Theoretical and computational modeling of a plasma wakefield BBU instability

Stephen Webb, Nathan Cook, David Bruhwiler

RadiaSoft, LLC., Boulder, CO

Alexei Burov, Valeri Lebedev, Sergei Nagaitsev

Fermi National Accelerator Laboratory, Batavia, IL

swebb@radiasoft.net

2018 Int'l. Computational Accelerator Physics Conference

24 October 2018

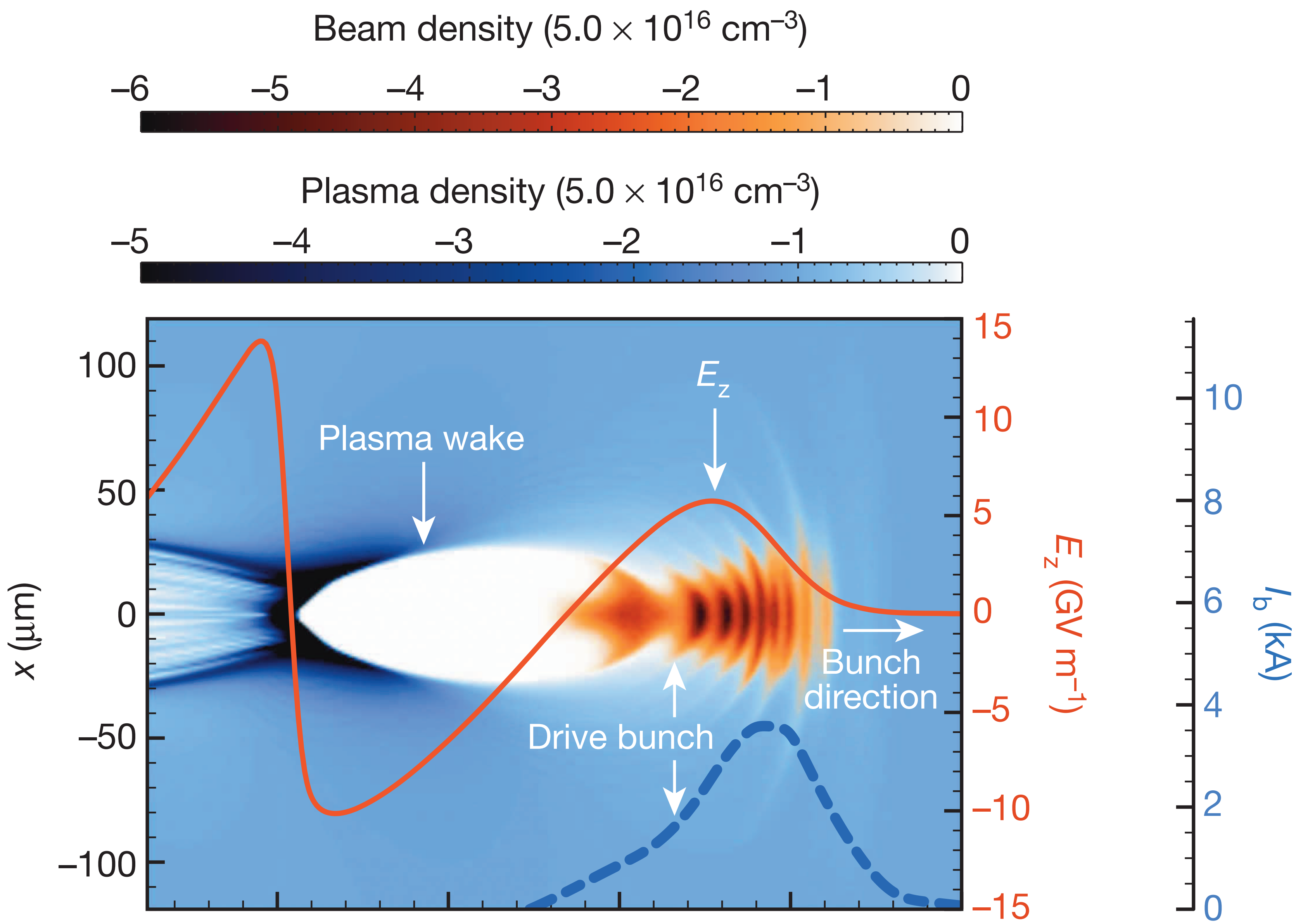
No project exists in a vacuum

Thanks to the FBPIC team, in particular Remi Lehe

https://github.com/fbpic/fbpic

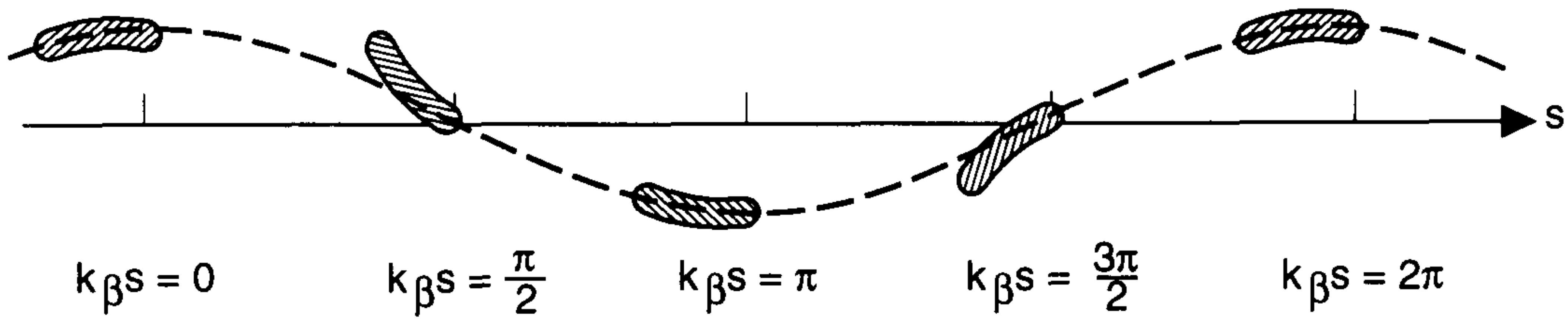
This work was supported by the Department of Energy, Office of Science, Office of High Energy Physics under contract no. DE-SC0018718

plasma accelerators at a glance



from M. Litos *et al.*, "High-efficiency acceleration of an electron beam in a plasma wakefield accelerator", *Nature* **92**, 515 (2014).

the beam break-up instability



$$\frac{d}{ds}\left[\gamma(s)\frac{dy_1}{ds}\right] + k_\beta^2\gamma y_1 = 0$$

$$\frac{d}{ds}\left[\gamma(s)\frac{dy_2}{ds}\right] + k_\beta^2\gamma y_2 = -\frac{N r_e W}{2\gamma(s)L} y_1$$

$$\sim a^{-4}$$

$$\Upsilon = -\frac{N r_e W_1(z) L_0}{4 k_\beta \gamma_i L} \ln\frac{\gamma_f}{\gamma_i}$$

fig. from A. Chao, *Physics of Collective Beam Instabilities in High Energy Physics*, J. Wiley & Sons (1993).

BBU is mitigated with BNS

a difference in betatron frequency
across the bunch detunes the instability

