte & Brandt in 1988 (BBU analysis)

- **Used in the past also for the PS at transition:** R. Cappi et al., Beam Break-Up Instability in the CERN PS at Transition (https://accelconf.web.cern.ch/accelconf/e00/PAPERS/WEP4A07.pdf)

Tail of the bunch at turn *n*

$$\frac{u_n}{u_0} = \frac{1}{2\sqrt{2\pi}} \times \left(\frac{\Omega L}{\beta c}\right)^{1/4} \times \frac{\beta c}{L \omega_r} \times e^{\frac{-\varepsilon L}{\beta c} + \sqrt{\frac{\Omega L}{\beta c}}}, \quad (1)$$

with

$$\frac{\Omega L}{\beta c} = \frac{N_b \, e \, \beta c}{\omega_u \left(E / e\right)} \times \frac{\omega_r \, R_{r,u}}{Q_r} \times n \, . \quad (2)$$

Be careful: there is a typo here in the paper from Gareyte & Brandt

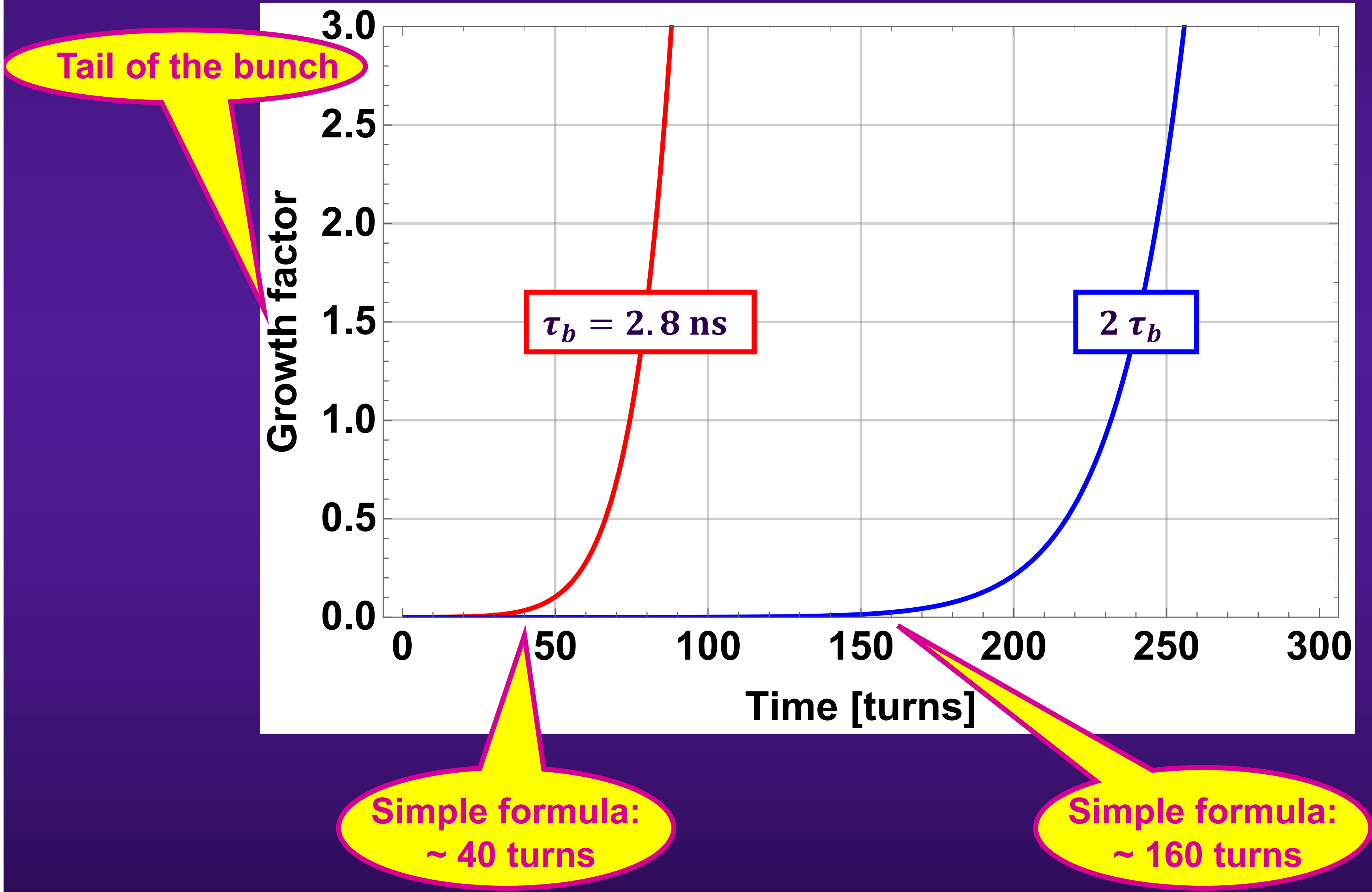
- Furthermore, a simple formula was given for the time when the growth starts (same as the instability rise-time deduced from simple model for TMCI, within a numerical factor => See later)

$$= \frac{\tau_{\mathrm{TMCI}}^{\mathrm{sm}}}{4}$$

$$\Delta t = T_0 \times \frac{\omega_u \left(E / e\right) \tau_b^2}{4 N_b \, e \, \beta c} \times \frac{\omega_r}{R_{r,u} \, Q_r}, \quad (3)$$

