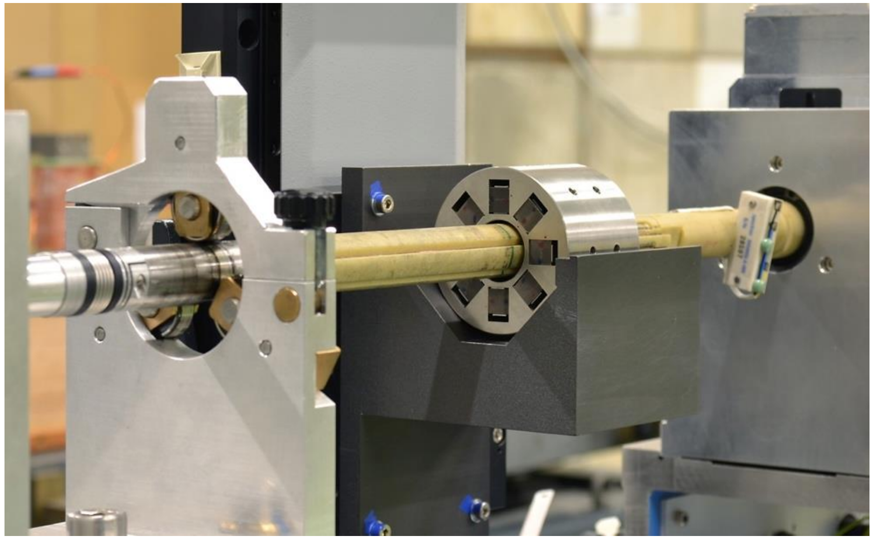
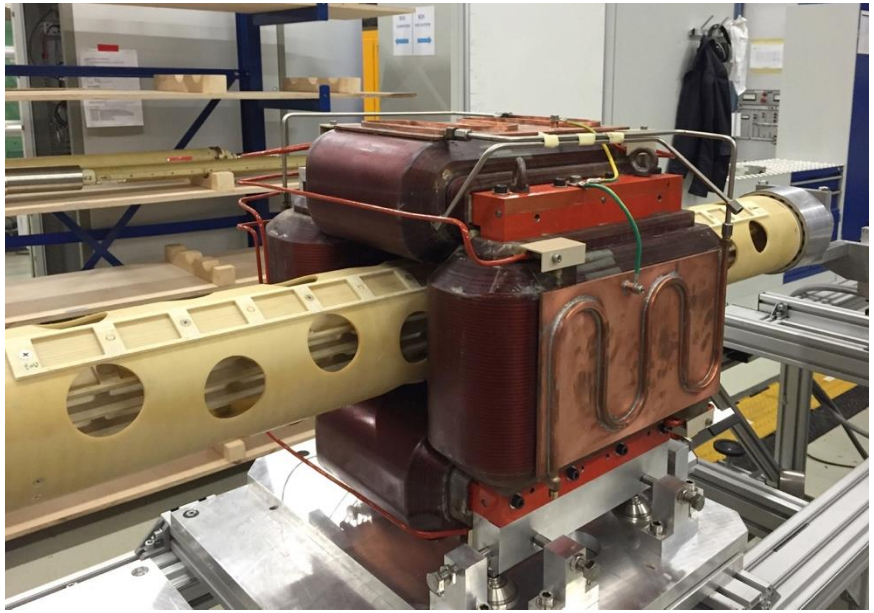
Challenges in Extracting Pseudo Multipoles from Magnetic Measurements

Stephan Russenschuck,
ICAP 2018

Rotating-Coil Magnetometers





Solving of Boundary Value Problems

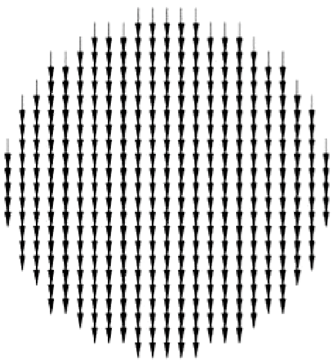
$$A_z(r,\varphi) = \sum_{n=1}^{\infty} r^n (\mathcal{A}_n \sin n\varphi + \mathcal{B}_n \cos n\varphi).$$

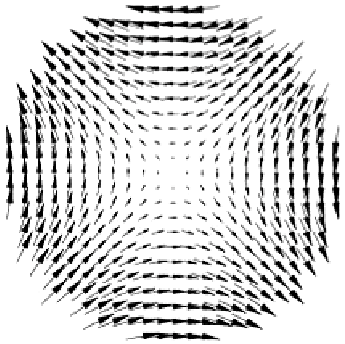
$$B_r(r,\varphi) = \frac{1}{r}\frac{\partial A_z}{\partial \varphi} = \sum_{n=1}^{\infty} n r^{n-1} (\mathcal{A}_n \cos n\varphi - \mathcal{B}_n \sin n\varphi),$$

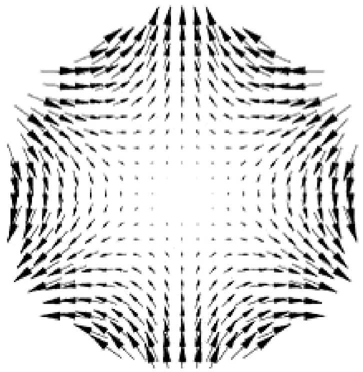
$$B_r(r_0,\varphi) = \sum_{n=1}^{\infty} (B_n(r_0) \sin n\varphi + A_n(r_0) \cos n\varphi),$$

$$\mathcal{A}_n = \frac{1}{n\, r_0^{n-1}} A_n(r_0)\,, \qquad \mathcal{B}_n = \frac{-1}{n\, r_0^{n-1}} B_n(r_0)\,.$$

$$B_r(r,\varphi) = \sum_{n=1}^{\infty} \left(\frac{r}{r_0}\right)^{n-1} (B_n(r_0) \sin n\varphi + A_n(r_0) \cos n\varphi)$$



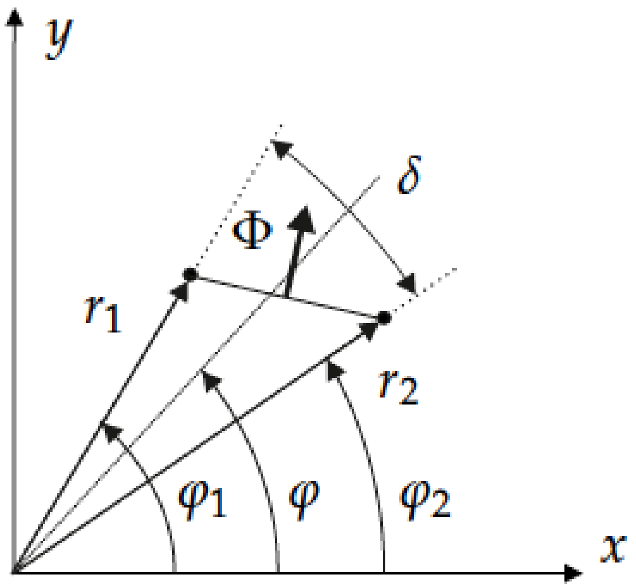




Rotating Coil Measurements (Sensitivity Factors)

$$\Phi(\varphi) = N \int_{\mathscr{A}} \mathbf{B} \cdot \mathrm{d}\mathbf{a} = N \int_{\mathscr{A}} \operatorname{curl} \mathbf{A} \cdot \mathrm{d}\mathbf{a} = N \int_{\partial \mathscr{A}} \mathbf{A} \cdot \mathrm{d}\mathbf{r}$$
$$= N\ell \left[A_z(\mathscr{P}_1) - A_z(\mathscr{P}_2) \right],$$

$$\Phi(\varphi) = N\ell \left[\sum_{n=1}^{\infty} \frac{r_0}{n} \left(\frac{r_2}{r_0} \right)^n (B_n(r_0) \cos n\varphi_2 - A_n(r_0) \sin n\varphi_2) - \sum_{n=1}^{\infty} \frac{r_0}{n} \left(\frac{r_1}{r_0} \right)^n (B_n(r_0) \cos n\varphi_1 - A_n(r_0) \sin n\varphi_1) \right],$$



