Mean-field density evolution of bunched particles with non-zero initial velocity

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NSF Grant: 1803719
and RC108666

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Outline

I. Literature review

II. New cylindrical expression and validation

III. Conclusions

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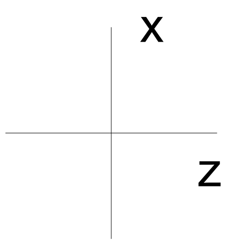
The pancake regime



Cigar (normal) : $\Delta z >> \Delta x$

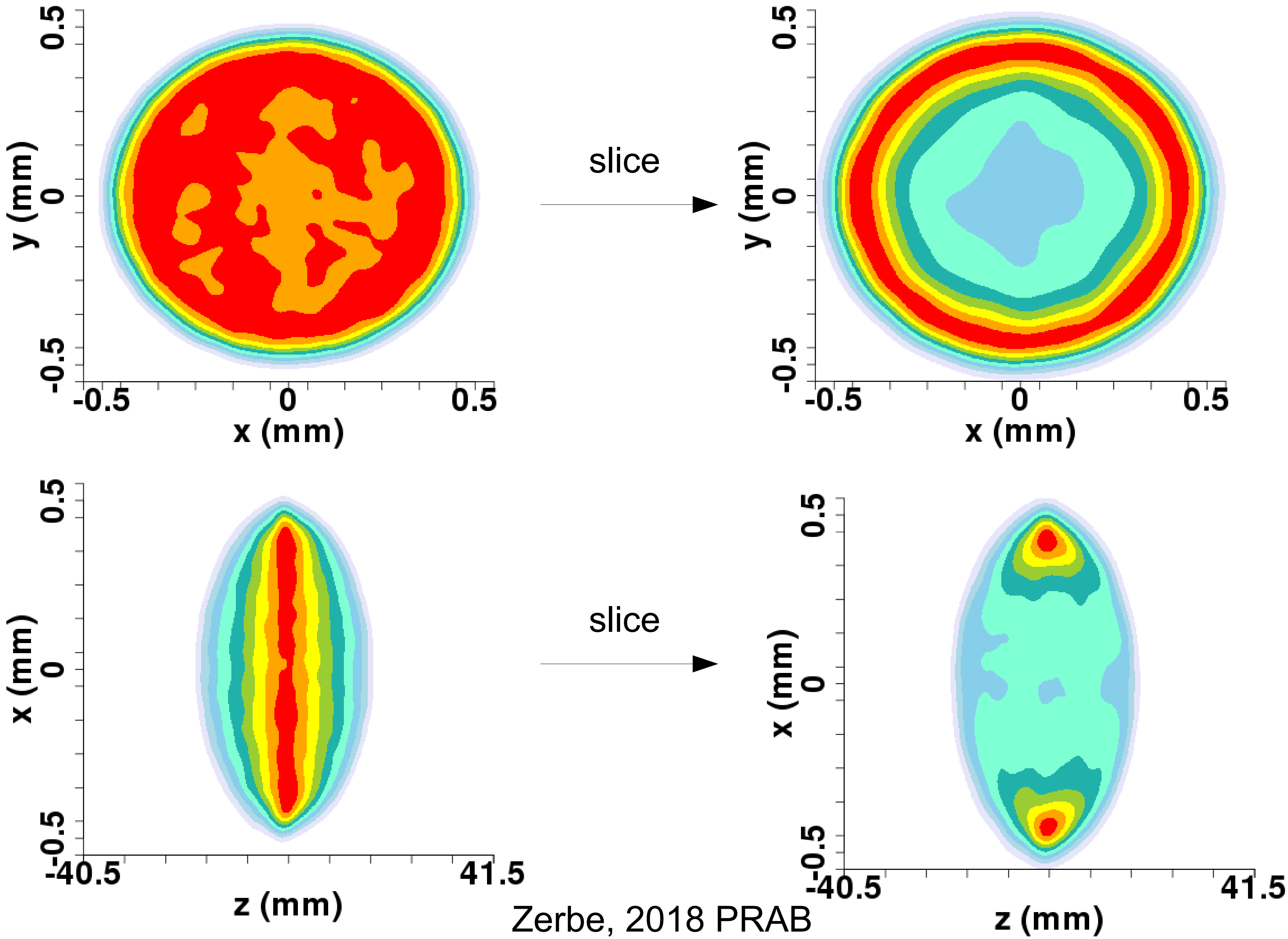
- Short bunches
- Denser bunches
  - Virtual cathode limit

    Valfells, 2002 Physics of Plasmas

  - Space-Charge dominated
    - Intense non-linearities
- Cost: Less overall particles