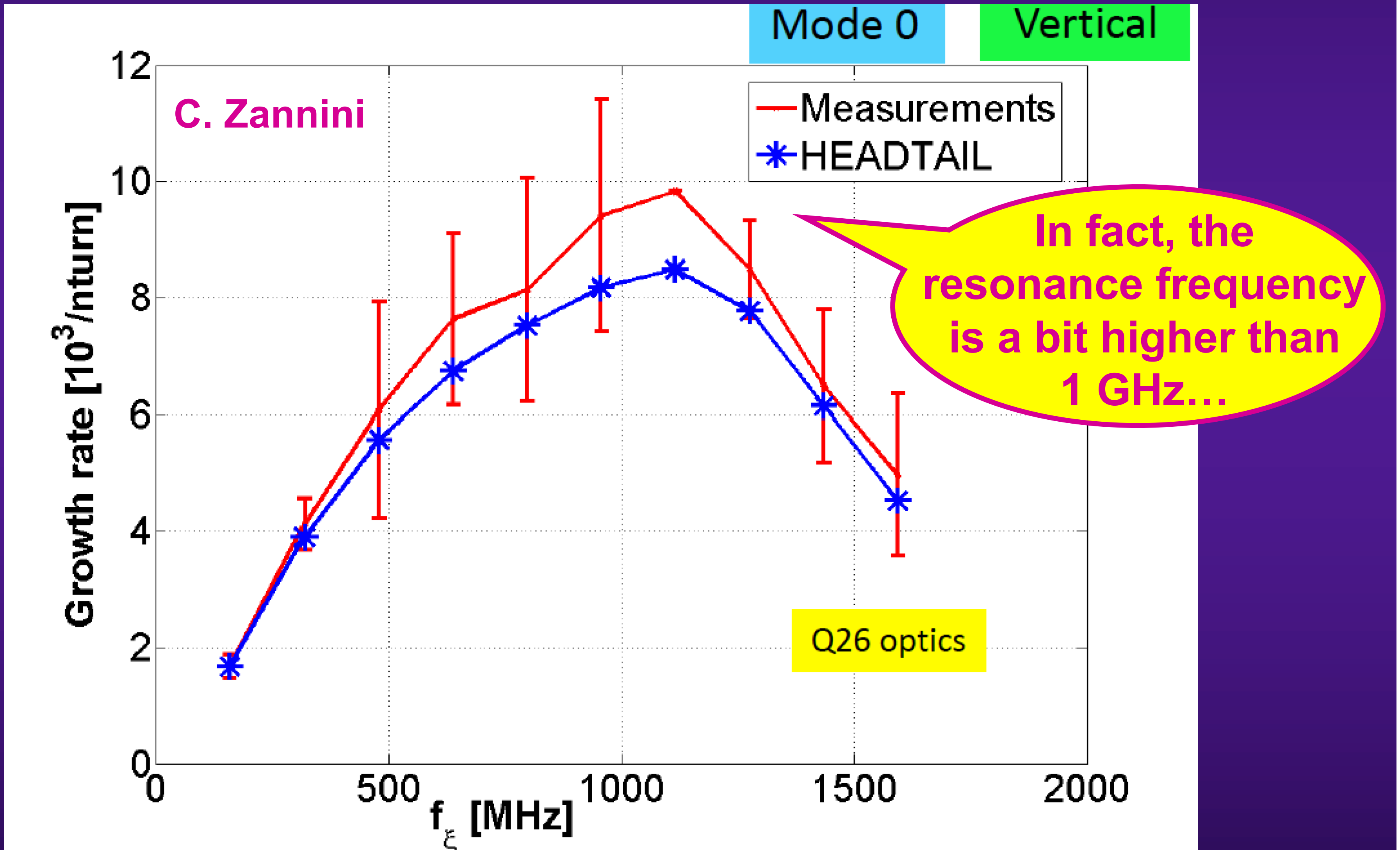
Fast instability of e^+ bunches in the SPS
$\Rightarrow$ Gareyte & Brandt in 1988 (BBU analysis)

- **Application to the case of SPS with p^+ and Q26** (same parameters as 2003)



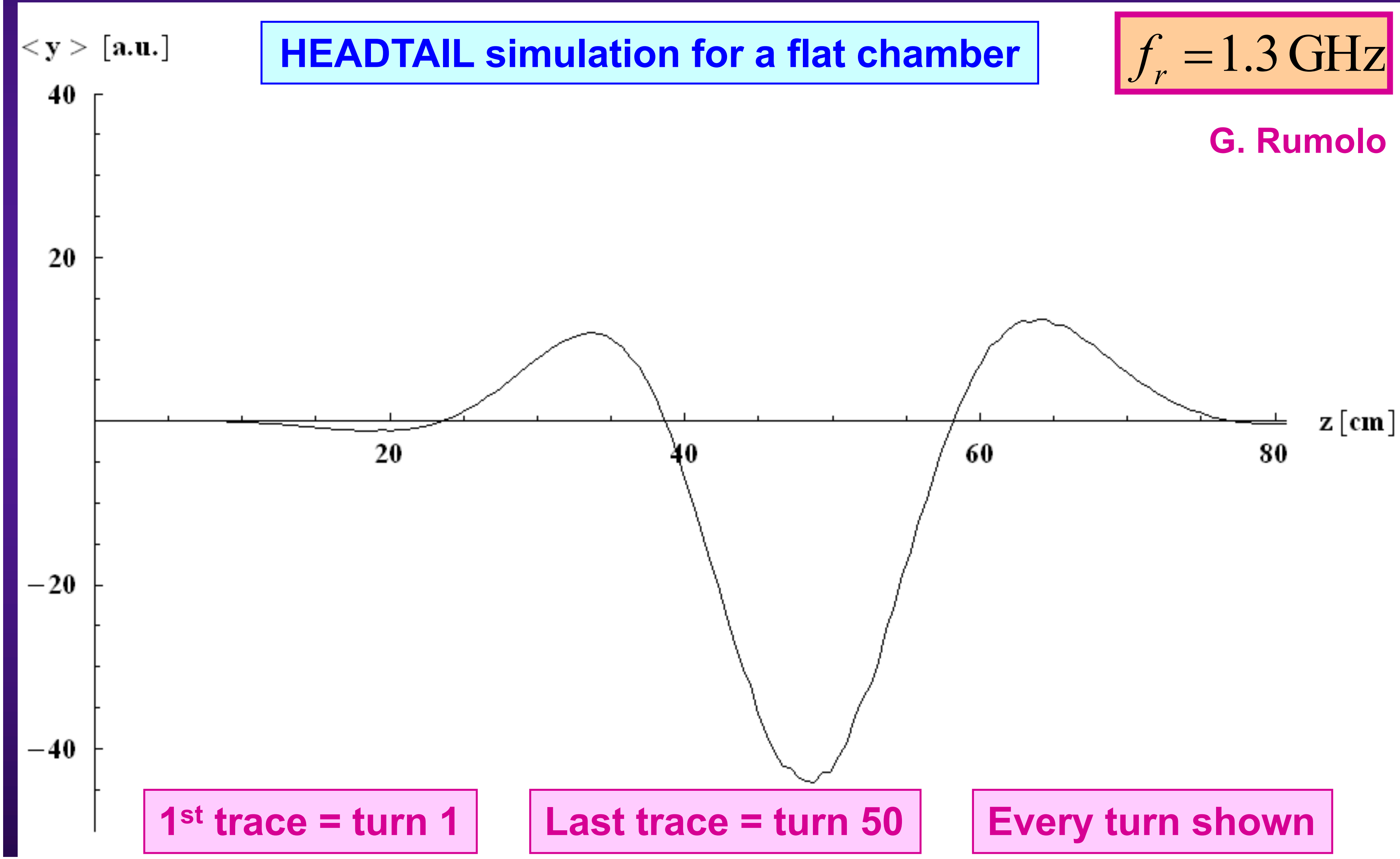
2^{nd} simulation studies (B. Salvant)…

- By the way, why should/could a Broad-Band resonator model be a good first approximation?