$$\frac{d}{ds}\left[\gamma(s)\frac{dy_2}{ds}\right] + (k_\beta + \Delta k_\beta)^2 \gamma y_2 = -\frac{N r_e W}{2\gamma(s) L} y_1$$

BBU does not grow if

$$\frac{\Delta k_\beta}{k_\beta} = \frac{\Upsilon}{k_\beta L_0} = \xi \frac{\delta p}{p}$$

Correlated energy spread
across the bunch damps BBU

chromaticity

see V. Balakin, A. Novokhatsky, and V. Smirnov, *Proc. 12th Int'l. Conf. High Energy Accel.* Fermilab (1983).

how this applies to plasma accelerators: hosing instability

We turn next to consider mechanisms which will tend to reduce growth. We observe from the dispersion relation of Eq. (8), that there are in principle two methods of "curing" the electron-hose. We may diminish the resonance at $\omega^2 \rightarrow \omega_0{}^2$, or at $k^2 \rightarrow k_\beta{}^2$. On the other hand, since focussing is typically weak, damping mechanisms relying on a spread or sweep in betatron wavenumber, $\Delta k_\beta \sim 1/L_g$, are ineffective, as they require an impractically large spread, $\Delta k_\beta / k_\beta \sim 1/k_\beta L_g > 1$. This rules out Landau damping due to a spread in energy within a beam slice,[24] and "BNS damping" due to a sweep in energy from head to tail.[26] This also rules out "phase-mix damping" of BBU growth due to nonlinear focussing, arising from a radially non-uniform plasma.[13]

from D. H. Whittum *et al.*, "Electron-hose instability in the ion-focused regime", *Phys. Rev. Lett.* **67**, 991 (1991).