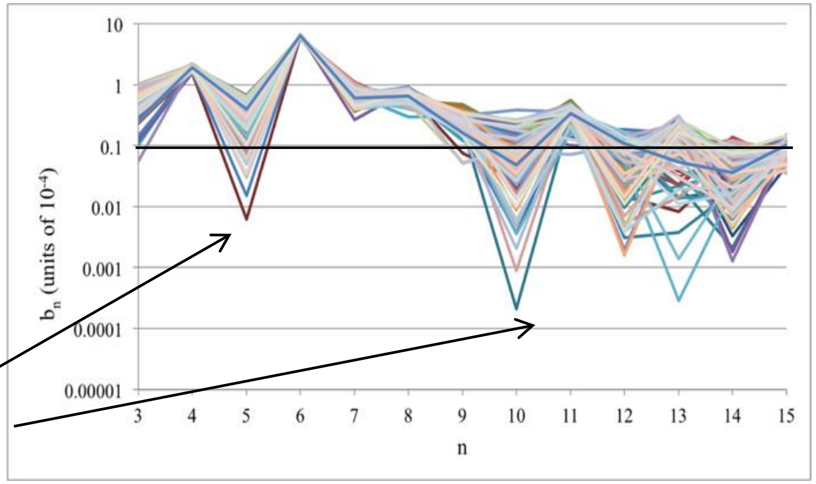
$$\Phi(\varphi) = \sum_{n=1}^{\infty} S_n^{\mathrm{rad}} \left(B_n(r_0) \cos n\varphi - A_n(r_0) \sin n\varphi \right) + S_n^{\tan} \left(B_n(r_0) \sin n\varphi + A_n(r_0) \cos n\varphi \right)$$

$$S_n^{\mathrm{rad}} = \frac{N\ell}{n r_0^{n-1}} \left[r_2^n \cos n(\varphi_2 - \varphi) - r_1^n \cos n(\varphi_1 - \varphi) \right],$$
$$S_n^{\tan} = -\frac{N\ell}{n r_0^{n-1}} \left[r_2^n \sin n(\varphi_2 - \varphi) - r_1^n \sin n(\varphi_1 - \varphi) \right],$$