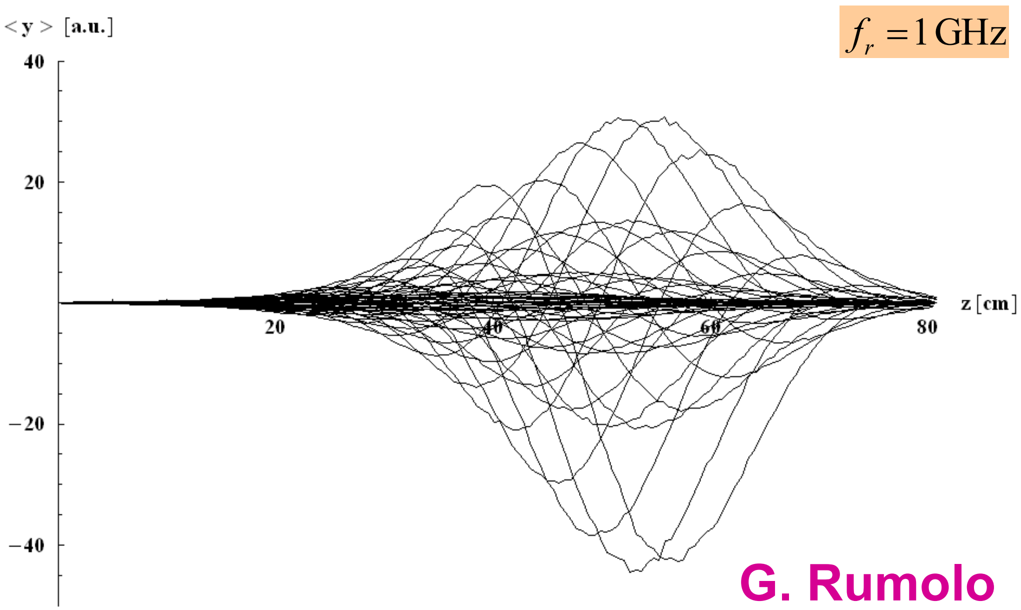


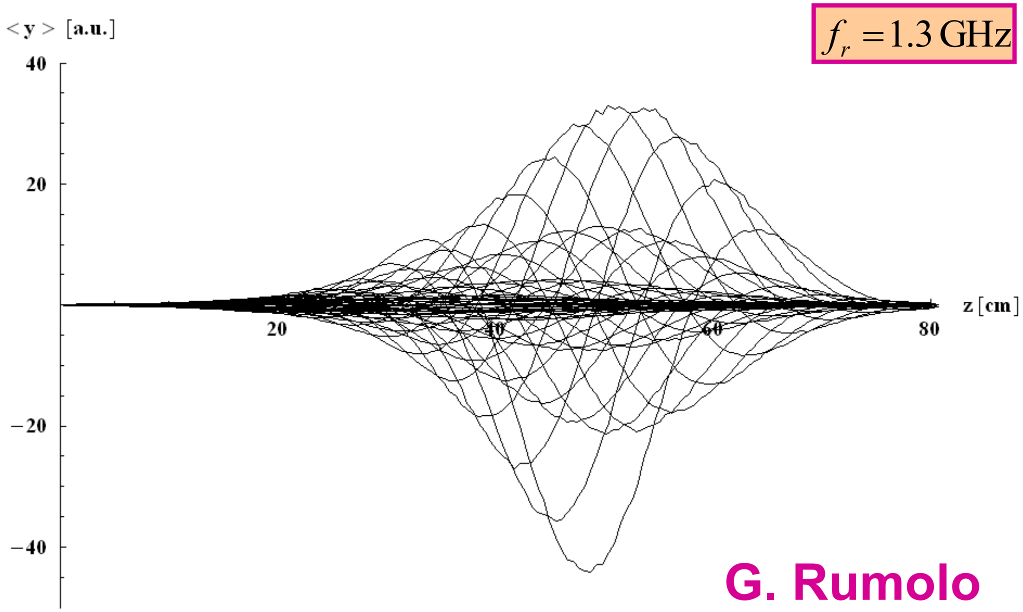
2^{nd} simulation studies (B. Salvant)…

- **However, 1^{st} simulations revealed only minor role** (for 1^{st} analyses…)



2nd simulation studies (B. Salvant)…

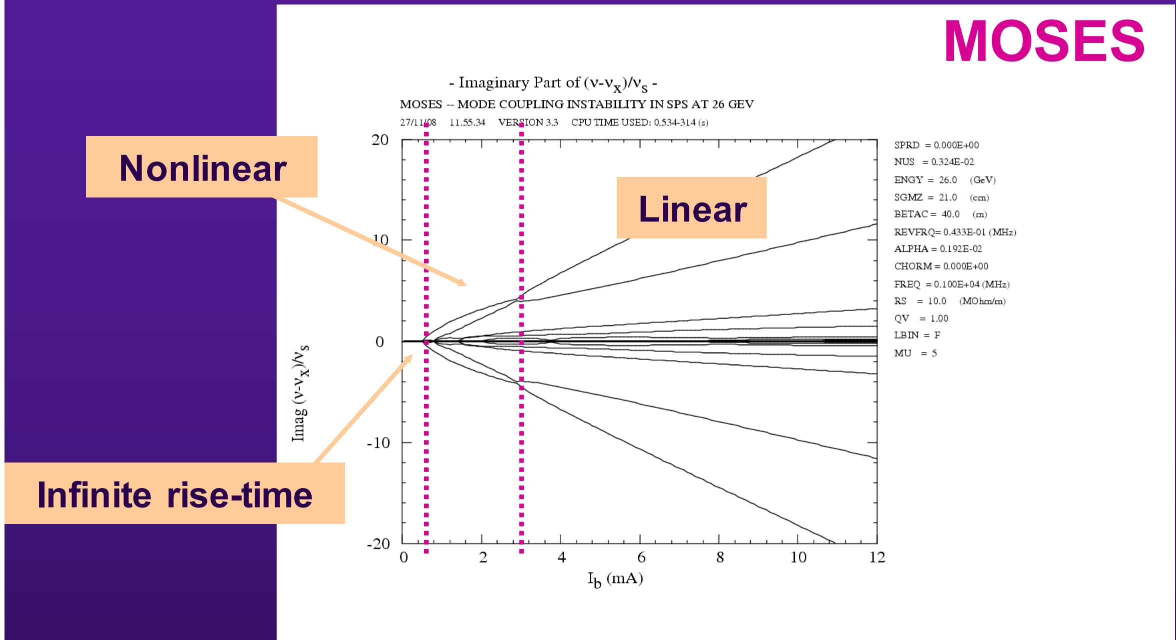


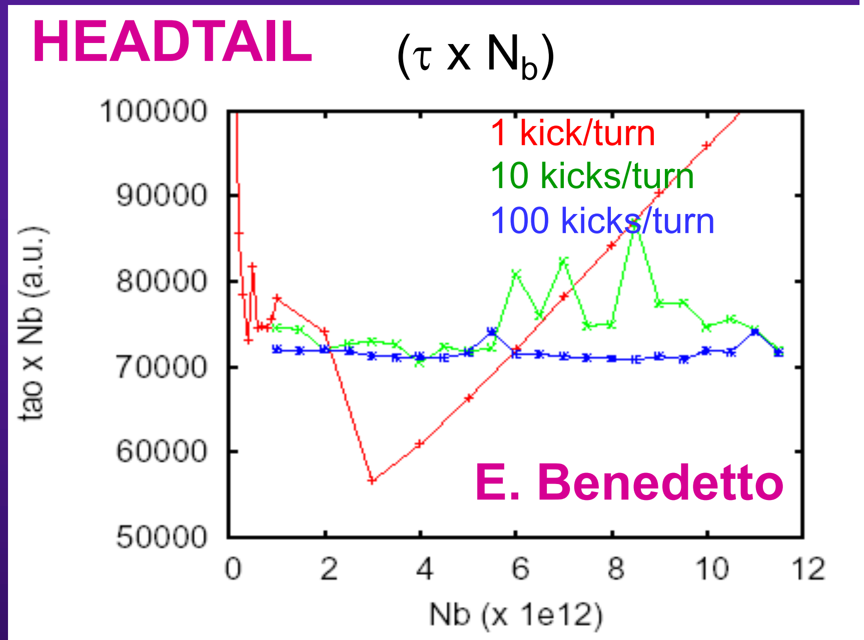


4th studies: Increasing the intensity threshold by increasing the slip factor (distance to transition)

