Constrained multi-objective shape optimization of superconducting RF cavities to counteract dangerous higher order modes

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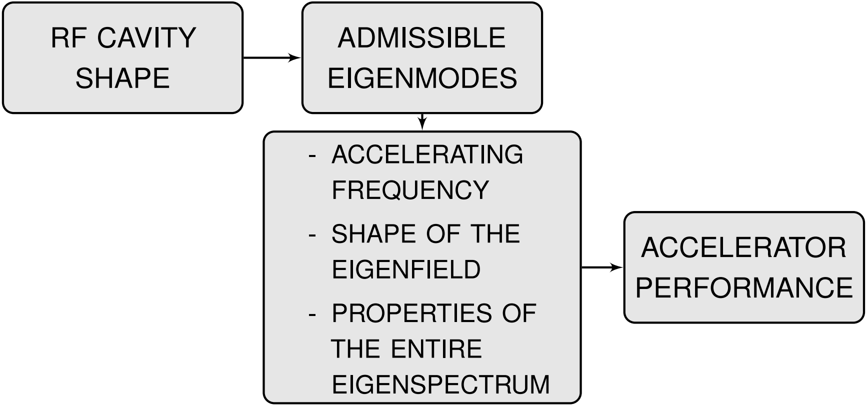
Shape optimization of RF cavities



RF CAVITY SHAPE

ADMISSIBLE EIGENMODES

- ACCELERATING FREQUENCY
- SHAPE OF THE EIGENFIELD
- PROPERTIES OF THE ENTIRE EIGENSPECTRUM

ACCELERATOR PERFORMANCE

- mainly optimized wrt. the properties of the fundamental mode[1]

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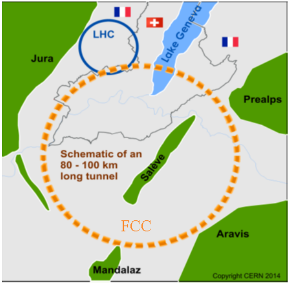
[1] V. Shemelin, S. Gorgi Zadeh, J. Heller and U. van Rienen, Systematical study on superconducting radio frequency elliptic cavity shapes applicable to future high energy accelerators and energy recovery linacs, Phys. Rev. Accel. Beams 19, 2016. `https://doi.org/10.1103/PhysRevAccelBeams.19.102002`

Future Circular Collider (FCC)

FCC-ee[2]

- circular lepton collider
- 100 km circumference, Geneva
- Z operating mode (FCC-ee-Z) (single-cell, Nb/Cu, 400.79 MHz)



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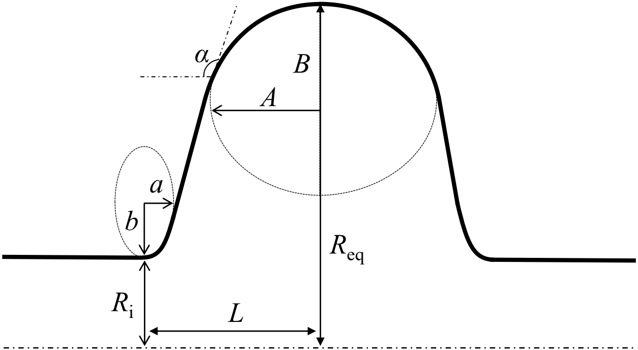
[2] S. Gorgi Zadeh, R. Calag, F. Gerigk and U. van Rienen, FCC-ee Hybrid RF Scheme, In Proceedings of IPAC2018, Vancouver, BC, Canada, 2018. `https://doi.org/10.18429/JACoW-IPAC2018-MOPMF036`

Single-cell elliptical cavity parameterization



- axisymmetric, variables R_i, L, A, B, a, b, $(R_{eq}) \rightarrow \alpha$

Constrained multi-objective optimization problem

- monopole and dipole modes major sources of beam instability

$$\min_{R_i, L, A, B, a, b} \quad (\overbrace{f_0 - f_1}^{F_1}, \overbrace{|f_1 - f_2|}^{F_2}, \overbrace{\frac{R}{Q}_{\perp 1} + \frac{R}{Q}_{\perp 2}}^{F_3}, \overbrace{-G_0 \cdot \frac{R}{Q}_0}^{F_4}),$$

$$\text{subject to} \quad f_0 = 400.79 \text{ MHz}, \quad \alpha \geq 90^\circ$$

- f_0 ... frequency of the fundamental mode
- f_1, f_2 ... frequency of the first and second dipole mode, resp.
- $\frac{R}{Q}_{\perp}$ ... transverse shunt impedance for the dipole modes[3]
- G_0 ... geometry factor [4]

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[3] B. P. Xiao et al., Higher Order Mode Filter Design for Double Quarter Wave Crab Cavity for the LHC High Luminosity Upgrade, In Proceedings of IPAC2015, Richmond, VA, USA , 2015. `https://doi.org/10.18429/JACoW-IPAC2015-WEPWI059`