Challenge: Spread and Noise Floor in Rotating Coil Measurements

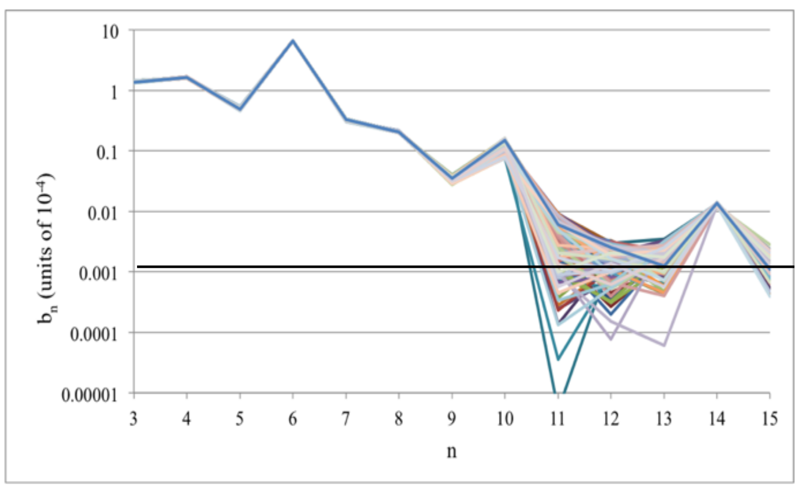
Minimum Signal Requirement: 10^{-8} Vs

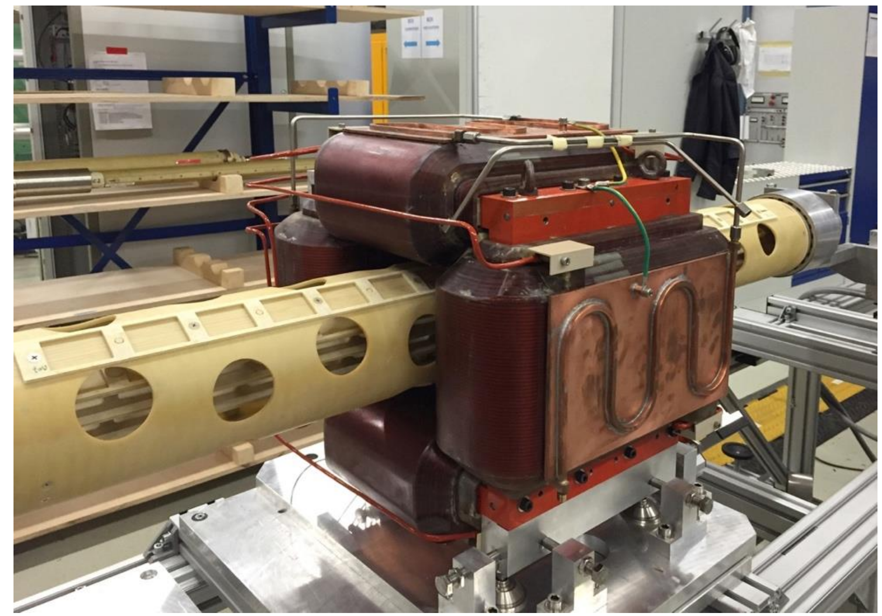
Noncompensated



Radial axes displacement

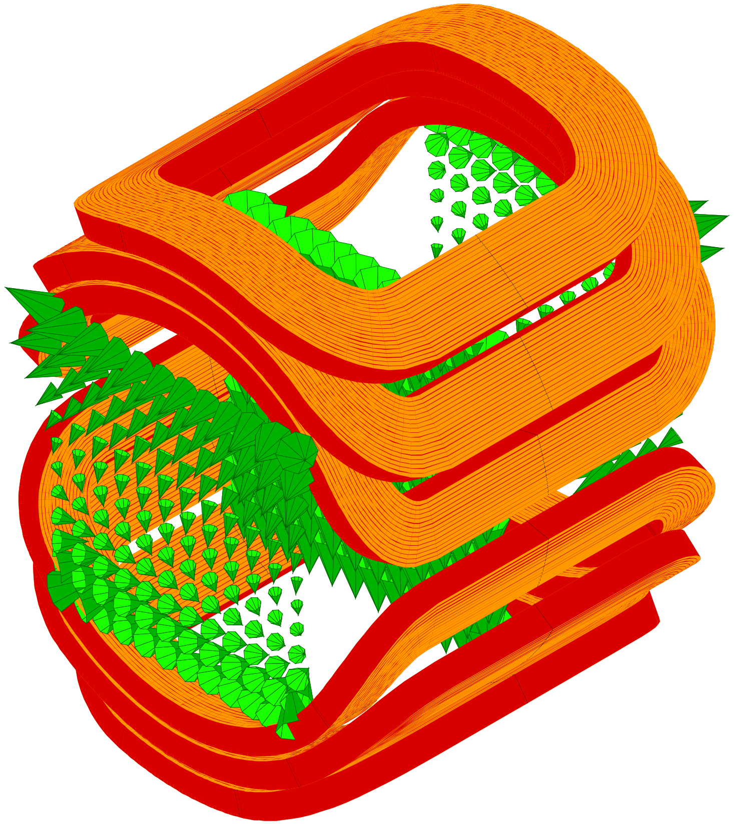
Compensated

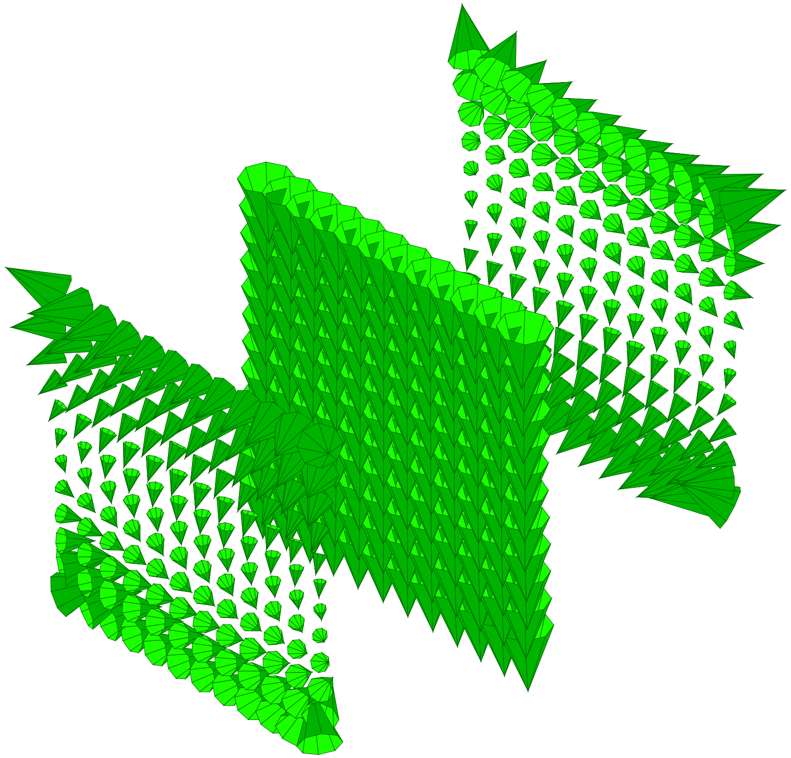




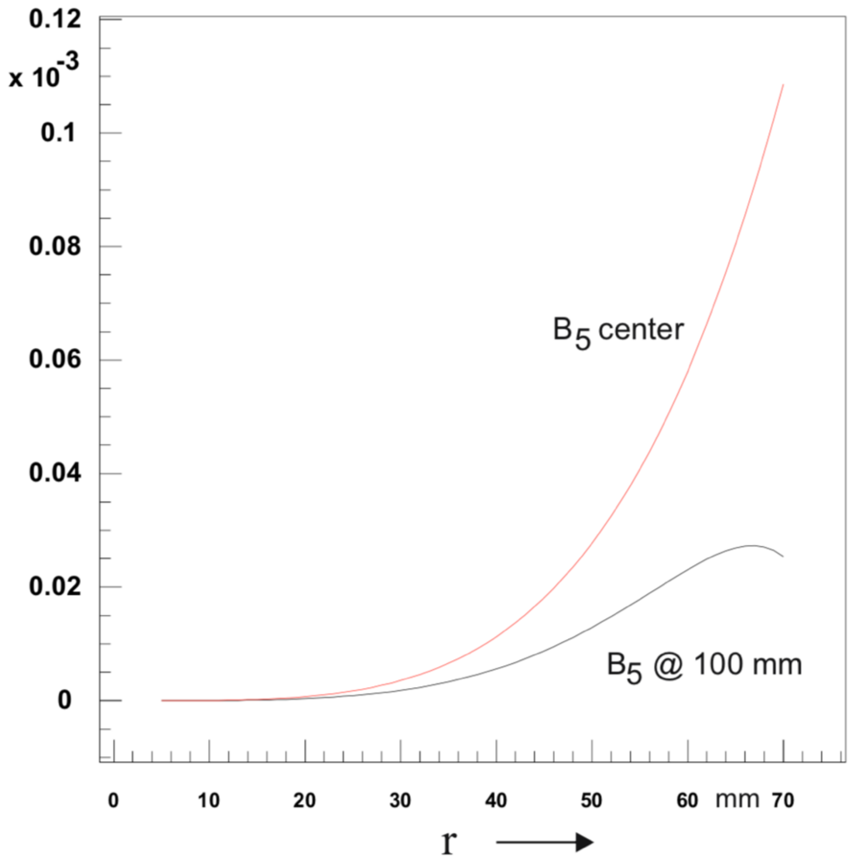
It makes no sense to consider relative harmonics smaller than 10^{-7}

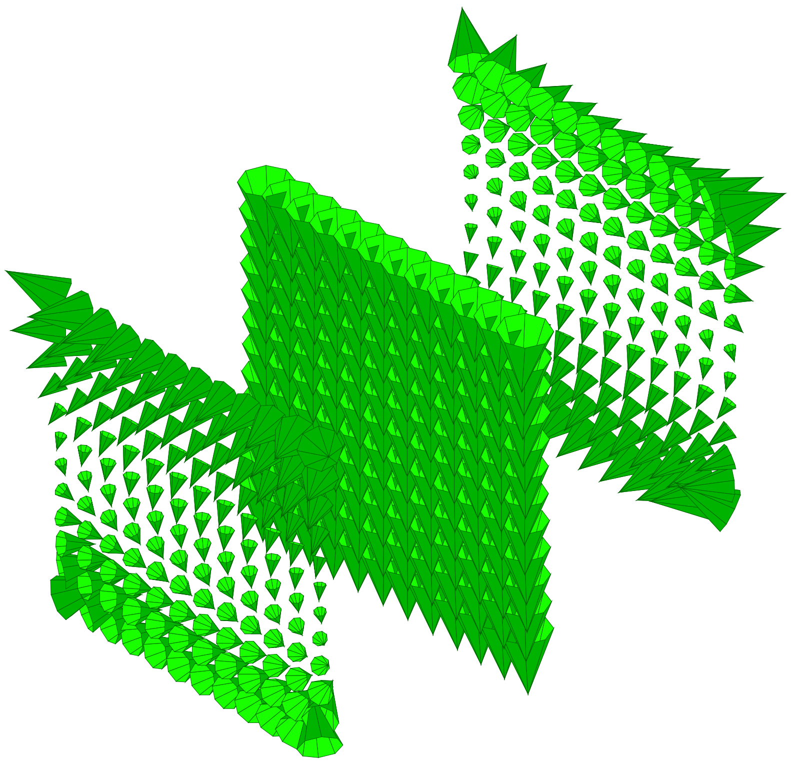
The 3D Field Problem (Elena Steerer)





The 3D Field Problem (Elena Steerer)