- **By the way, the simple formula giving the instability rise-time well above TMCI threshold (which was checked with MOSES and HEADTAIL, within the same factor 2 as before => See http://www-linux.gsi.de/~boine/CERN-GSI-2009/benedetto.ppt) can be written as**

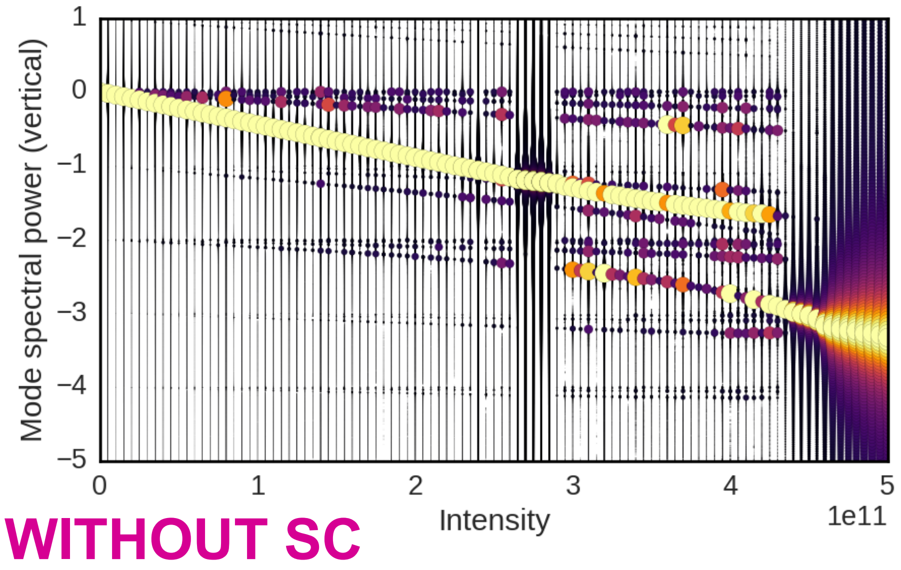
$$\tau_{\mathrm{TMCI}}^{\mathrm{sm}} = \frac{T_s}{\pi} \times \frac{N_{b,th}}{N_b}$$

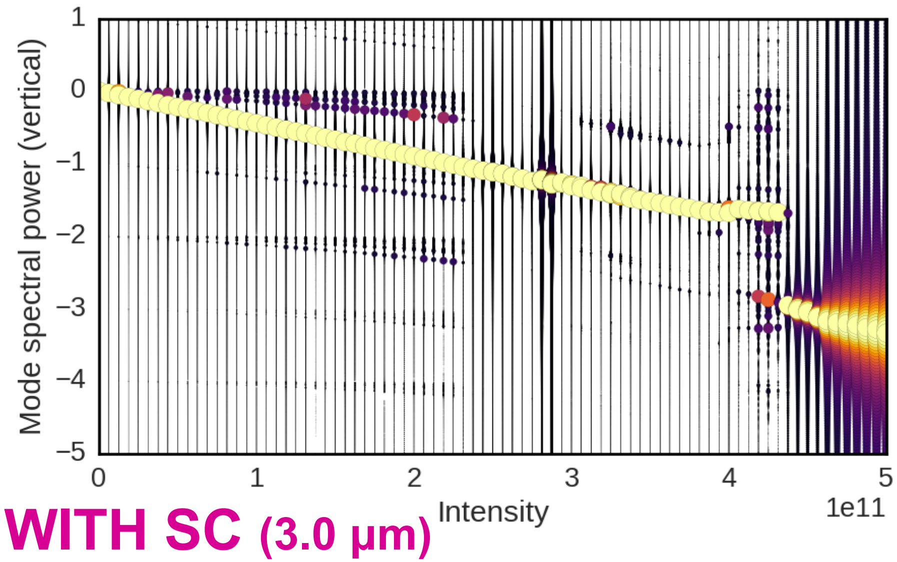


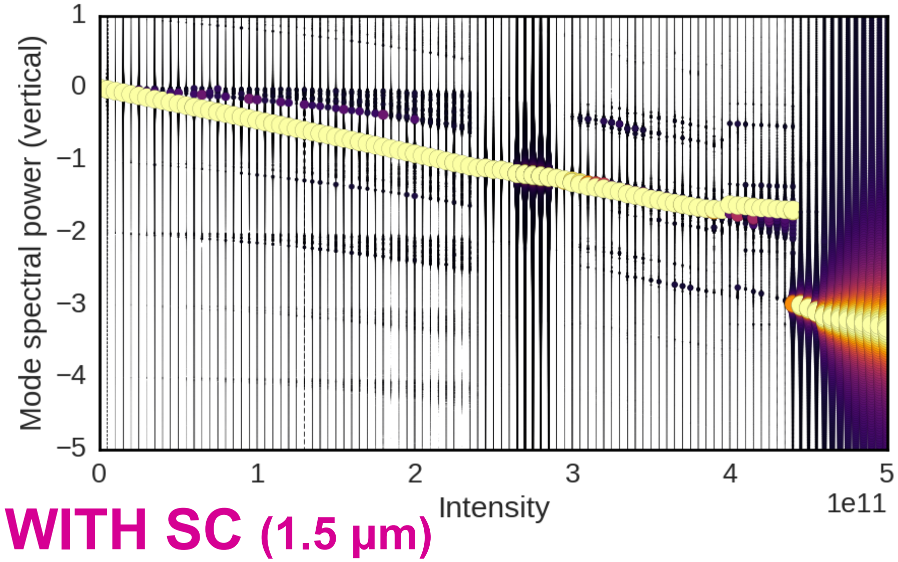


4th studies: Increasing the intensity threshold by increasing the slip factor (distance to transition)

- **Good agreement also with past pyHEADTAIL simulations** using a frozen SC model **for Q20 (considering the Broad-Band resonator model)**