Pancake (UEM) : $\Delta z << \Delta x$

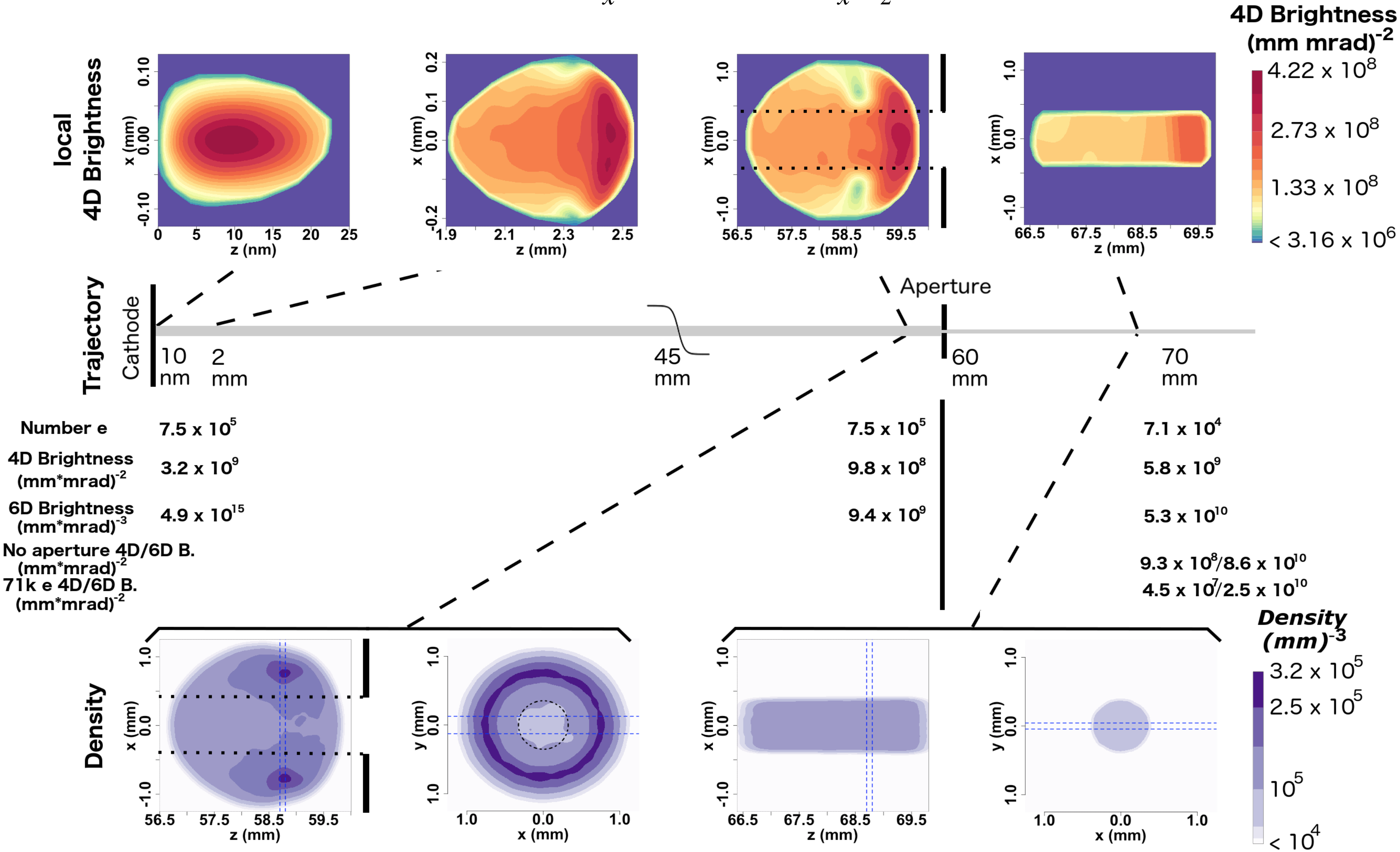
Density Projection Slicing



Zerbe, 2018 PRAB

Aperture → Brightness increase

$$\epsilon_x^2 = |COV(x, p_x)| \qquad B_{4D} = \frac{N}{\epsilon_x^2} \qquad B_{6D} = \frac{N}{\epsilon_x^2 \epsilon_z}$$



Williams, 2017 Presentation

Fluid models for this?

Focussing force

$$\frac{F_e}{mv^2} = -k^2 x$$

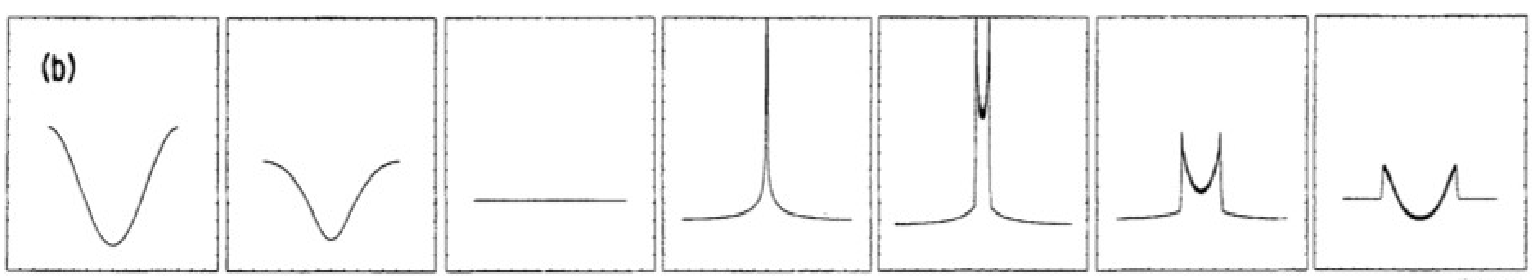
$\rho_0 =$ initial probability-like density

$$\rho_u = \frac{k^2 m v^2}{4\pi e^2} = \frac{N}{Perveance / k^2}$$

$$\rho(r, z) = \frac{\rho_u}{1 + \left(\frac{\rho_u}{\rho_0} - 1\right)\cos(kz)}$$

Formation of shock (simulations only – laminar theory does not see this phenomenon)

ρ



r @ different t's

Anderson, 1987 Part. Accel.

Fluid models for this?

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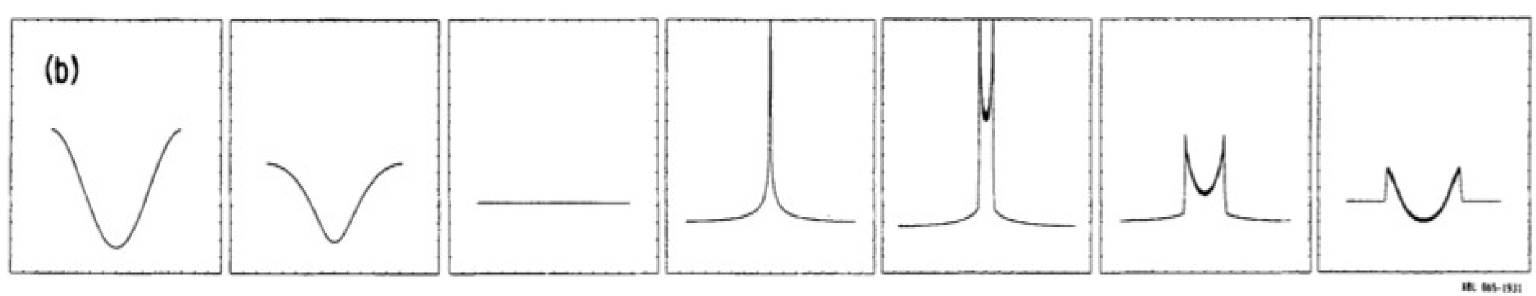


r @ different t's

Anderson, 1987 Part. Accel.