efficiency vs. instability

power transferred

$$\eta_p = \frac{P_t}{P}$$

power in

wake defocusing

$$\eta_t = -\frac{F_t}{F_r}$$

ion channel focusing

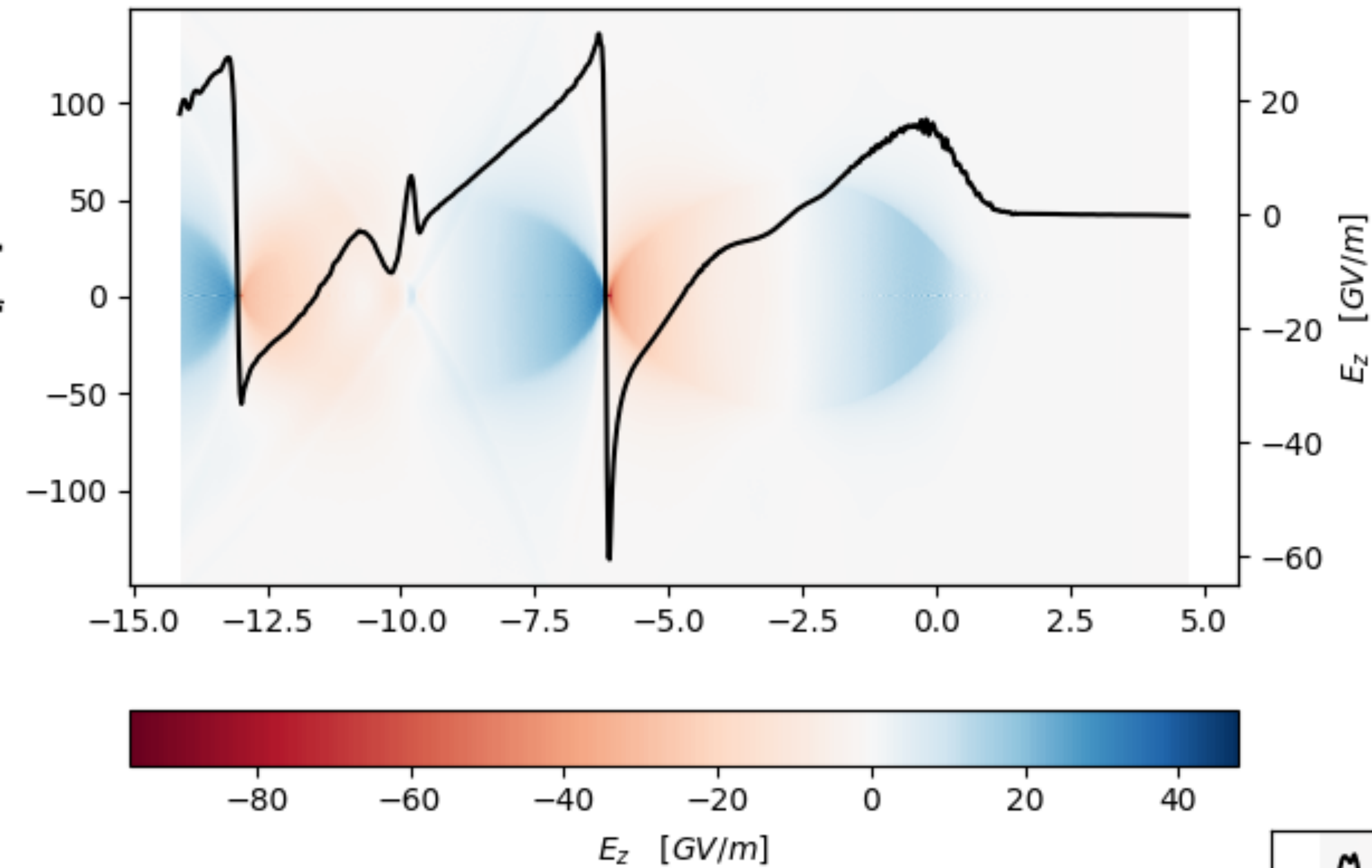
in a plasma accelerator, the focusing and accelerating structure is the same, and these quantities are related.

$$\eta_t \approx \frac{\eta_p^2}{4(1-\eta_p)}$$

Achieving high efficiency in a plasma accelerator makes the BBU instability in the witness bunch worse.

see V. Lebedev, A. Burov, S. Nagaitsev, "Efficiency versus instability in plasma accelerators" *Phys. Rev. Acc. Beams* **20**, 121301 (2017).

How does all this apply to a plasma wakefield accelerator?