[4] J. Sekutowicz et al., Cavities for JLAB's 12 GEV Upgrade, In Proceedings of PAC2003, Portland, OR, USA, 2003. `https://doi.org/10.1109/PAC.2003.1289717`

Forward solver

- Maxwell's equations
  - frequency domain
  - axisymmetric domain in 3D[5,6]
  - vacuum; no external fields, sources or charges; PEC
- FEM → a GEVP for each azimuthal mode number $m \in \mathbb{N}_0$
- smallest eigenpair for (using half of the cross section)
  - $m = 0$, PEC → properties of the fundamental mode (TM_{010})
  - $m = 1$, PEC → properties of the dipole mode TM_{110}
  - $m = 1$, PMC → properties of the dipole mode TE_{111}

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[5] P. Arbenz, O. Chinellato, On solving complex-symmetric eigenvalue problems arising in the design of axisymmetric VCSEL devices, Appl. Numer. Math. 58 (4): 381-394, 2008. `https://doi.org/10.1016/j.apnum.2007.01.019`

[6] O. Chinellato, The complex-symmetric Jacobi–Davidson algorithm and its application to the computation of some resonance frequencies of anisotropic lossy axisymmetric cavities, ETH Zurich (Diss. ETH No. 16243), 2005. `https://doi.org/10.3929/ethz-a-005067691`

Pareto optimality

Definition

A point $\boldsymbol{d_1} = (R_{i,1}, L_1, A_1, B_1, a_1, b_1)$ dominates $\boldsymbol{d_2}$ if

$$\forall i \in \{1, \dots 4\}, \quad F_i(\boldsymbol{d_1}) \leq F_i(\boldsymbol{d_2}) \quad \text{and}$$
$$\exists i \in \{1, \dots 4\}, \quad F_i(\boldsymbol{d_1}) < F_i(\boldsymbol{d_2}).$$

Definition

A point $\boldsymbol{d}$ is Pareto optimal if it is not dominated by any other point.

Evolutionary algorithm (EA)

- evaluate a random population of individuals I_i, $i = 1, \ldots, N$
- for a predetermined number of generations do
  - variator: for pairs of individuals I_i, I_{i+1}, perform:

    $$\mathit{crossover}(I_i, I_{i+1}), \quad \mathit{mutation}(I_i), \quad \mathit{mutation}(I_{i+1})$$
  - evaluate new individuals
  - selector: choose N fittest individuals for the next generation

- massively parallel implementation[7] also used in OPAL
- combined with the axisymmetric Maxwell eigensolver[8]

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[7] Y. Ineichen et al., A fast and scalable low dimensional solver for charged particle dynamics in large particle accelerators, Comput. Sci. Res. Dev. 28 (2) (2013) 185-192. `https://doi.org/10.1007/s00450-012-0216-2`

[8] M. Kranjčević, A. Adelmann, P. Arbenz, A. Citterio and L. Stingelin, Multi-objective shape optimization of radio frequency cavities using an evolutionary algorithm, ArXiv e-prints arXiv:1810.02990, 2018.

Constraint handling

- f_0 = 400.79 MHz
  - given $\boldsymbol{d} = (R_i, L, A, B, a, b)$, find R_{eq} s.t. f_0 = 400.79 MHz
  - root-finding method[9] on (in mm) [325, 375]
  - if $|f_0 - 400.79\text{ MHz}| \geq 1$ MHz, fine mesh eigensolve avoided (on average, 4 fine eigensolves for each $\boldsymbol{d}$)
- $\alpha \geq 90^\circ$ ... otherwise, the individual is discarded

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[9] G. E. Alefeld, F. A. Potra and Y. Shi, Algorithm 748: Enclosing Zeros of Continuous Functions, ACM Trans. Math. Softw. 21 (3) (1995) 327-344. `https://doi.org/10.1145/210089.210111`

Results

- Euler cluster[10] (Euler I and II) of ETH Zurich

FORWARD SOLVE:

- coarse eigensolves ... 10'000 triangles, 2 s
- fine eigensolves ... 300'000 triangles, 90 s
  (24 s meshing, 64 s eigenpairs, 2 s objective function values)
- 4 fine eigensolves to find R_{eq} and the properties of TM_{010}
- 2 fine eigensolves to find the properties of TM_{110} and TE_{111}
  (no remeshing)

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[10] https://scicomp.ethz.ch/wiki/Euler