Here k = 0

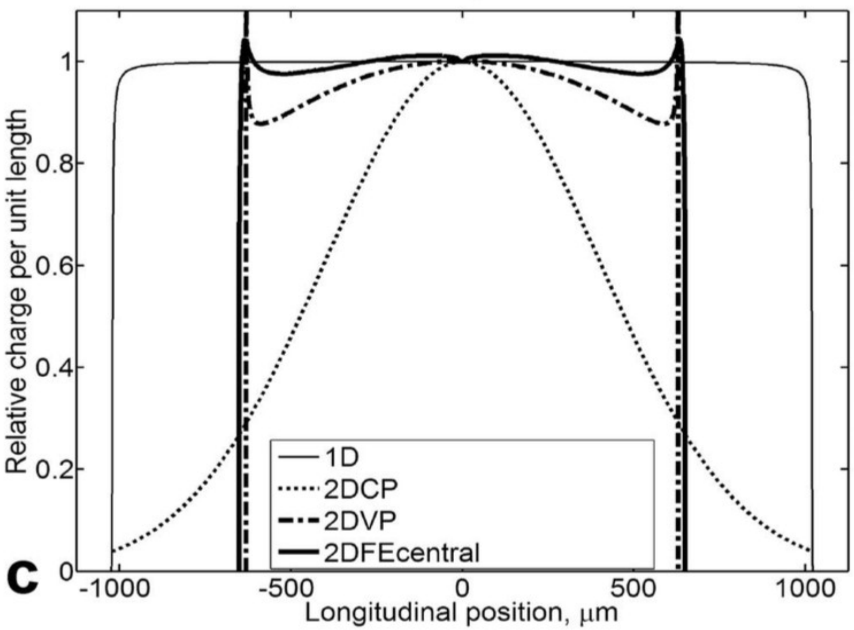
Fluid models for this?

Evolution of pancake width – A 1D problem

Time evolved instead of along trajectory

Longitudinal → $$\rho(z;t)=\frac{\rho_0}{1+\frac{q\Sigma_{tot}}{2m\epsilon_0}\rho_0 t^2}$$ 1D in figure

ρ



z

Reed, 2006 J. Appl. Phys.

Fluid models for this?

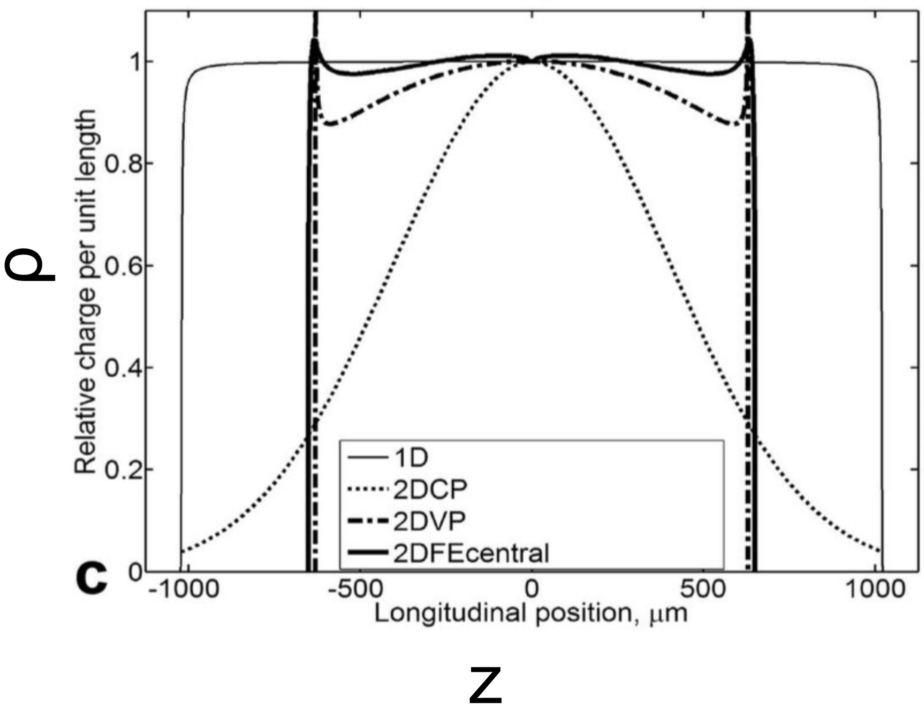
Evolution of pancake width – A 1D problem

Time evolved instead of along trajectory

Longitudinal →

$$\rho(z;t)=\frac{\rho_0}{1+\frac{q\,\Sigma_{tot}}{2\,m\,\epsilon_0}\rho_0 t^2}$$

1D in figure



Attempt to model for dense cylindrical (cigar) beams to see if shock can be observed

Reed, 2006 J. Appl. Phys.