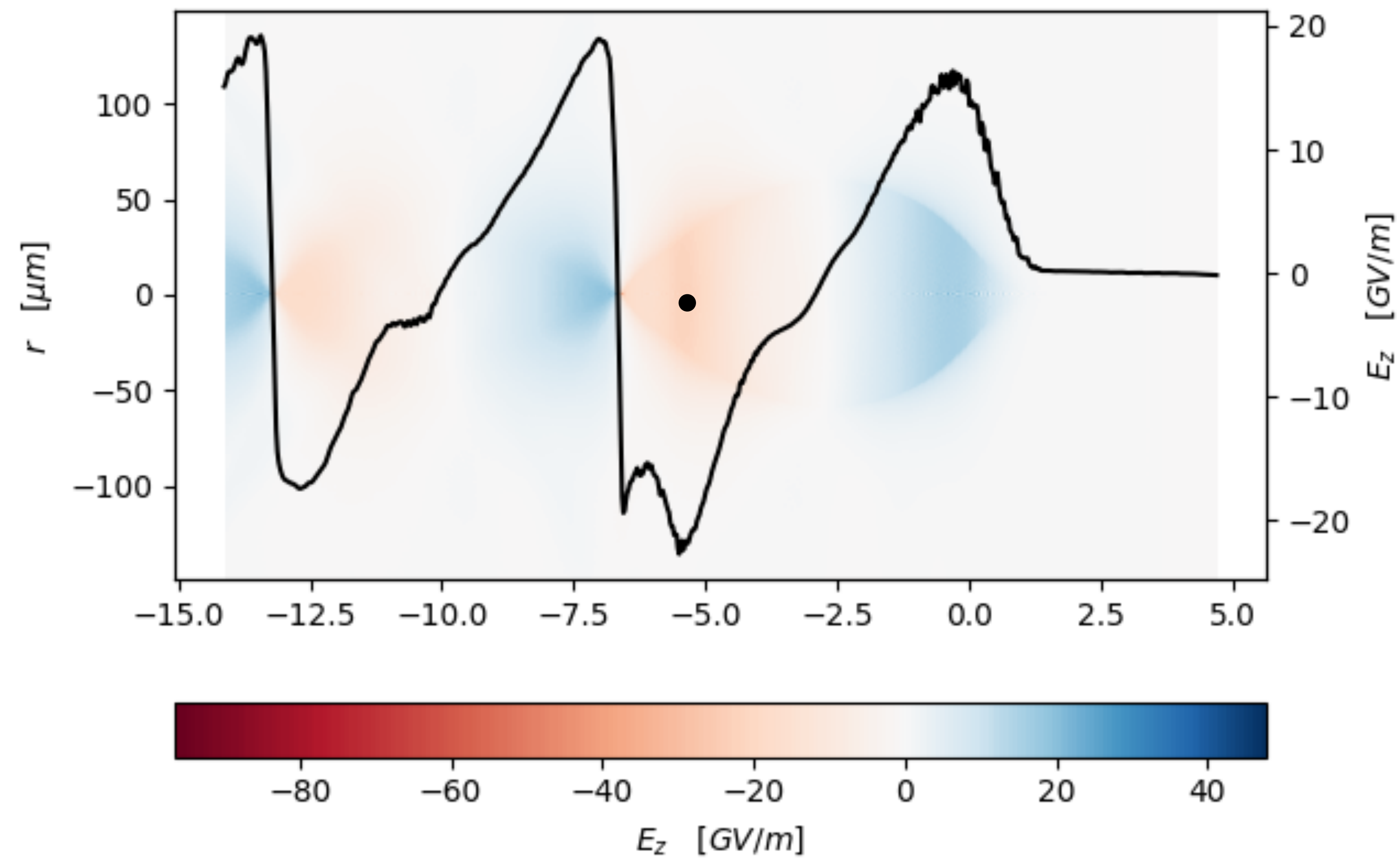


unloaded wake

parameter	value
n_{pe}	$4.0 \times 10^{19}\,\mathrm{cm}^{-3}$
λ_p	$166.9\,\mu\mathrm{m}$
N_e	10^{10}
σ_r	$3.65\,\mu\mathrm{m}$
σ_z	$12.77\,\mu\mathrm{m}$