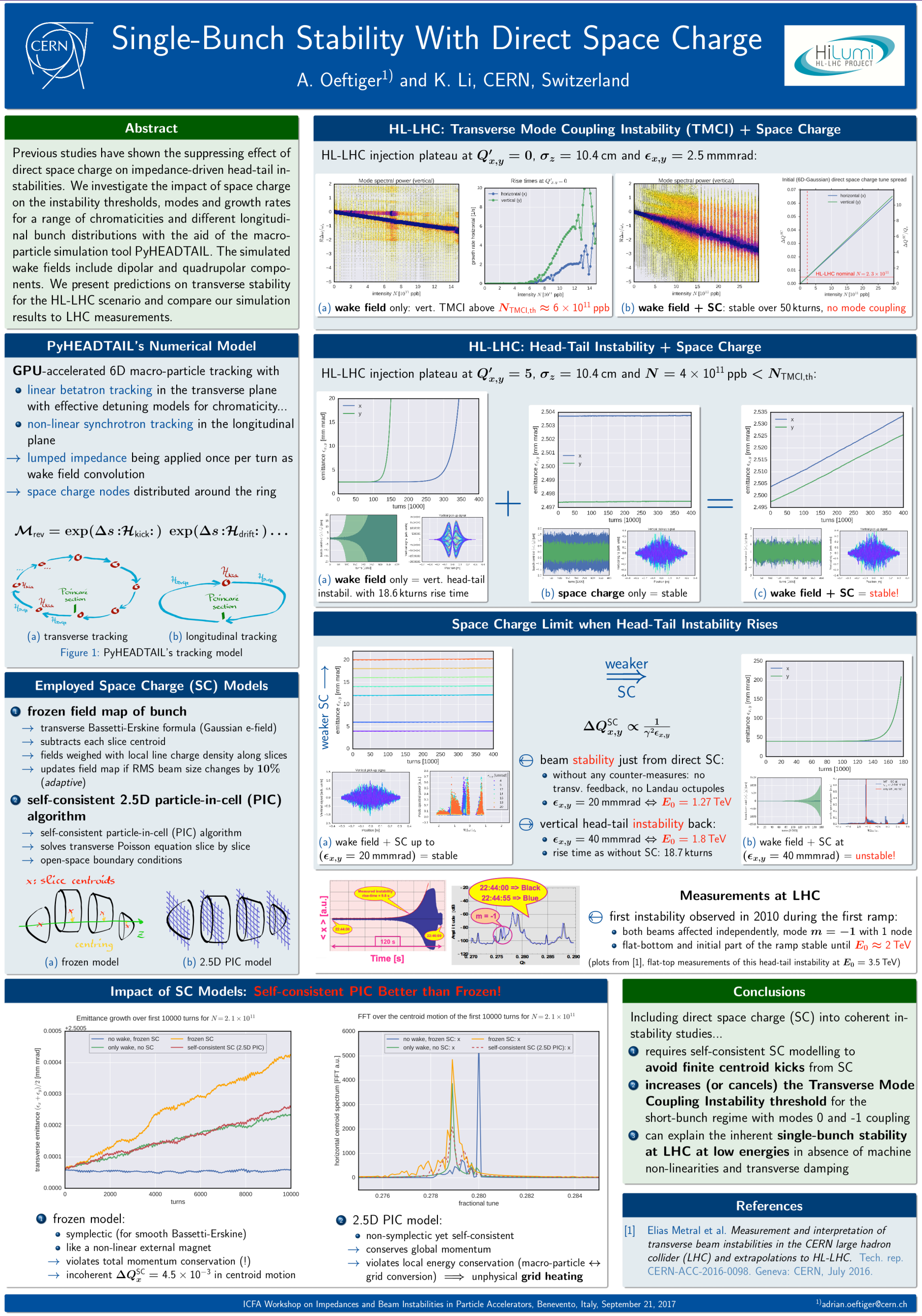






K. Li

SC simulations for both LHC (left) and SPS (right)



Single-Bunch Stability With Direct Space Charge

A. Oeftiger[1] and K. Li, CERN, Switzerland

Abstract

Previous studies have shown the suppressing effect of direct space charge on impedance-driven head-tail instabilities. We investigate the impact of space charge on the instability thresholds, modes and growth rates for a range of chromaticities and different longitudinal bunch distributions with the aid of the macro-particle simulation tool PyHEADTAIL. The simulated wake fields include dipolar and quadrupolar components. We present predictions on transverse stability for the HL-LHC scenario and compare our simulation results to LHC measurements.

PyHEADTAIL's Numerical Model

GPU-accelerated 6D macro-particle tracking with

- linear betatron tracking in the transverse plane with effective detuning models for chromaticity...
- non-linear synchrotron tracking in the longitudinal plane

→ lumped impedance being applied once per turn as wake field convolution

→ space charge nodes distributed around the ring

$$\mathcal{M}_{\text{rev}} = \exp(\Delta s : \mathcal{H}_{\text{kick}}:)\ \exp(\Delta s : \mathcal{H}_{\text{drift}}:)\ \ldots$$

(a) transverse tracking (b) longitudinal tracking

Figure 1: PyHEADTAIL's tracking model

Employed Space Charge (SC) Models

- **frozen field map of bunch**
  - → transverse Bassetti-Erskine formula (Gaussian e-field)
  - → subtracts each slice centroid
  - → fields weighed with local line charge density along slices
  - → updates field map if RMS beam size changes by 10% (*adaptive*)
- **self-consistent 2.5D particle-in-cell (PIC) algorithm**
  - → self-consistent particle-in-cell (PIC) algorithm
  - → solves transverse Poisson equation slice by slice
  - → open-space boundary conditions

(a) frozen model (b) 2.5D PIC model

HL-LHC: Transverse Mode Coupling Instability (TMCI) + Space Charge

HL-LHC injection plateau at $Q'_{x,y} = 0$, $\sigma_z = 10.4$ cm and $\epsilon_{x,y} = 2.5$ mmmrad:

(a) **wake field** only: vert. TMCI above $N_{\text{TMCI,th}} \approx 6 \times 10^{11}$ ppb (b) **wake field + SC**: stable over 50 kturns, no mode coupling

HL-LHC: Head-Tail Instability + Space Charge

HL-LHC injection plateau at $Q'_{x,y} = 5$, $\sigma_z = 10.4$ cm and $N = 4 \times 10^{11}$ ppb $< N_{\text{TMCI,th}}$:

(a) **wake field** only → vert. head-tail instabil. with 18.6 kturns rise time + (b) **space charge** only → stable = (c) **wake field + SC** → stable!

Space Charge Limit when Head-Tail Instability Rises

weaker SC

$$\Delta Q^{\text{SC}}_{x,y} \propto \frac{1}{\gamma^2 \epsilon_{x,y}}$$

- beam stability just from direct SC:
  - without any counter-measures: no transv. feedback, no Landau octupoles
  - $\epsilon_{x,y} = 20$ mmmrad ⇔ $E_0 = 1.27$ TeV
- vertical head-tail instability back:
  - $\epsilon_{x,y} = 40$ mmmrad ⇔ $E_0 = 1.8$ TeV
  - rise time as without SC: 18.7 kturns

(a) wake field + SC up to ($\epsilon_{x,y} = 20$ mmmrad) = stable (b) wake field + SC at ($\epsilon_{x,y} = 40$ mmmrad) = unstable!