Our fluid models

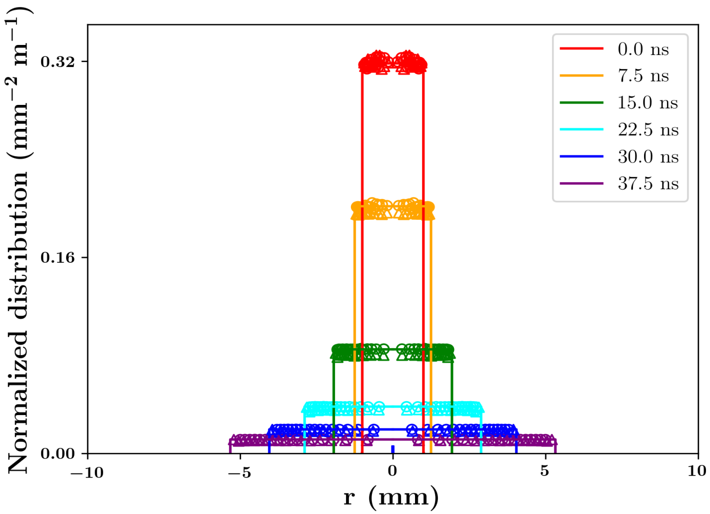
Cylindrical

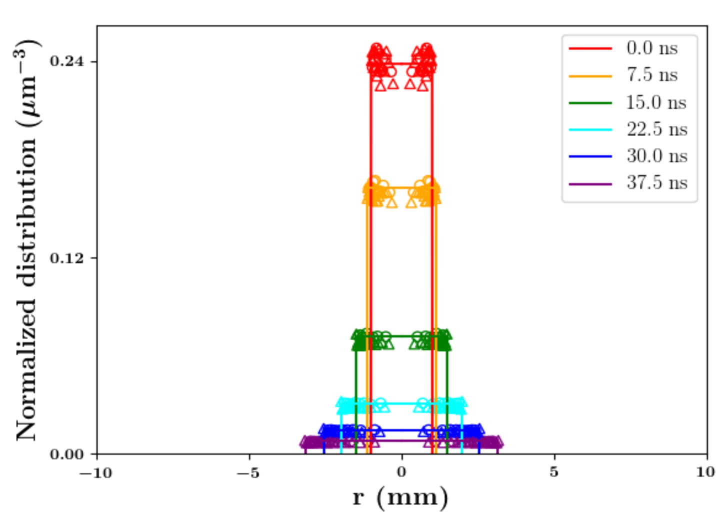
$$\rho(r;t)=\frac{r_0^2}{r^2}\frac{\rho_{02}}{1+\left(\frac{\rho_{02}}{\bar{\rho}_{02}}-1\right)2\sqrt{\ln\left(\frac{r}{r_0}\right)}F\left(\sqrt{\ln\left(\frac{r}{r_0}\right)}\right)}$$

Spherical

$$\rho(r;t)=\frac{r_0^3}{r^3}\frac{\rho_{03}}{1+\frac{3}{2}\left(\frac{\rho_{03}}{\bar{\rho}_{03}}-1\right)\left(\frac{r_0}{r}\sqrt{1-\frac{r_0}{r}}\tanh^{-1}\left(\sqrt{1-\frac{r_0}{r}}\right)+1-\frac{r_0}{r}\right)}$$

uniform



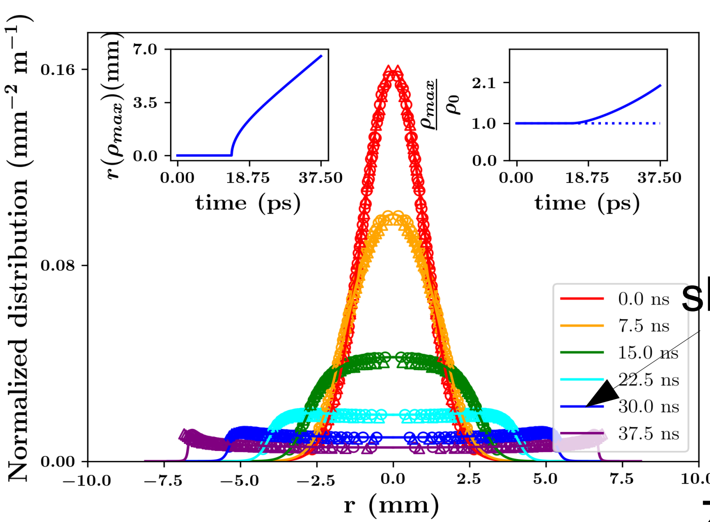


Solid lines: Theory

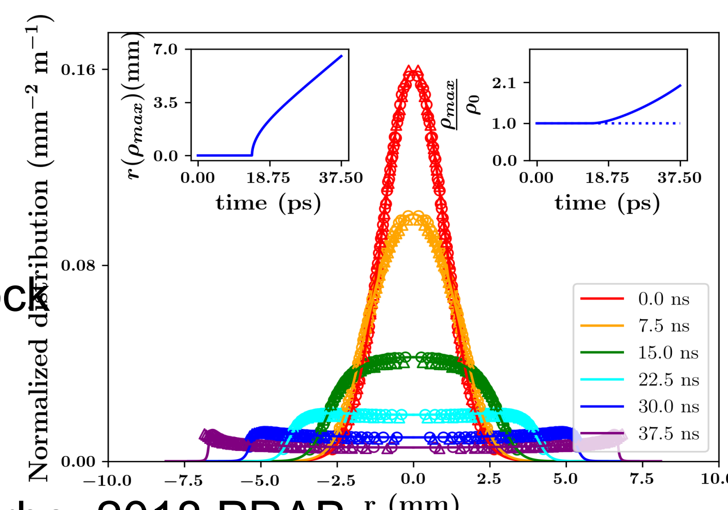
Circles: PIC (Warp)

Triangles: N-particle (COSY Infinity and home-made)

Gaussian



shock



shock

Zerbe, 2018 PRAB

Take aways

- Step away from UEM
  - Discuss cylindrical evolution
- PIC (Vlasov) and N-particle (Coulomb) typical
  - New self-consistent laminar density evolution
    - Validation
    - Identify when it breaks down
- Our previous fluid model has no velocity
  - I fixed that

Laminar beams

Wangler's Prin. RF Linac

The ideal beam with highest beam quality is called the laminar beam because it exhibits laminar-like flow. A laminar beam represents the ideal of a highly ordered and coherent beam, which is never exactly realized.