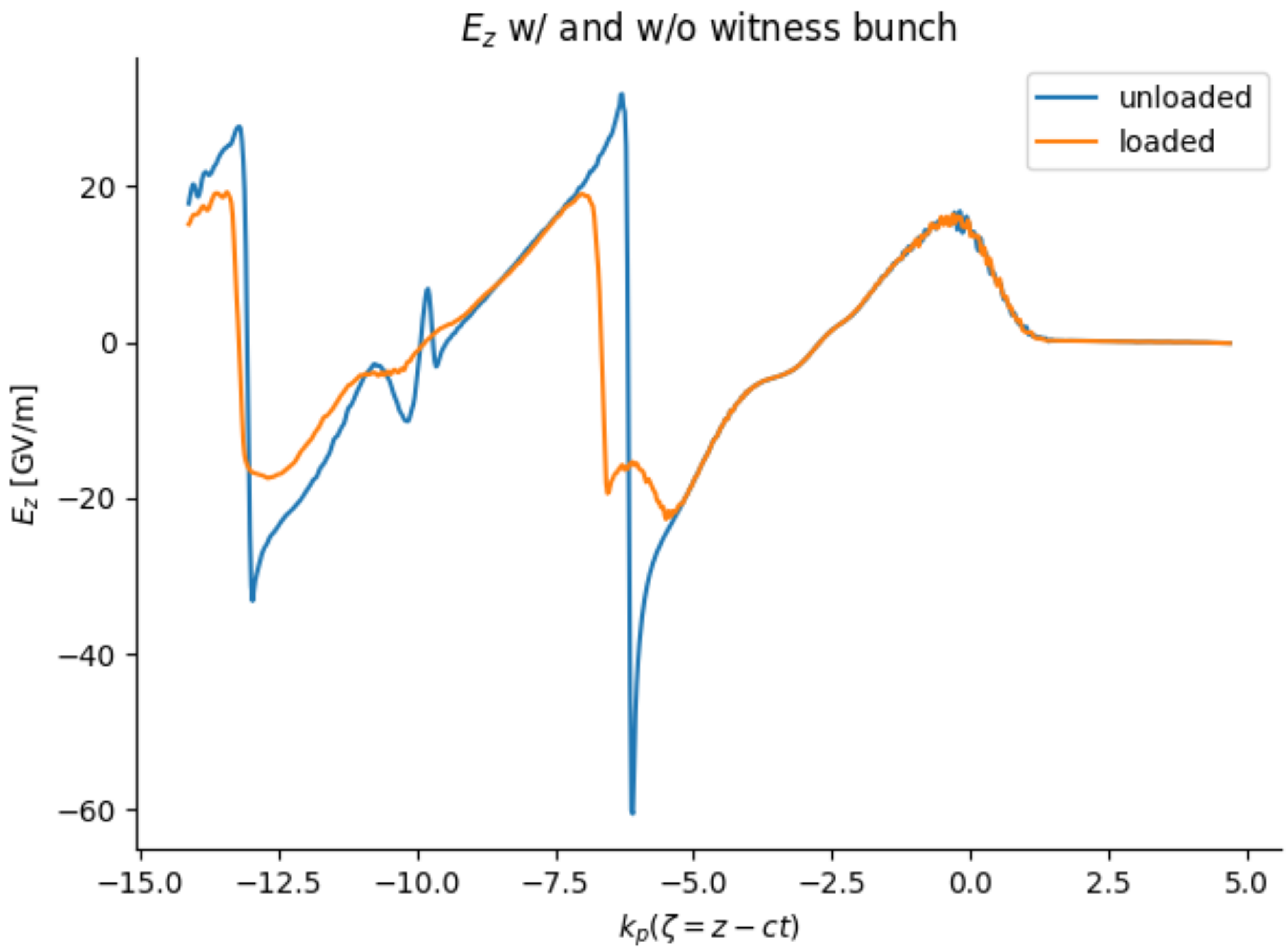
loaded wake

parameter	value
N_e	4.3×10^{9}
σ_r	$3.65\,\mu\mathrm{m}$
σ_z	$6.38\,\mu\mathrm{m}$
z_0	$150\,\mu\mathrm{m}$



can we describe this different with wake functions?

$$E_z \propto \int d\zeta' W(\zeta - \zeta') I(\zeta')$$



extracting wake functions from PIC simulations

run PIC simulation
of PWFA and compute fields

$$(F_z, \mathbf{F}_\perp)(\zeta, \mathbf{x}_\perp)$$

$$\mathrm{FFT}\left[(F_z, \mathbf{F}_\perp)(\zeta, \mathbf{x}_\perp)\right] =$$

$$(\tilde{F}_z, \tilde{\mathbf{F}}_\perp)(k, \mathbf{x}_\perp)$$

"impedance ansatz"

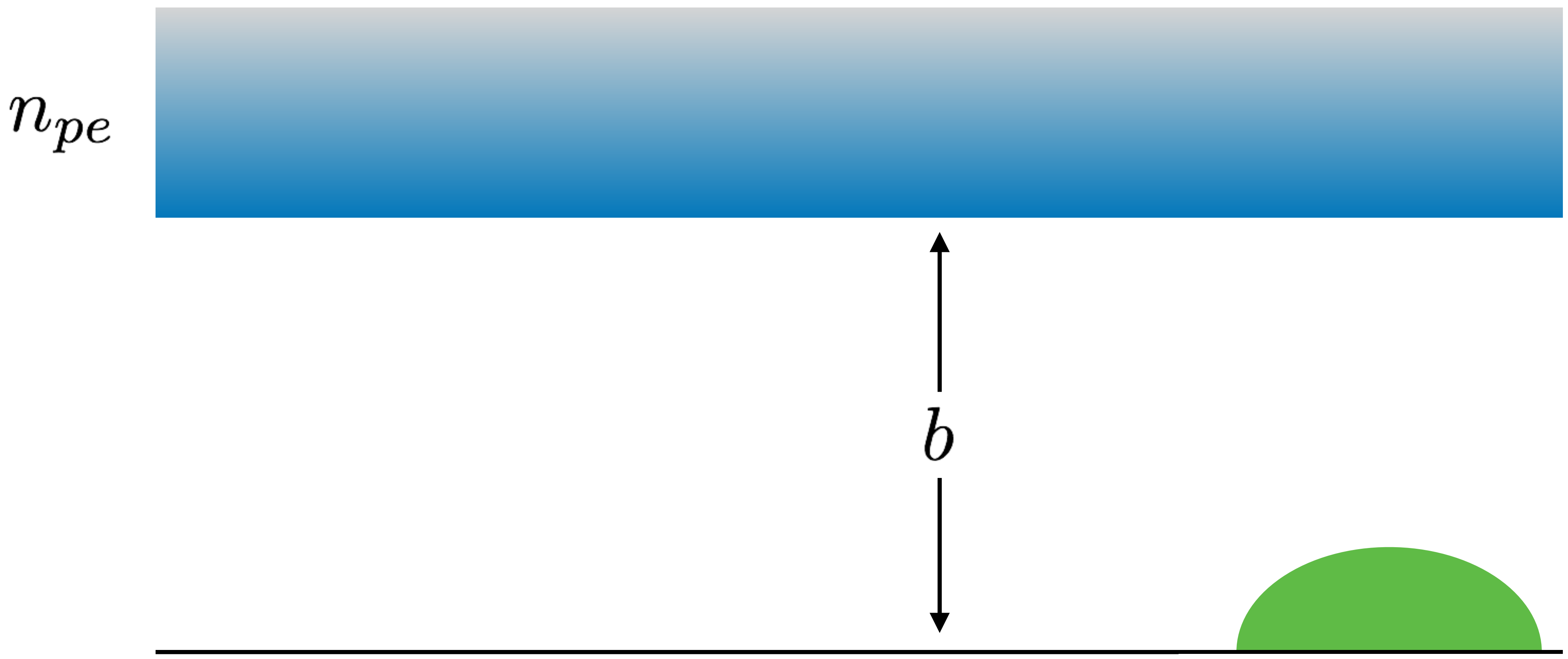
$$(\tilde{F}_z, \tilde{\mathbf{F}}_\perp)(k, \mathbf{x}_\perp) \propto$$