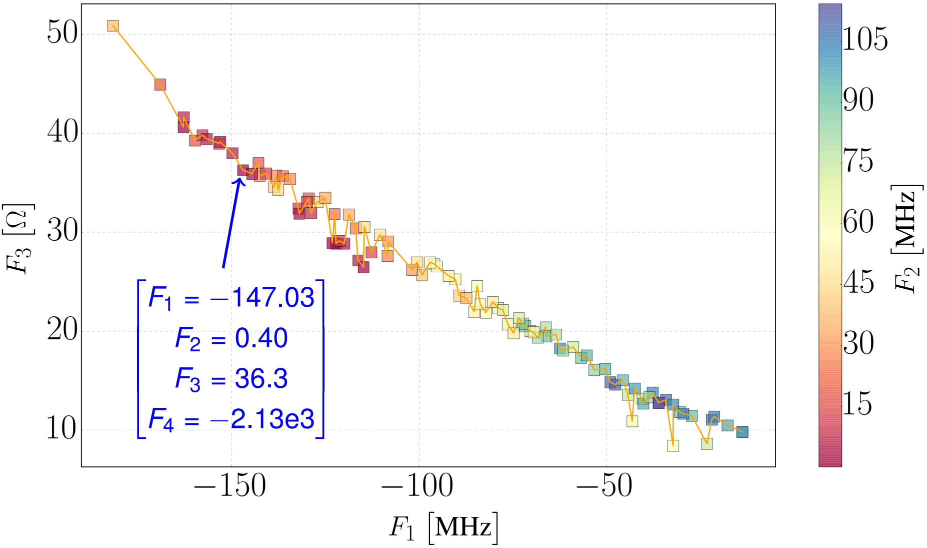
Results

OPTIMIZATION:

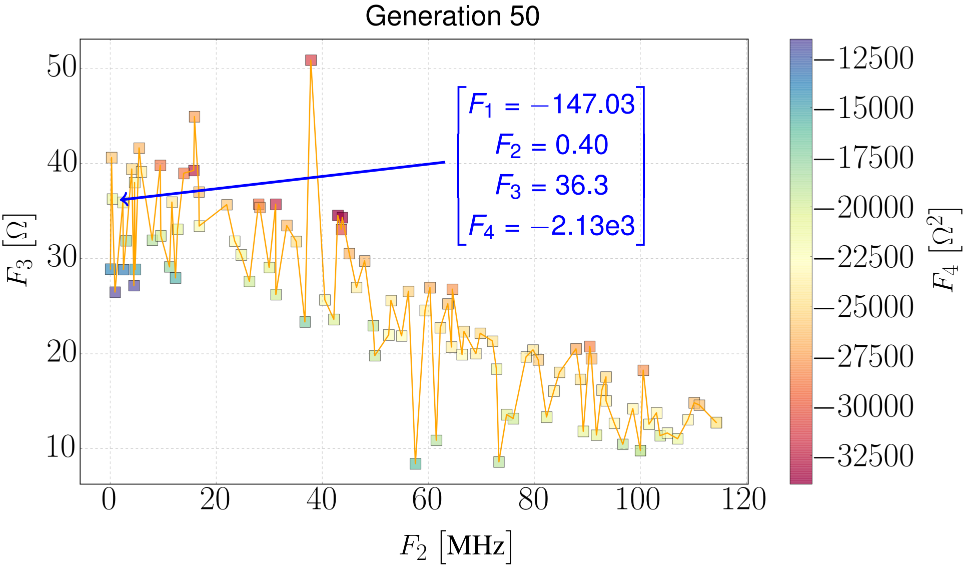
- 13 h for 50 generations with $N = 100$ on 96 processes (30% of the individuals discarded)
- initial design variable bounds:

Variable	R_i	L	A	B	a	b
Lower bound [mm]	145	120	40	40	10	10
Upper bound [mm]	160	190	140	140	70	70

Generation 50



$$\begin{bmatrix} F_1 = -147.03 \\ F_2 = 0.40 \\ F_3 = 36.3 \\ F_4 = -2.13\text{e}3 \end{bmatrix}$$



$$\begin{bmatrix} F_1 = -147.03 \\ F_2 = 0.40 \\ F_3 = 36.3 \\ F_4 = -2.13\text{e}3 \end{bmatrix}$$

Fundamental mode of the chosen RF cavity

Description of the chosen RF cavity

Variable	R_i [mm]	L [mm]	A [mm]	B [mm]
Value	141.614	146.270	103.54	127.521

Variable	a [mm]	b [mm]	R_{eq} [mm]	α [°]
Value	41.921	45.812	339.166	91.697

Objective	F_1 [MHz]	F_2 [MHz]	F_3 [Ω]	F_4 [Ω^2]
Value	-147.03	0.40	36.3	$-2.13e3$

TM_{010}	$f_0 = 400.79$ MHz	$R/Q_0 = 94.9\ \Omega$
	$E_{pk}/E_{acc} = 1.92$	$B_{pk}/E_{acc} = 4.16$ mT/(MV/m)
TE_{111}	$f_1 = 547.82$ MHz	$R/Q_{\perp 1} = 5.10\ \Omega$
TM_{110}	$f_2 = 548.22$ MHz	$R/Q_{\perp 2} = 31.2\ \Omega$

Conclusions

- optimized the shape of the superconducting RF cavity for the FCC-ee-Z wrt. the fundamental mode and the first dipole band
- new optimization algorithm
  - EA + constraint handling
  - other axisymmetric RF structures
  - HOMs corresponding to arbitrary azimuthal mode numbers

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