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Derivation idea

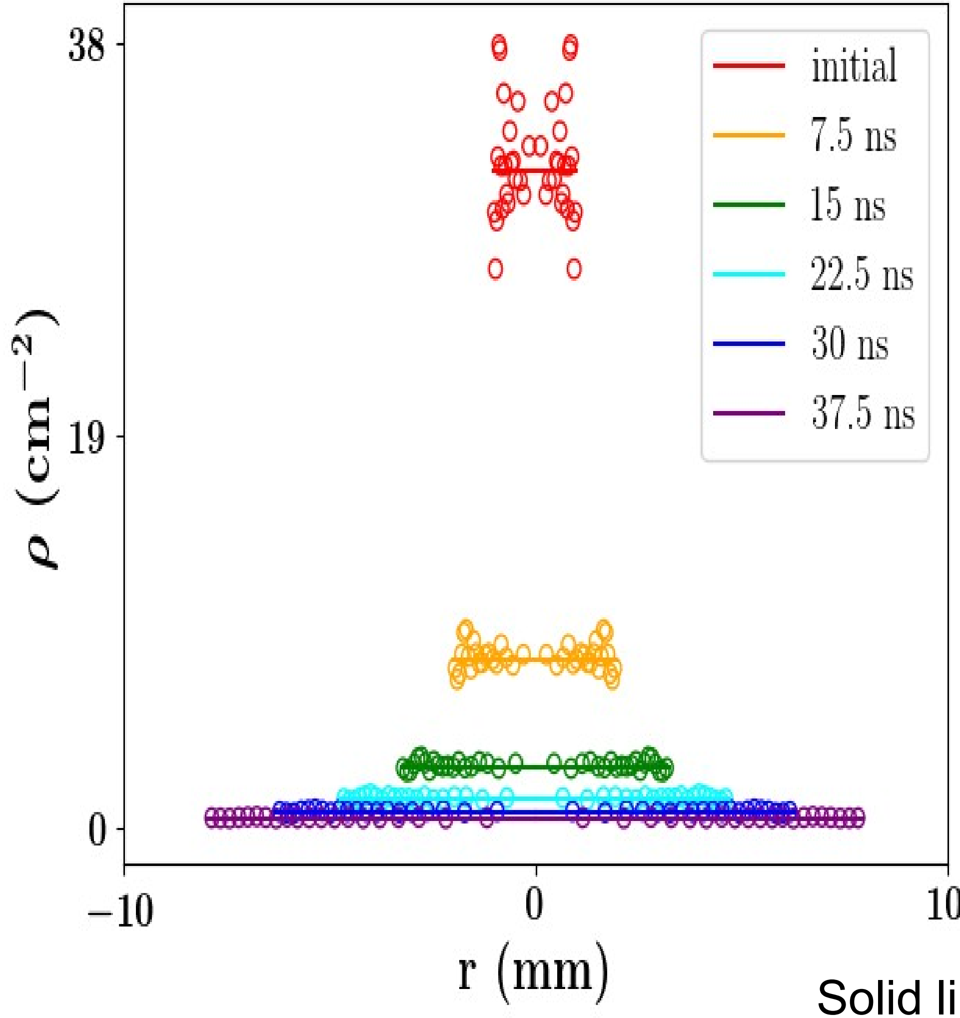
- Lagrangian particle with initial velocity can be mapped to Lagrangian particle at different time that has zero velocity
  - v_{r2}: velocity scale $v_{r2} = \sqrt{\frac{q \Lambda_{tot} \bar{\rho}_{02}}{m \epsilon}} r_0$
  - r_{t2}: turn-around radius $r_{t2} = r_0 e^{-v_0^2 / v_{r2}^2}$
  - t_{ft2}: time-position free expansion relation from turn-around point under spherical symmetry
  - t_{t2} : time to turn around point (- if $v_0 < 0$)
- Time position-relation can be summarized as