$$\sum_n (Z_0, Z_\perp)_n(k)(x + iy)^n \times \langle \tilde{I}(k) \rangle_n$$

$$\mathrm{IFFT}\left[(Z_0, Z_\perp)_n(k)\right] =$$

$$(W_0, W_\perp)_n$$

the plasma hollow channel: a test case



the hollow channel has analytic solutions

$$W_{\parallel m}(\zeta-\zeta') = \frac{2\kappa_m}{b^{2m}}\cos\left[\Omega_m k_p(\zeta-\zeta')\right]$$

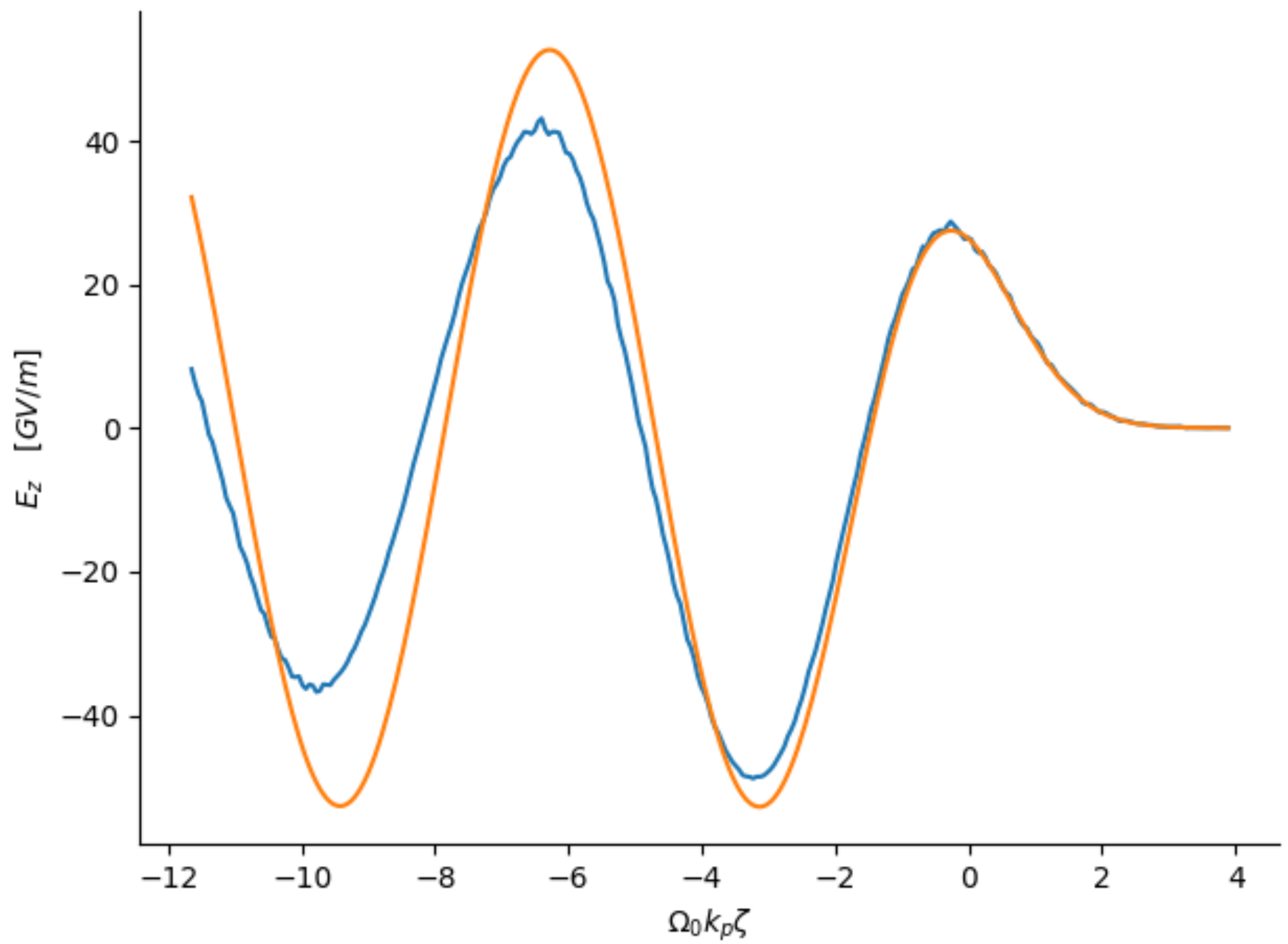
$$W_{\perp m}(\zeta-\zeta') = \frac{2m\kappa_m}{b^{2m}\Omega_m k_p}\sin\left[\Omega_m k_p(\zeta-\zeta')\right]$$