Pseudo-Multipoles (Fourier Bessel Series)

$$\phi_{\mathrm{m}}(r,\varphi,z) = \left\{ \begin{array}{c} \cos\, n\varphi \\ \sin\, n\varphi \end{array} \right\} I_n(pr) \left\{ \begin{array}{c} \cos\, pz \\ \sin\, pz \end{array} \right\}$$

$$I_n(pr) = \sum_{k=0}^{\infty} \frac{1}{k!\,\Gamma(k+n+1)} \left(\frac{pr}{2}\right)^{n+2k}$$

$$\phi_{\mathrm{m}} = \sum_{k=0}^{\infty} \sum_{n=1}^{\infty} r^{n+2k} (\mathcal{C}_{n+2k,n}(z) \sin\, n\varphi + \mathcal{D}_{n+2k,n}(z) \cos\, n\varphi)$$

Pseudo-Multipoles II

$$
\begin{aligned}
\phi_{\mathrm{m}} = \sum_{n=1}^{\infty} \Big\{ & \mathcal{C}_{n,n}(z) - \frac{\mathcal{C}_{n,n}^{(2)}(z)}{4(n+1)} r^2 \\
& + \frac{\mathcal{C}_{n,n}^{(4)}(z)}{32(n+1)(n+2)} r^4 - \frac{\mathcal{C}_{n,n}^{(6)}(z)}{384(n+1)(n+2)(n+3)} r^6 + \dots \Big\} r^n \sin n\varphi \\
+ \sum_{n=1}^{\infty} \Big\{ & \mathcal{D}_{n,n}(z) - \frac{\mathcal{D}_{n,n}^{(2)}(z)}{4(n+1)} r^2 \\
& + \frac{\mathcal{D}_{n,n}^{(4)}(z)}{32(n+1)(n+2)} r^4 - \frac{\mathcal{D}_{n,n}^{(6)}(z)}{384(n+1)(n+2)(n+3)} r^6 + \dots \Big\} r^n \cos n\varphi \,,
\end{aligned}
$$

Mid-Plane Field and the Translating Fluxmeter

$$\frac{-1}{\mu_0} B_y(x, y=0, z) \approx$$

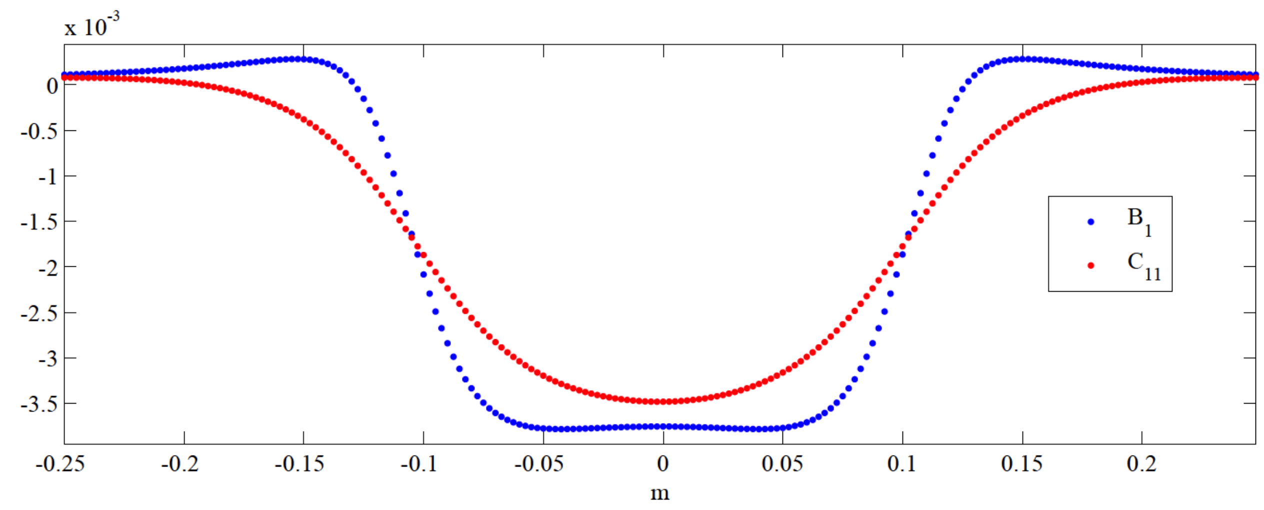
$$\mathcal{C}_{1,1}(z) \quad - \frac{\mathcal{C}_{1,1}^{(2)}(z)}{8} x^2 \quad + \frac{\mathcal{C}_{1,1}^{(4)}(z)}{192} x^4 \quad - \frac{\mathcal{C}_{1,1}^{(6)}(z)}{9216} x^6$$

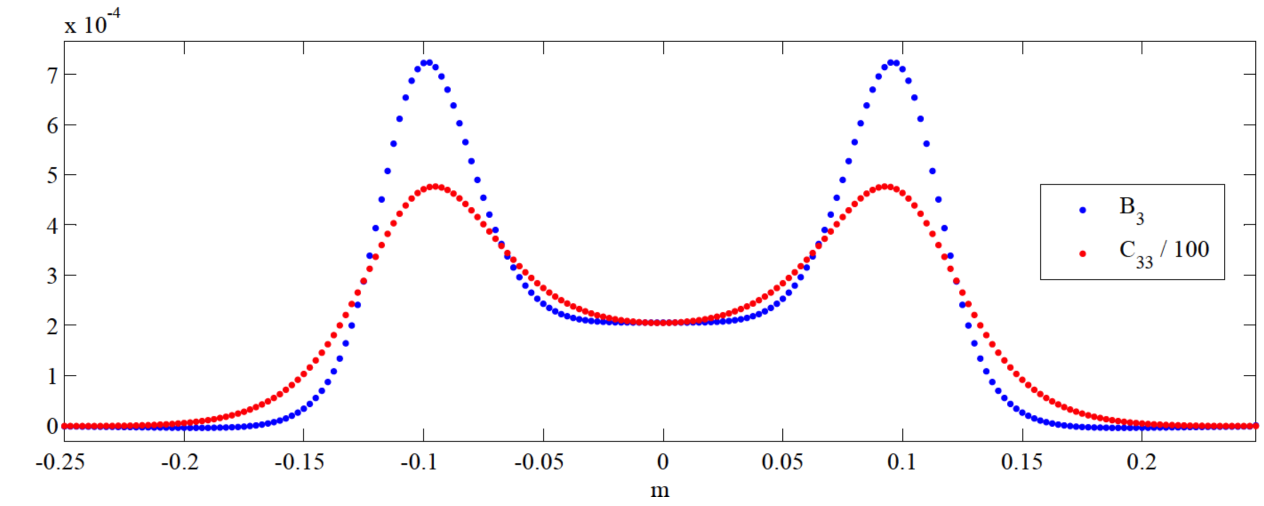
$$+ 3\,\mathcal{C}_{3,3}(z)\,x^2 - \frac{3\,\mathcal{C}_{3,3}^{(2)}(z)}{16} x^4 + \frac{3\,\mathcal{C}_{3,3}^{(4)}(z)}{640} x^6$$

$$+ 5\,\mathcal{C}_{5,5}(z)\,x^4 - \frac{5\,\mathcal{C}_{5,5}^{(2)}(z)}{24} x^6$$