Measurements at LHC

- first instability observed in 2010 during the first ramp:
  - both beams affected independently, mode $m = -1$ with 1 node
  - flat-bottom and initial part of the ramp stable until $E_0 \approx 2$ TeV

(plots from [1], flat-top measurements of this head-tail instability at $E_0 = 3.5$ TeV)

Impact of SC Models: Self-consistent PIC Better than Frozen!

- frozen model:
  - symplectic (for smooth Bassetti-Erskine)
  - like a non-linear external magnet
  - → violates total momentum conservation (!)
  - → incoherent $\Delta Q^{\text{SC}}_x = 4.5 \times 10^{-3}$ in centroid motion
- 2.5D PIC model:
  - non-symplectic yet self-consistent
  - → conserves global momentum
  - → violates local energy conservation (macro-particle ↔ grid conversion) ⟹ unphysical **grid heating**

Conclusions

Including direct space charge (SC) into coherent instability studies...

- requires self-consistent SC modelling to **avoid finite centroid kicks** from SC
- **increases (or cancels) the Transverse Mode Coupling Instability threshold** for the short-bunch regime with modes 0 and -1 coupling
- can explain the inherent **single-bunch stability at LHC at low energies** in absence of machine non-linearities and transverse damping

References

[1] Elias Metral et al. *Measurement and interpretation of transverse beam instabilities in the CERN large hadron collider (LHC) and extrapolations to HL-LHC.* Tech. rep. CERN-ACC-2016-0098. Geneva: CERN, July 2016.

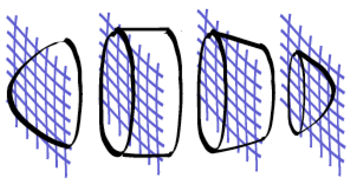
ICFA Workshop on Impedances and Beam Instabilities in Particle Accelerators, Benevento, Italy, September 21, 2017

[1] adrian.oeftiger@cern.ch

Set-up of space charge with PIC: ***A. Oeftiger***

- 3×10^6 macro-particles
- smooth approximation (constant beta functions around machine)
- 200 space charge kicks along ring
- simulate for 20 000 turns
- 1 impedance kick per turn with 500 slices
- 2.5D space charge PIC: 100 transverse grids equally distributed over $6\sigma_z$ along bunch line charge density to solve free-space Poisson eq.

  ⟶ transverse grid size fixed to 10 or $20\sigma_{x,y}$ total width (128×128 cells)



✓ cross-check with 3D model: same qualitative behaviour with growing instability towards end of bunch at 9×10^6 macro-particles and 300 longitudinal mesh points (2.5D PIC resolution ×3)!

2 – PARTICLE MODEL FOR TMCI ($Q' = 0$) WITH SC AND/OR ReaD

Reactive transverse damper

2 – PARTICLE MODEL

- Following the same formalism as Chin-Chao-Blaskiewicz_2016 (PRAB 19, 014201 (2016): http://journals.aps.org/prab/pdf/10.1103/PhysRevAccelBeams.19.014201): "Two particle model for studying the effects of space-charge force on strong head-tail instabilities"

- Adding a reactive transverse damper

2 – PARTICLE MODEL

- **Chin-Chao-Blaskiewicz_2016: WF + SC with constant wake and zero chromaticity**

$$y_1'' + \left(\frac{\omega_\beta}{c}\right)^2 y_1 = K\,(y_1 - y_2) + W\,y_2$$

$$y_2'' + \left(\frac{\omega_\beta}{c}\right)^2 y_2 = K\,(y_2 - y_1)$$

Wake Field (WF)

Space Charge (SC)

- **Discussed here: WF + SC + TD**

$$y_1'' + \left(\frac{\omega_\beta}{c}\right)^2 y_1 = K\,(y_1 - y_2) + W\,y_2 + g_{TD}\,(y_1 + y_2)$$

$$y_2'' + \left(\frac{\omega_\beta}{c}\right)^2 y_2 = K\,(y_2 - y_1) + g_{TD}\,(y_1 + y_2)$$

Transverse Damper (TD)

2 – PARTICLE MODEL

- Chin-Chao-Blaskiewicz_2016 (with WF + SC)

Dimensionless parameter α W / T_s *

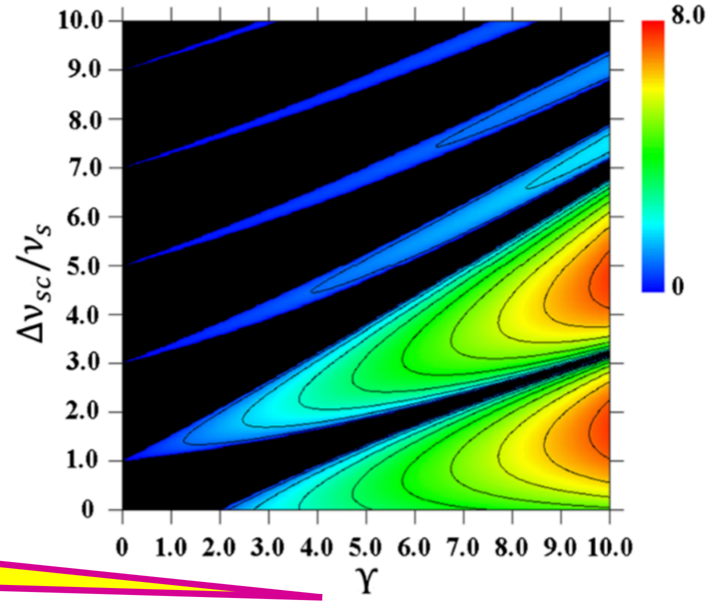


FIG. 4. Flat contour plot for the growth factor $g \times T_s$ as a function of ϒ and $\frac{\Delta\nu_{sc}}{\nu_s}$.