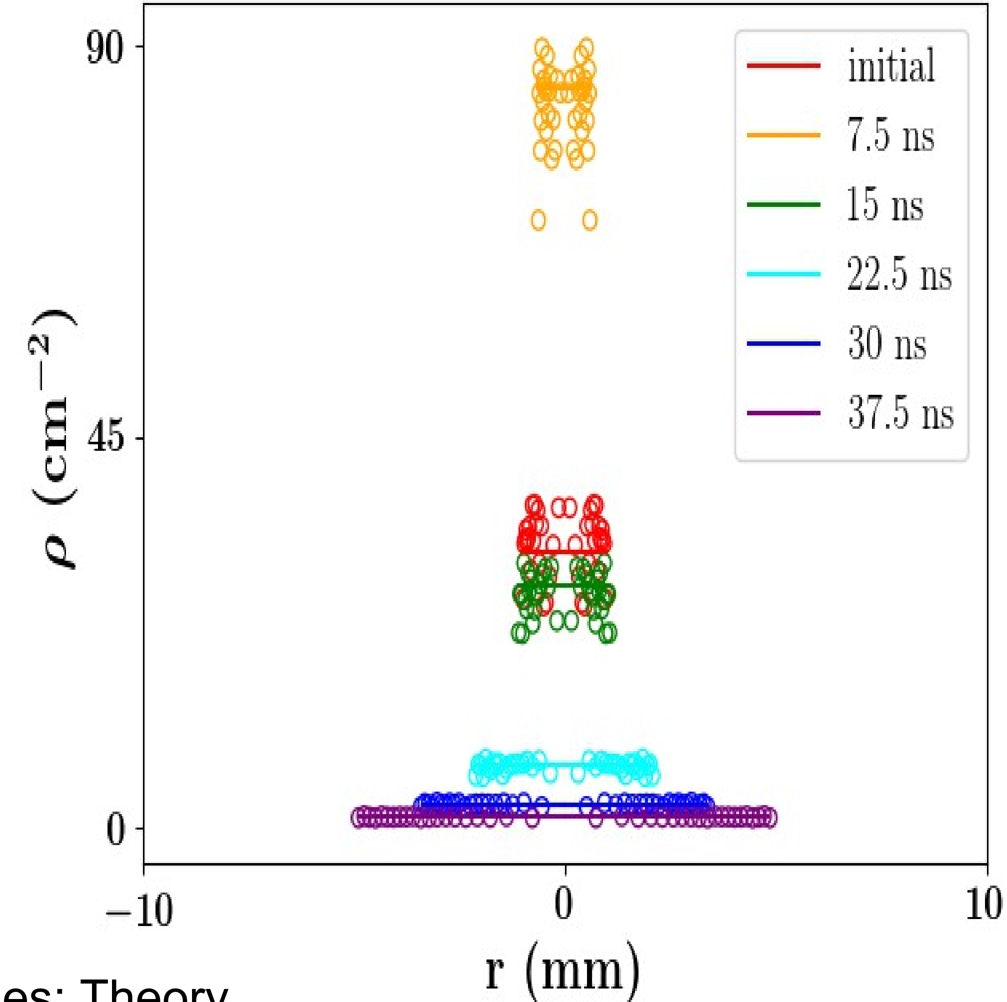
$$t = \pm t_{ft2} - t_{t2}$$

- Evolution derived exactly like previously

Cylindrically-symmetric uniform distribution with non-zero initial velocity

$$v_0 = C\frac{r_0}{R} \qquad v_{r2} = 10^5\frac{m}{s}\frac{r_0}{R}$$





Solid lines: Theory

Circles: PIC (Warp)

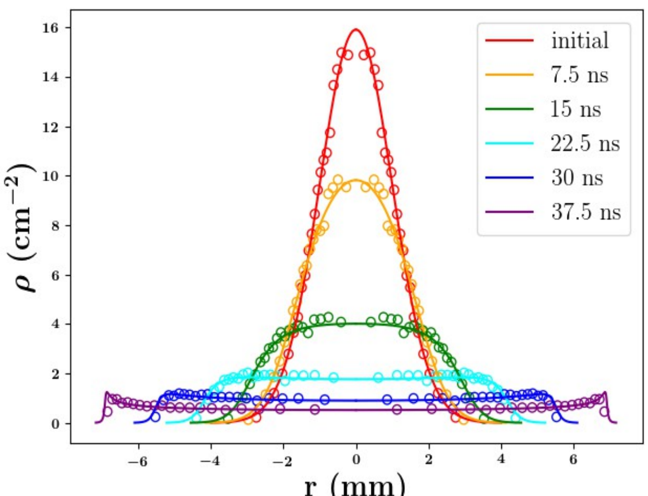
C = 10^5 m/s

Large outward

C = -10^5 m/s

Large inward

Analogous Gaussian



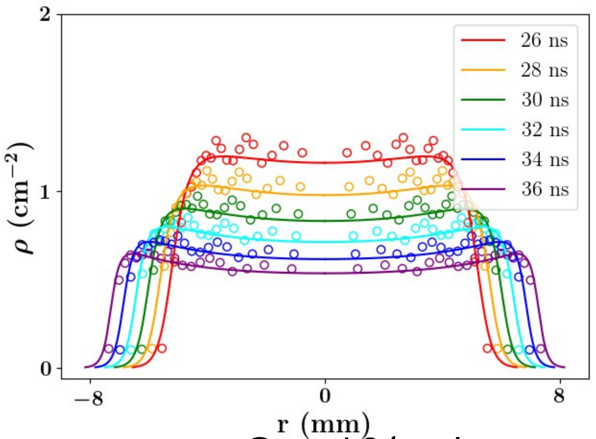
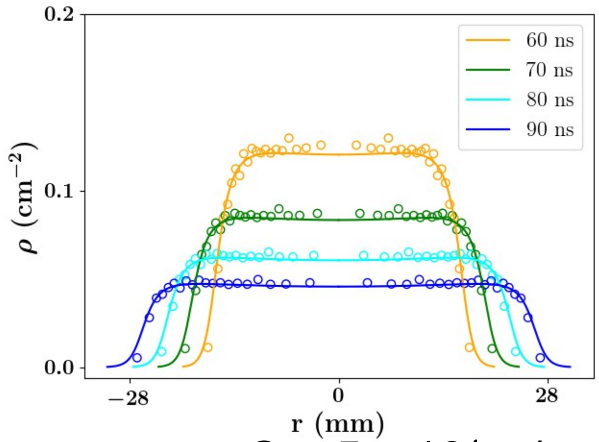
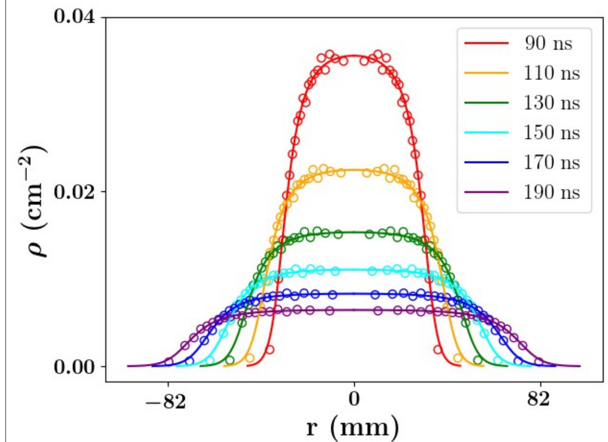
C = 0

$$v_0 = C \frac{r_0}{\sigma_r}$$

$$v_{r2} = 1.4\, x\, 10^5 \frac{m}{s} \sqrt{1 - e^{\frac{-r_0^2}{2\sigma_r^2}}}$$

Solid lines: Theory

Circles: PIC (Warp)