$$+ 7\,\mathcal{C}_{7,7}(z)\,x^6$$

The Leading Term is NOT the Measured One





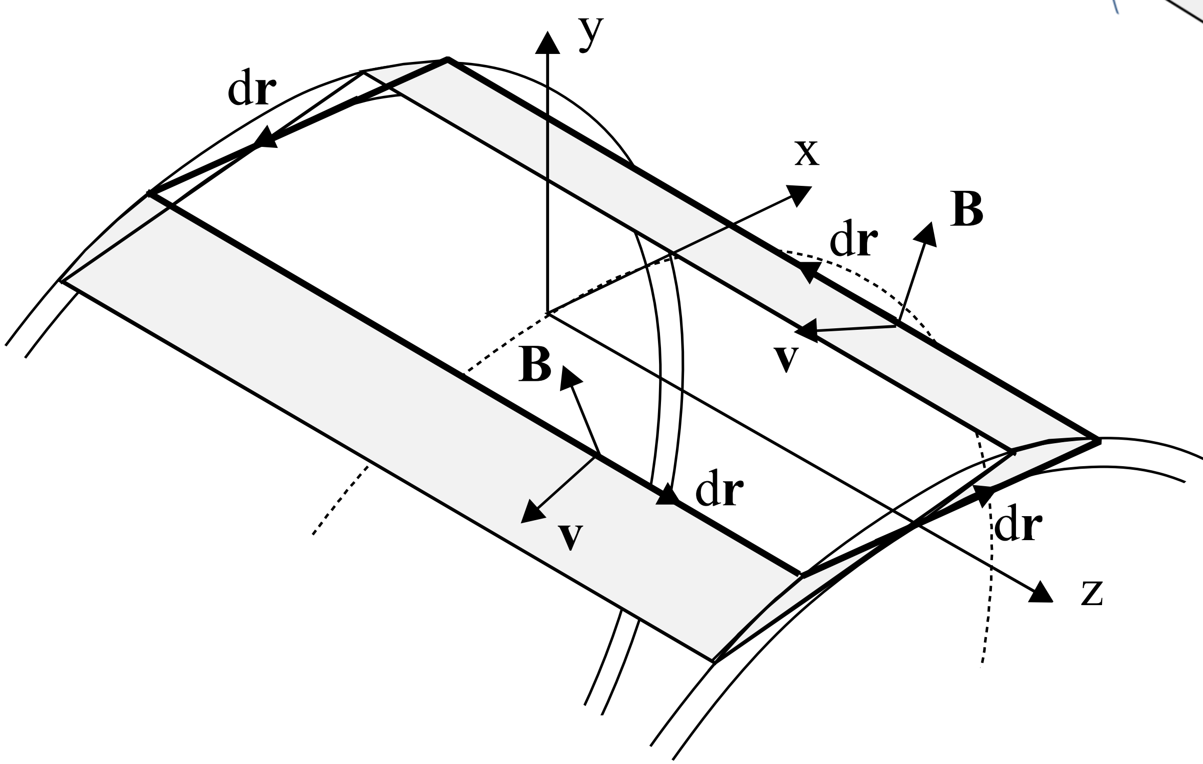
Fourier Transform for the Extractions of Cn,n

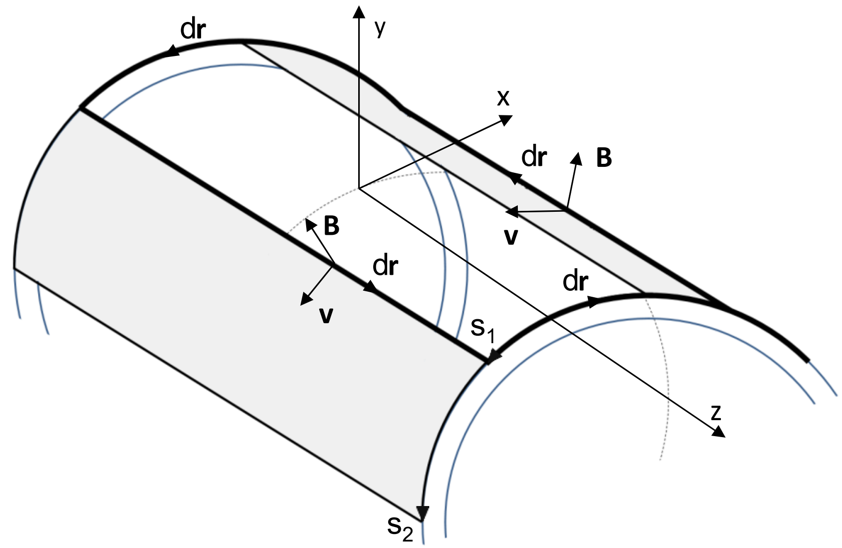
$$B_n(r_0,z) = -\mu_0\, r_0^{n-1}\overline{\mathcal{C}}_n(r_0,z) =$$

$$-\mu_0\, r_0^{n-1}\left(n\,\mathcal{C}_{n,n}(z) - \frac{(n+2)\mathcal{C}_{n,n}^{(2)}(z)}{4(n+1)}r_0^2 + \frac{(n+4)\mathcal{C}_{n,n}^{(4)}(z)}{32(n+1)(n+2)}r_0^4 - \dots\right).$$

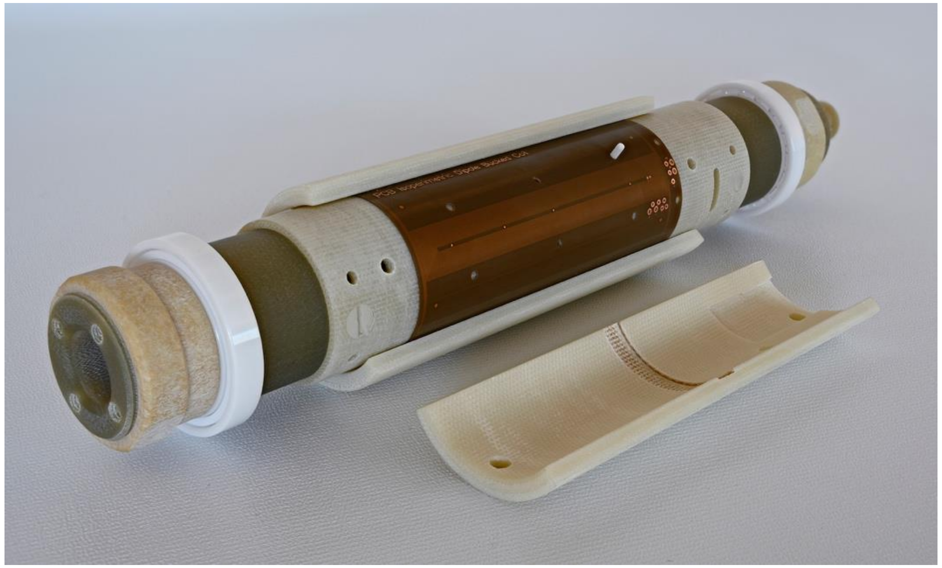
$$\mathcal{F}\{\mathcal{C}_{n,n}(z)\} = \frac{-\mathcal{F}\{B_n(r_0,z)\}}{\mu_0\, r_0^{n-1}\left(n - \frac{(n+2)(i\omega)^2}{4(n+1)}r_0^2 + \frac{(n+4)(i\omega)^4}{32(n+1)(n+2)}r_0^4 - \dots\right)}$$