* T_s is the synchrotron period

g = Instability growth rate

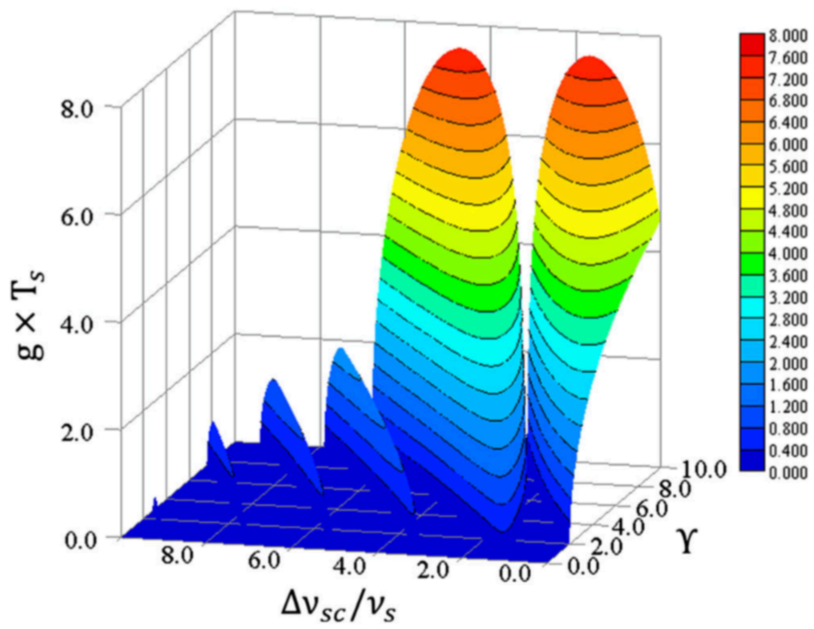
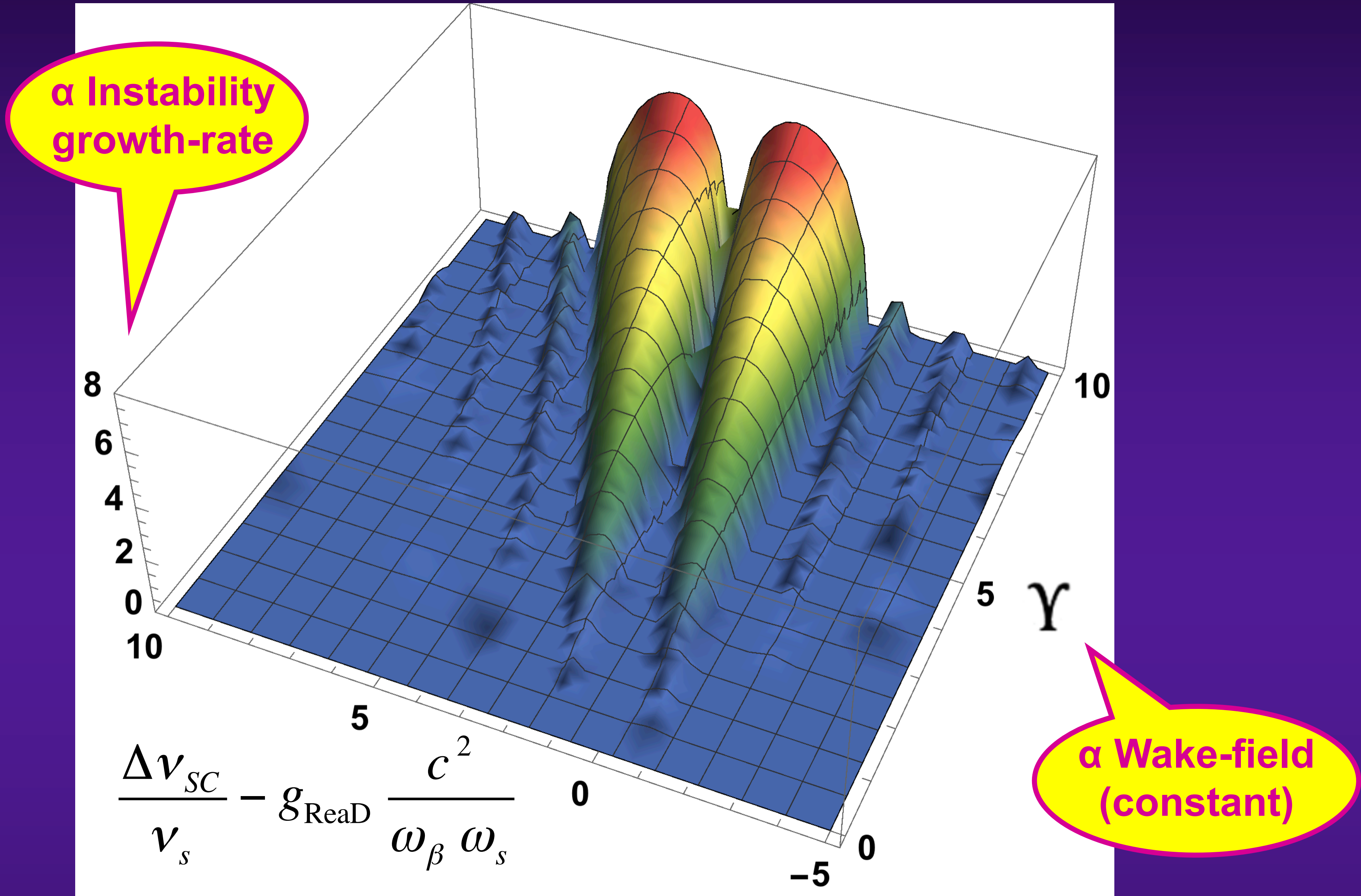


FIG. 5. Three-dimensional contour plot for the growth factor $g \times T_s$ as a function of ϒ and $\frac{\Delta\nu_{sc}}{\nu_s}$.



- Results from Burov_2016 (using a ReaD only) and Chao-Chin-Blaskiewicz_2016 (using SC only) have been recovered and combined
- Both SC and ReaD affect TMCI in a similar way and can suppress it

2 – PARTICLE MODEL

- **Similar result as Burov_2016 in his paper "Efficiency of feedbacks for suppression of transverse instabilities of bunched beams" (https://arxiv.org/abs/1605.06198), where he considered the case of a reactive damper (on Fig. 1) but without space charge**

