Analytical Calculations for
Thomson Backscattering Based Light Sources

Paul Volz
Introduction: Thomson Scattering

- scattering a high intensity laser off a relativistic electron beam
- Doppler shift allows for dramatic increase in photon energy
\[ \nu' = \nu_0 \frac{1 - \beta \cos \theta_i}{1 - \beta \cos(\theta_f) + \frac{\nu_0}{\gamma mc^2} (1 - \cos \theta_p)} \]

schematic taken from [1]
• conventional X-ray sources require GeV electron beams
• Thomson sources can achieve high photon energies with relatively low energy electron beams (e.g. 40 keV photons from 50 MeV electrons)
• tunability
Properties: Tunability

- scattered photons travel in direction of electron beam
- cone of half angle $1/\gamma$
incident angle dictates Doppler shift and frequency of scattered photons
Monte Carlo methods like CAIN [2] exist
  - high computation cost for large amounts of particles

other analytical codes exist, but we couldn’t find any with coherent treatment of radiation

FEL codes include coherent treatment but the scenario doesn’t match

goal: quick code to prove concepts for new machines and prototyping
substantial work done by S. K. Ride, E. Esarey and others [3] [4]
analytical integration of Liénard-Wiechert potentials using Bessel identities
code does not integrate numerically
evaluation of complex Bessel functions
written in python
Normalized intensity for counterpropagating linearly polarized laser \((N = 7, \lambda = 500 \text{ nm})\) and relativistic electron \((\gamma = 100)\) as a function of photon energy and scattering angle.
Normalized intensity of radiation scattered by a relativistic electron ($\gamma = 10$) from a counterpropagating high intensity ($a_o = 2$) linearly polarized laser pulse, viewed in plane of the detector. The detector is located at $z'$ and centered on the electron beam axis. Distances in $x'$, $y'$ are measured in units $\gamma_0(x'/z'), \gamma_0(y'/z') \propto \gamma_0 \theta$. The first three harmonics are shown.
Normalized Intensity of transversely scattered radiation by a relativistic electron ($\gamma = 10$) traveling in plane of polarization of a high intensity ($a_0 = 2$) linearly polarized laser pulse, viewed in plane of the detector. The detector is located at $z'$ and centered on the electron beam axis. Distances in $x'$, $y'$ are measured in units $\gamma_0(x'/z'), \gamma_0(y'/z') \propto \gamma_0 \theta$. The first three harmonics are shown.
Normalized Intensity of radiation scattered by a relativistic electron ($\gamma = 10$) from a counterpropagating high intensity ($a_0 = 2$) circularly polarized laser pulse, viewed in plane of the detector. The detector is located at $z'$ and centered on the electron beam axis. Distances in $x'$, $y'$ are measured in units $\gamma_0(x'/z')$, $\gamma_0(y'/z') \propto \gamma_0 \theta$. The first three harmonics are shown.
Normalized Intensity of transversely radiation scattered by a relativistic electron \( (\gamma = 10) \) from a high intensity \( (a_o = 2) \) circularly polarized laser pulse, viewed in plane of the detector. The detector is located at \( z' \) and centered on the electron beam axis. Distances in \( x', y' \) are measured in units \( \gamma_0 (x'/z'), \gamma_0 (y'/z') \propto \gamma_0 \theta \). The first three harmonics are shown.
Intensity of radiation in arbitrary units produced by a single electron on the right and by two identical electrons treated coherently on the left. The coherent addition of radiation increases the intensity four-fold.
laser field can be compared to undulator field
laser pulse introduces an additional phase factor
- laser field can be compared to undulator field
- laser pulse introduces an additional phase factor

Coherent addition of radiation produced by two electrons half a wavelength apart. Laser phase factor has been omitted on the right.
Intensity of radiation produced by a Gaussian bunch ($\gamma = 69.5$) interacting with a circularly polarized laser pulse ($\lambda = 2.665$ cm). Incoherent addition on the left, coherent addition on the right.
First Results: Gaussian Bunch

- 200 k particles simulated with $80 \times 80$ pixel detector resolution
- Gaussian energy spread and longitudinal distribution
- no transverse momenta or displacement
- incoherent increase in intensity matches expectations for number of particles
- coherent addition is very noisy
  - most radiation cancels out
  - spectrum is dominated by minority of particles
  - pseudo-random generation of bunch amplifies this effect
First Results: Microbunching

Energy spread and longitudinal electron distribution of the simulated bunch.
Intensity of radiation produced by a microbunched beam ($\gamma = 69.5$) interacting with a circularly polarized laser pulse ($\lambda = 2.665$ cm). Relatively low energy spread on the left, larger energy spread on the right.
First Results: Microbunching

- 200 k particles simulated with $80 \times 80$ pixel detector resolution
- no transverse momenta or displacement
- strong microbunching of 60% with non-zero bandwidth
- runtime: about 2000 s on a workstation CPU
fast analytical code
correctly handles coherent addition of radiation for two electrons
emittance needs to be fully implemented
noise levels of coherent addition need to be improved
Thank you for your attention!


Backup Slides
substantial work done by S. K. Ride, E. Esarey and others [3] [4]

integration of Liénard-Wiechert potentials

\[
\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} \left| \int_{-T/2}^{T/2} \left[ \vec{n} \times (\vec{n} \times \vec{\beta}) \right] e^{i\omega \left( t - \frac{\vec{n} \cdot \vec{r}}{c} \right)} dt \right|^2
\]
integral can be split by polarization

\[
\frac{d^2 I_\theta}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} \left| \int_{\eta_0}^{\eta_0} \left[ \frac{d\vec{r}}{d\eta} \hat{e}_\theta \right] e^{i\psi} d\eta \right|^2
\]

\[
\frac{d^2 I_\phi}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} \left| \int_{\eta_0}^{\eta_0} \left[ \frac{d\vec{r}}{d\eta} \hat{e}_\phi \right] e^{i\psi} d\eta \right|^2
\]
Integral can be split by polarization

\[
\begin{align*}
\frac{d^2 I_\theta}{d\omega d\Omega} &= \frac{e^2 \omega^2}{4\pi^2 c} \left| \int_{\eta_0}^{\eta_0} \left[ \frac{d\vec{r}}{d\eta} \hat{e}_\theta \right] e^{i\Psi} \, dt \right|^2 \\
\frac{d^2 I_\phi}{d\omega d\Omega} &= \frac{e^2 \omega^2}{4\pi^2 c} \left| \int_{\eta_0}^{\eta_0} \left[ \frac{d\vec{r}}{d\eta} \hat{e}_\phi \right] e^{i\Psi} \, dt \right|^2
\end{align*}
\]

Solution via Bessel identity

\[
\exp \left( i b \sin \sigma \right) = \sum_{n=-\infty}^{\infty} J_n(b) \exp \left( i n\sigma \right)
\]