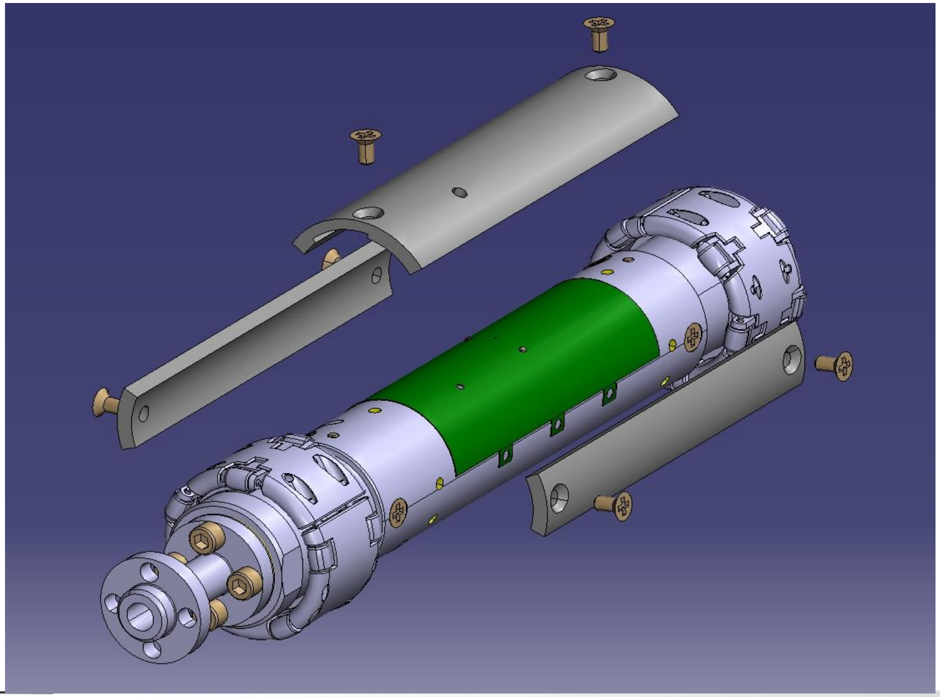
Challenge: Classical Induction Coils Intercept the B_z Field Component