Note: Different notations used

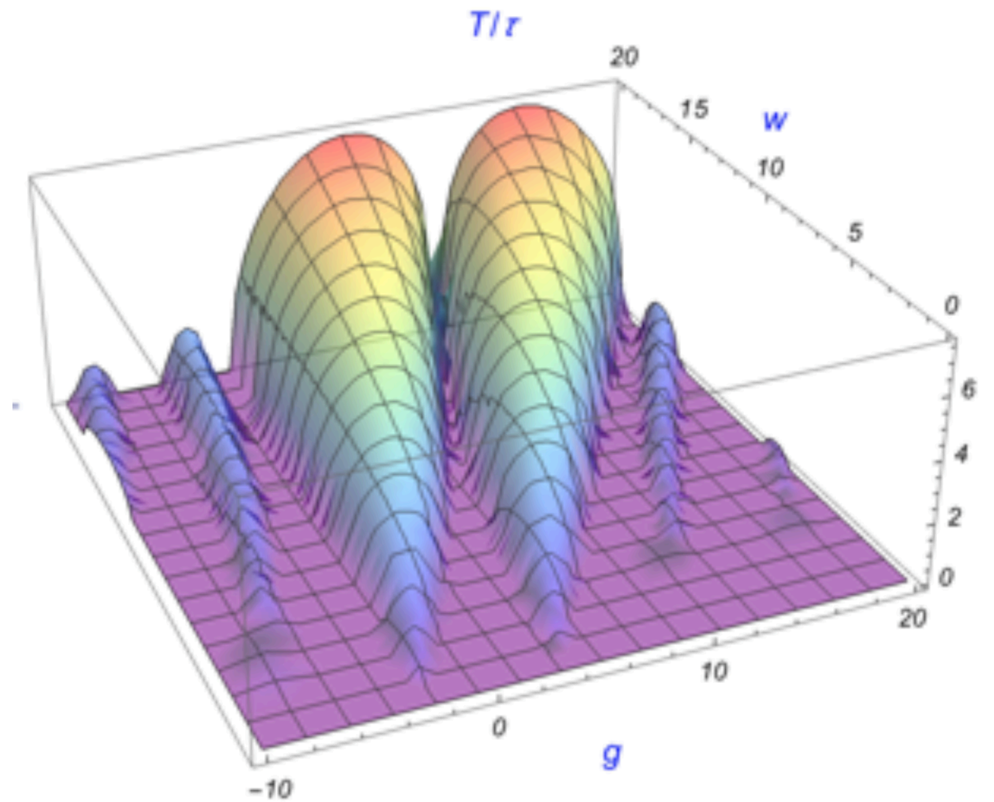
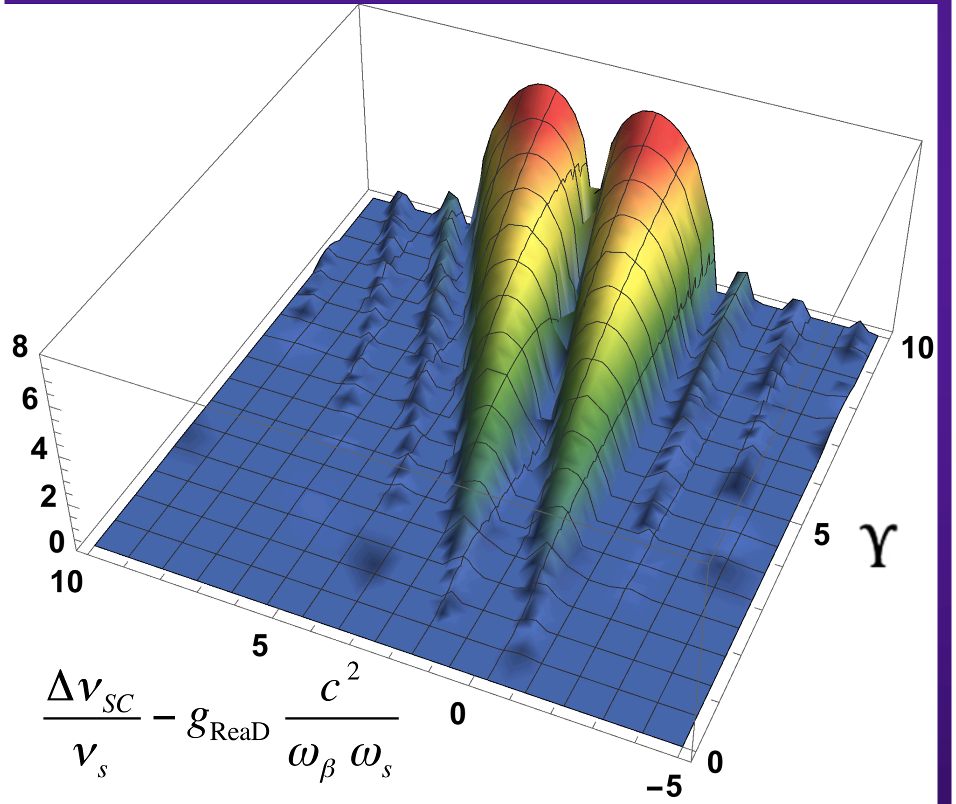


Fig. 1: Two-particle growth rate versus gain g and constant wake value w for reactive damper and zero chromaticity. All the values are in the units of the inverse synchrotron period $1/T$.

2 – PARTICLE MODEL

=> To be able to compare to Burov_2016, we need to divide by

- **2 the WF axis**
- **π the SC + TD_reactive axis**



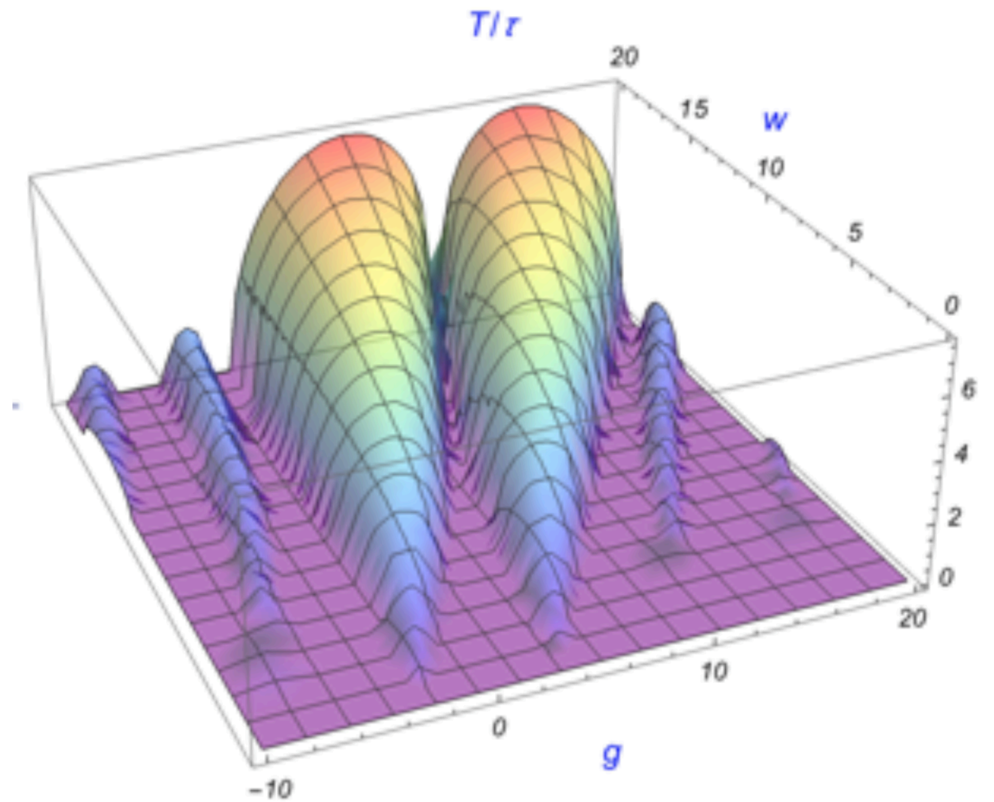


Fig. 1: Two-particle growth rate versus gain g and constant wake value w for reactive damper and zero chromaticity. All the values are in the units of the inverse synchrotron period $1/T$.

◆ GALACTIC: GArnier-LAclare Coherent Transverse Instabilities Code

- Uses a decomposition on the low-intensity eigenvectors (as proposed by Garnier-Laclare in 1987) => "Water-bag" longitudinal distribution (for now)

- Effect of transverse damper recently added (to study destabilizing effect of resistive transverse damper) => IPAC18 paper (http://accelconf.web.cern.ch/AccelConf/ipac2018/papers/thpaf048.pdf)

- Remark: 2 other codes (Vlasov solvers) including the transverse damper were developed in the recent years

  - A. Burov developed a Nested Head-Tail Vlasov Solver (**NHTVS**) with transverse damper in **2014**

  - N. Mounet solved Sacherer integral equation with transverse damper, using a decomposition over Laguerre polynomials of the radial functions (**DELPHI** code, **2015**)

** Sacherer integral equation was also solved using a decomposition over Laguerre polynomials of the radial functions by Besnier in 1974 and Y.H. Chin in 1985 in the code MOSES*

Without transverse damper