	Small velocity	**Medium velocity**	**Large velocity**
Outward	 C = 10^4 m/s	 C = 5 x 10^4 m/s	 C = 10^5 m/s

What is going on here?

$$r_{t2} = r_0 e^{-v_0^2/v_{r2}^2}$$

If $v_0 << v_{r2}$

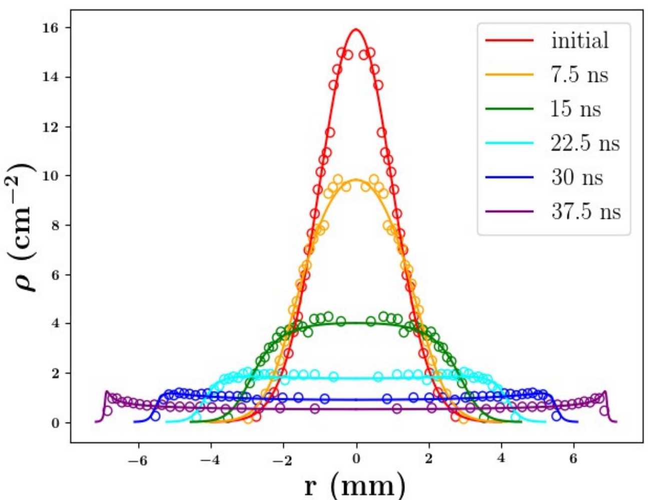
$r_{t2} \approx r_0$ → Earlier, cold SC-dominated model approximate – slightly delays shock

If $v_0 >> v_{r2}$

Initial velocity profile very important

→ Here: transforms Gaussian to uniform-like, i.e. it looses the shock

Analogous Gaussian



C = 0

$$v_0 = C\frac{r_0}{\sigma_r} \qquad v_{r2} = 1.4\,x\,10^5\,\frac{m}{s}\sqrt{1 - e^{\frac{-r_0^2}{2\sigma_r^2}}}$$

Solid lines: Theory

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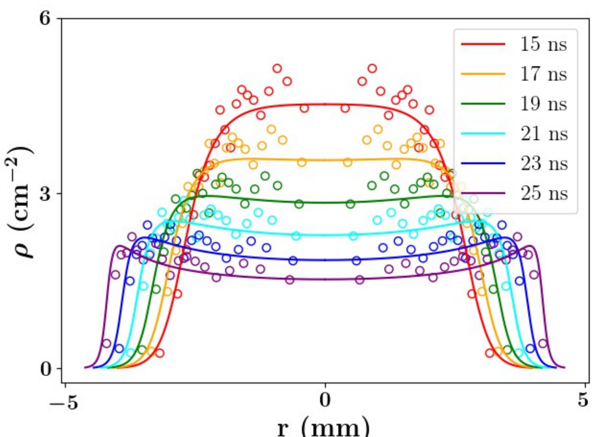
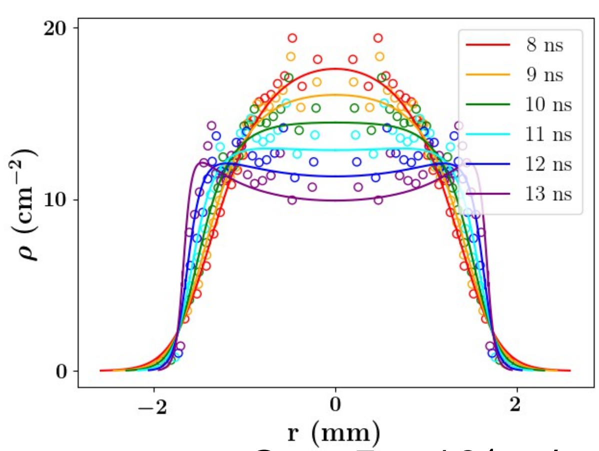
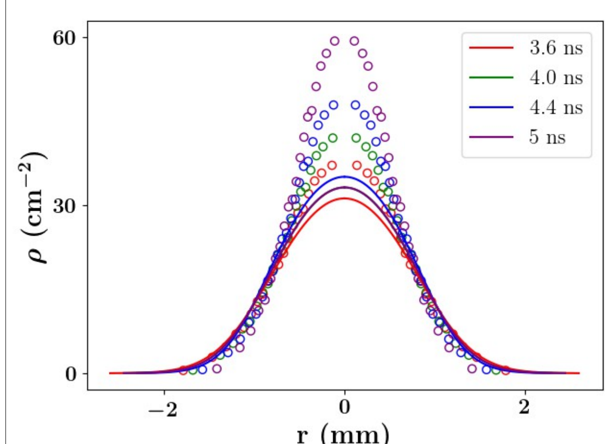
	Small velocity	Medium velocity	Large velocity
Inward	 C = -10^4 m/s	 C = -5 x 10^4 m/s	 C = -10^5 m/s

What is going on here?

$$r_{t2} = r_0 e^{-v_0^2/v_{r2}^2}$$

If $v_0 << v_{r2}$

$$r_{t2} \approx r_0$$

→ Earlier, cold SC-dominated model still approximate – slightly moves shock earlier