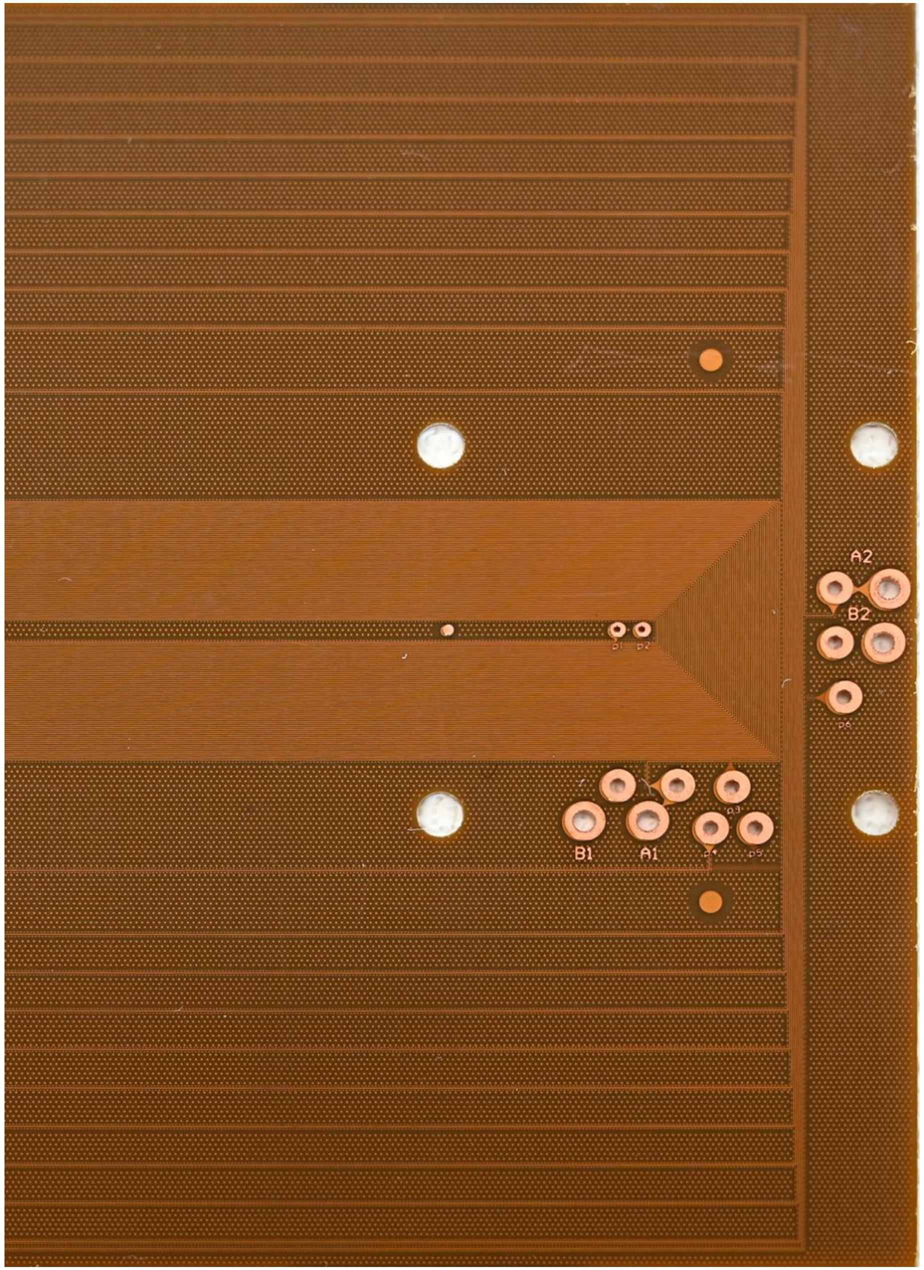




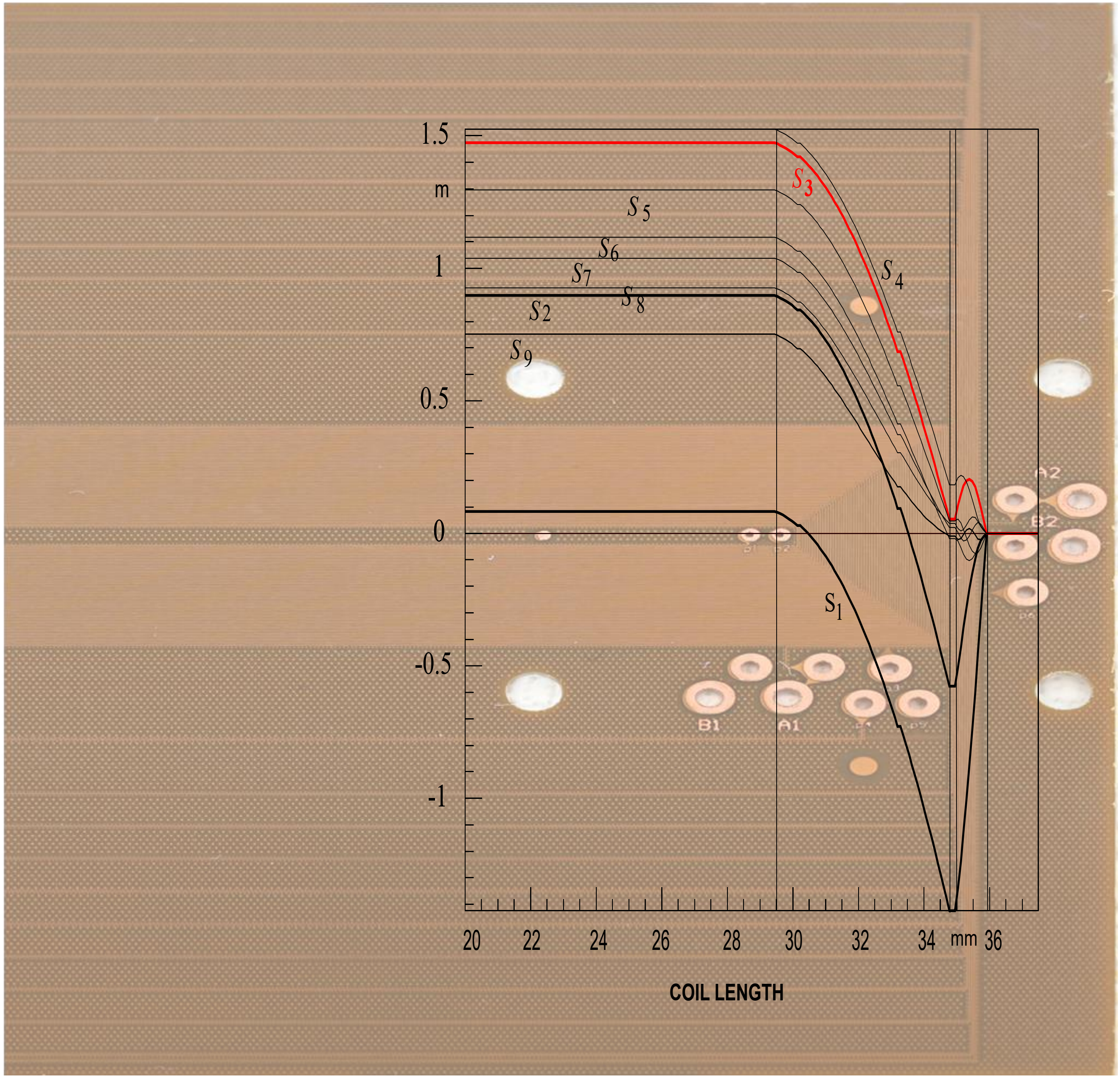
Saddle-shaped, Iso-Perimetric Induction Coil







Challenge: Sensitivity Function as Test Function of Convoluted Signal



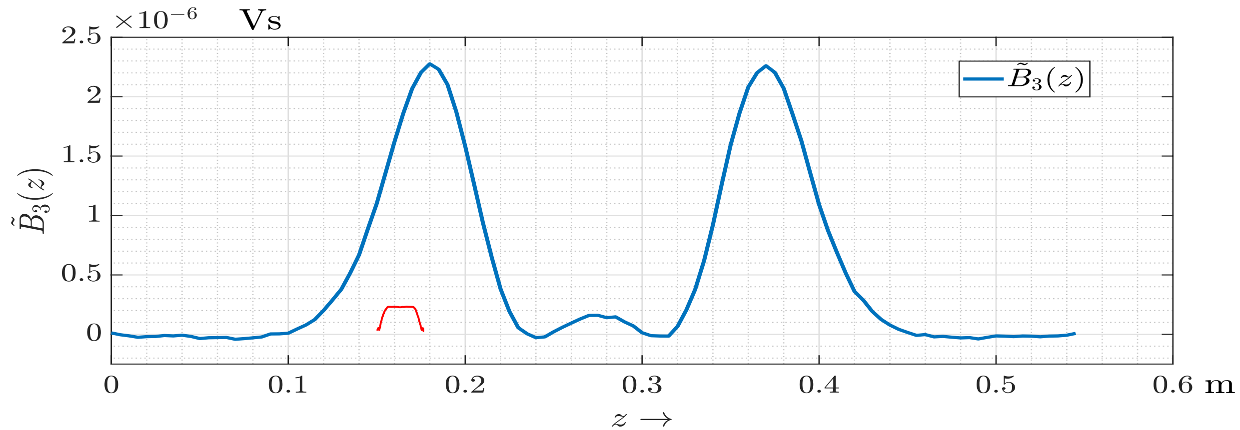
Challenge 4:

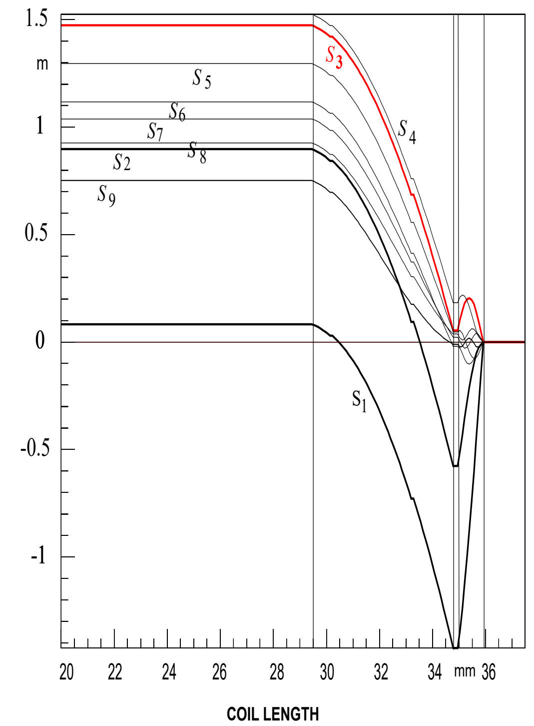
Sufficiently long for a sufficiently large number of turns –

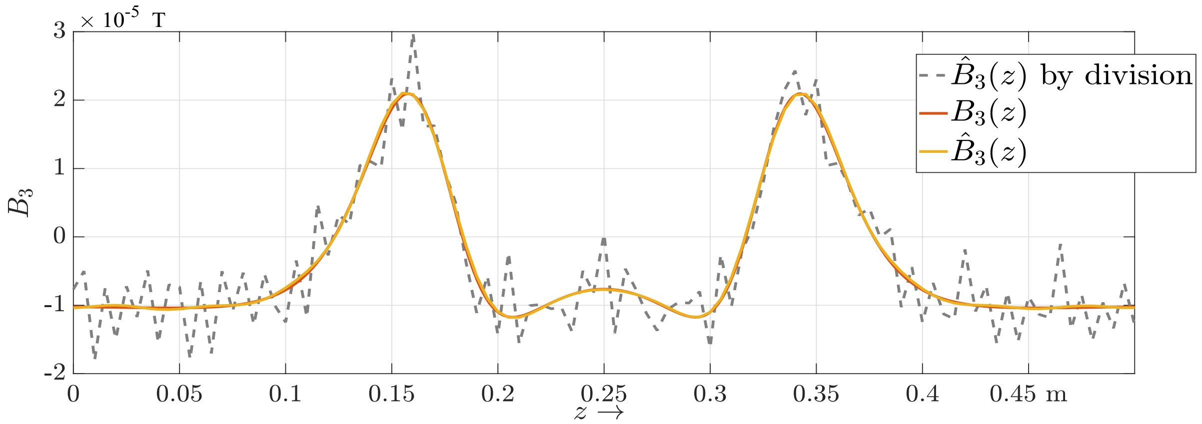
Sufficiently short for sensitivity to high spatial frequencies