$$\Omega_m = \left[\frac{(1+\delta_{m0})(m+1)K_{m+1}(k_p b)}{2(m+1)K_{m+1}(k_p b)+k_p b K_m(k_p b)}\right]^{1/2}$$

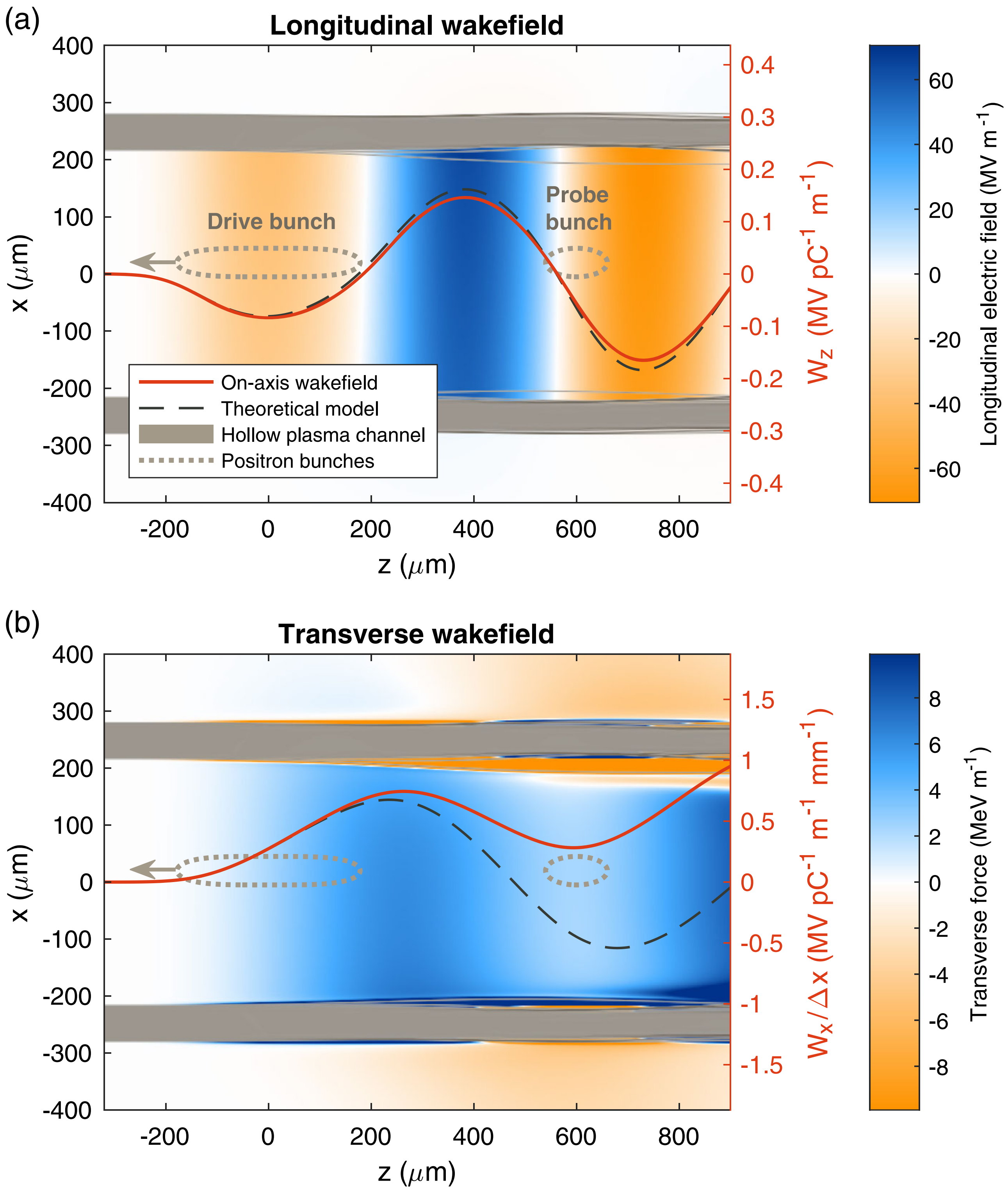
$$\kappa_m = k_p^2\left[\frac{K_m(k_p b)}{k_p b K_{m+1}(k_p b)}\right]\left[1+\frac{k_p b K_m(k_p b)}{2(m+1)K_{m+1}(k_p b)}\right]^{-1}$$