If $v_0 >> v_{r2}$

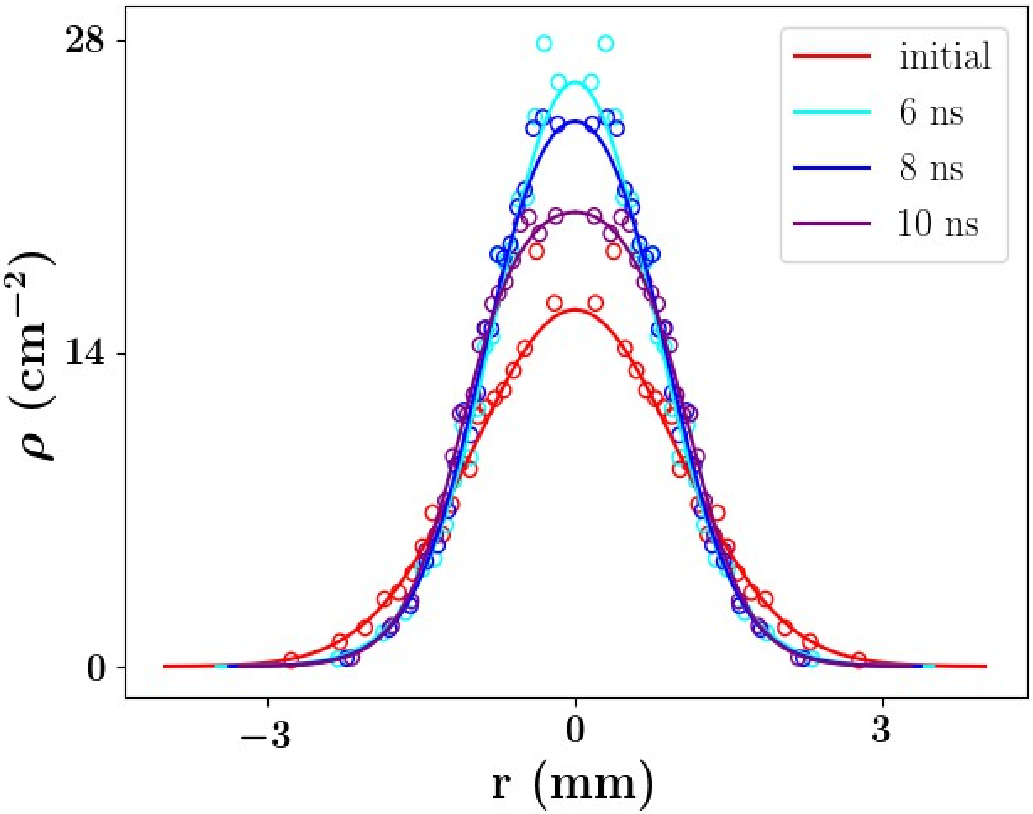
Initial velocity profile very important

→ Here: linear velocity breaks laminar assumption in the middle of the bunch, so bunch bounces back faster in model than reality

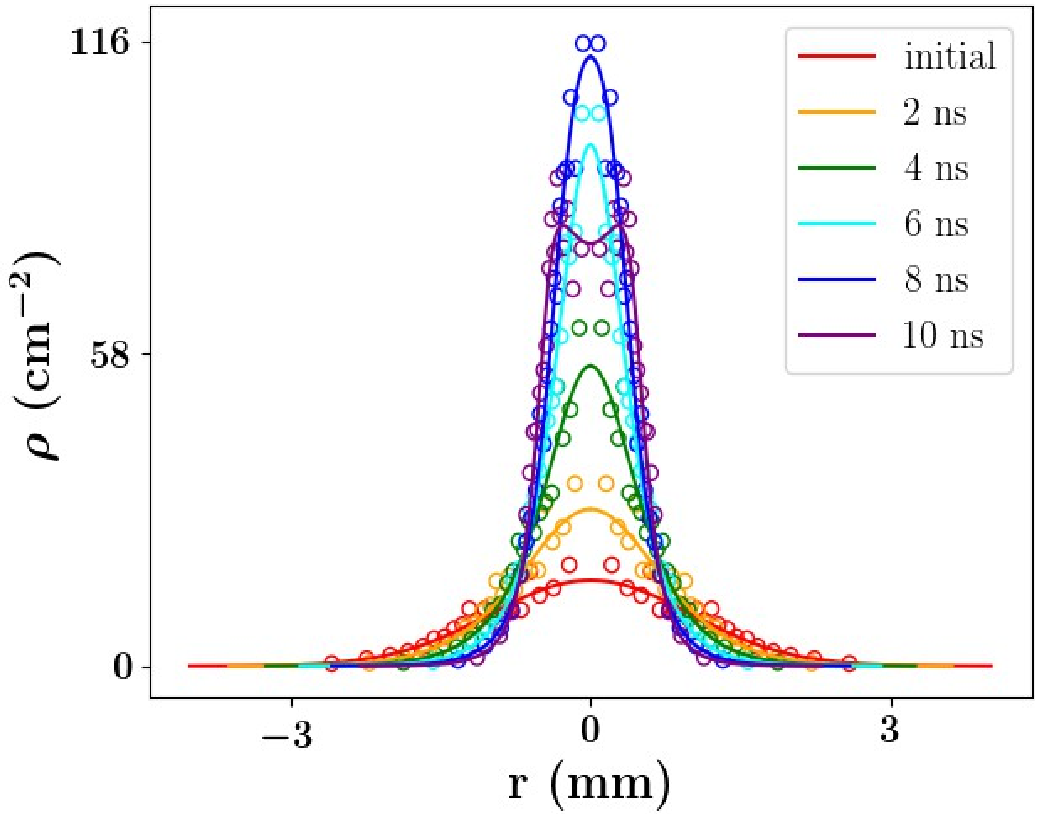
Cylindrical Gaussian with spatially non-linear initial velocity

$$v_0 = C\sqrt{1-e^{\frac{-r_0^2}{2\sigma_r^2}}}$$

$$v_{r2} = 1.4\,x\,10^5\,\frac{m}{s}\sqrt{1-e^{\frac{-r_0^2}{2\sigma_r^2}}}$$



C = -10^5 m/s



C = -2 x 10^5 m/s

Solid lines: Theory

Circles: PIC (Warp)

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Conclusions

- Surprisingly accurate self-consistent analytic model that predicts laminar density evolution
  - VERY fast
  - Able to predict through crossovers, i.e. focii
- Predicts when beam becomes non-laminar
  - Temperature?
- Physics captured by velocity scale, v_{r2}
- Spherical case in paper (similar)
- Expect pancake regime to be qualitatively similar to higher dimensions