Challenge: Deconvolution of Noisy Signals

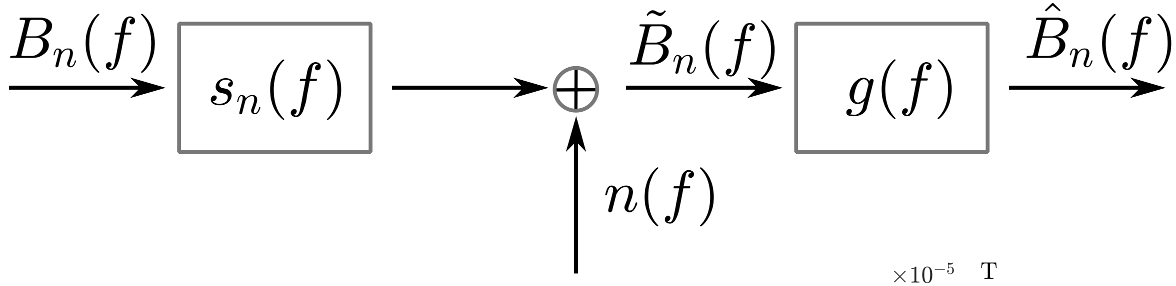
$$\mathcal{F}\{\mathcal{C}_{n,n}(z)\} = \frac{\mathcal{F}\{\tilde{B}_n(r_0,z)\}}{\mathcal{F}\{s_n(r_0,z)\}} \frac{-1}{\mu_0\, r_0^{n-1} \left(n - \frac{(n+2)(i\omega)^2}{4(n+1)} r_0^2 + \frac{(n+4)(i\omega)^4}{32(n+1)(n+2)} r_0^4 - \dots\right)}$$



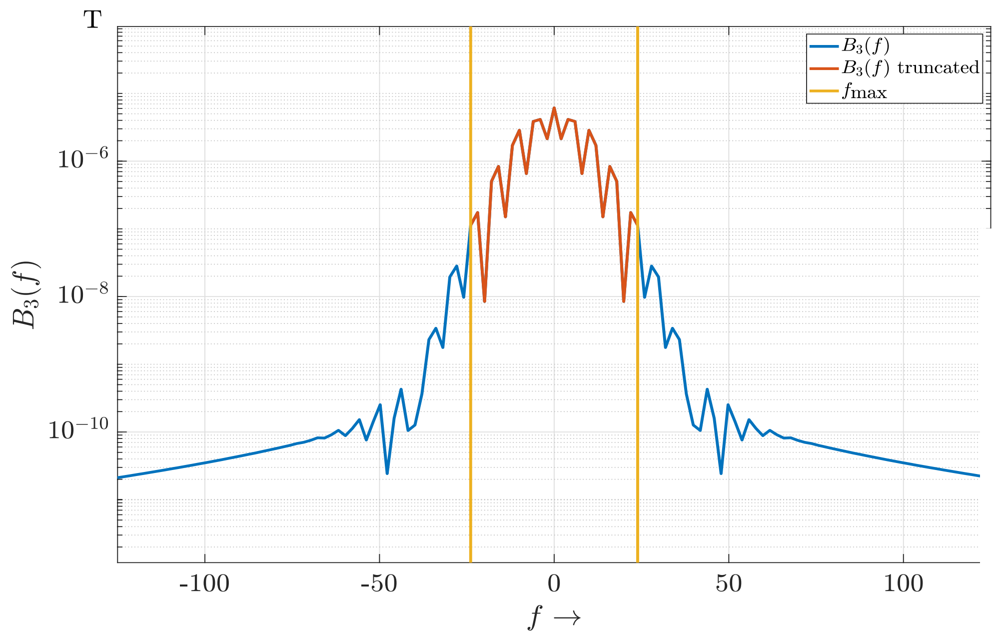


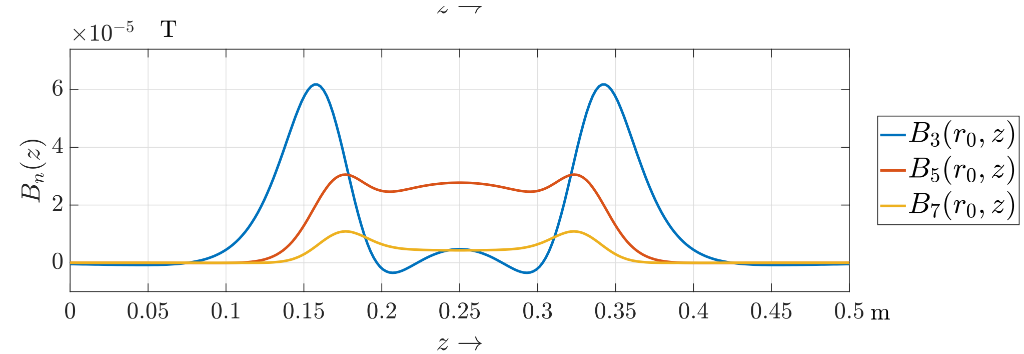


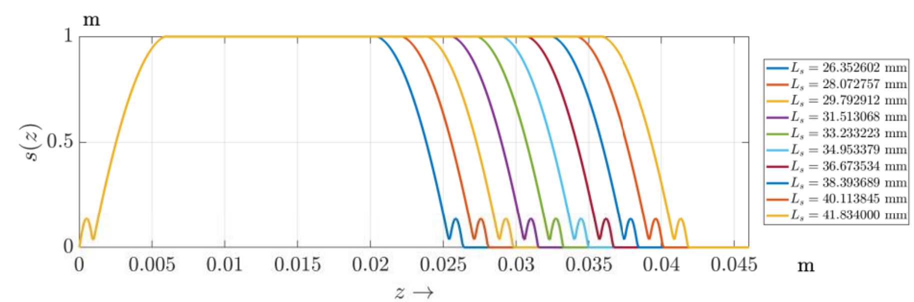
Challenge: Design of Experiment

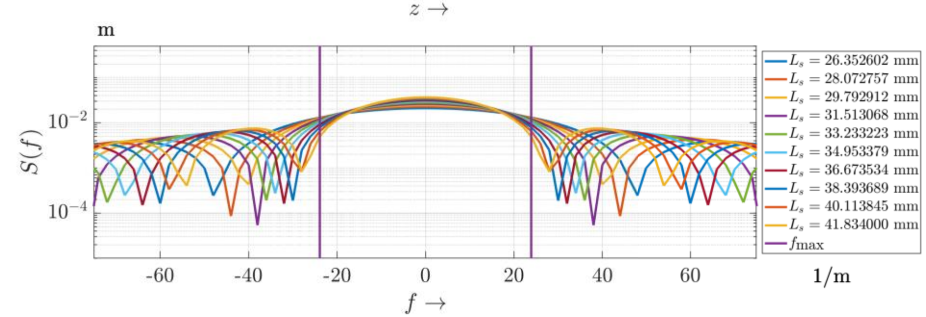


Wiener-Filter









Challenge: Which norm?

Summary

Pseudo-multipole extraction from magnetic measurements requires a careful **design of experiment**. Compute the highest order of multipoles and pseudo-multipoles required for the reproduction of the field distribution. Check that your saddle-shaped (!) sensor is large enough and has a sufficient number of turns (flux linkage larger than 10^{-8} Vs) but is short enough so that its zero-sensitivity harmonics are high enough. Scan the magnet and deconvolute with a Wiener filter.

Trust the numerical (FEM) model. Compute the field distribution, convolute it with the sensitivity function of the transducer, measure at one position in the magnet ends and compare. If consistent, trust the simulations and the magnet production process. Use simulated data for beam tacking.

You cannot use 3D black-box measurements for beam tracking. Nobody believes in simulations but the guy who did them. Everybody believes in measurements but the guy who did them.