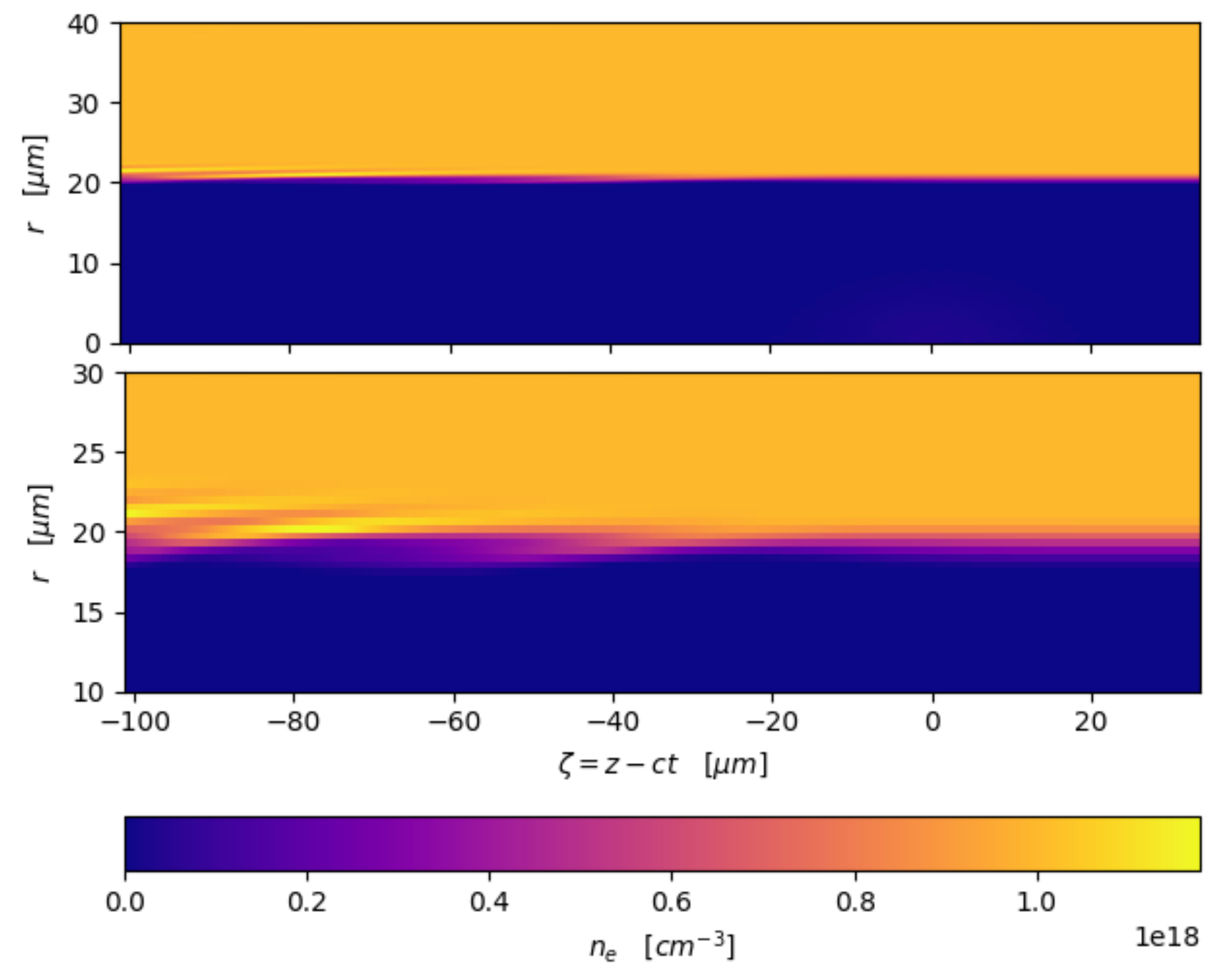
see C. B. Schroeder, D. H. Whittum, and J. S. Wurtele, “Multimode Analysis of the Hollow Plasma Channel Wakefield Accelerator”, *Phys. Rev. Lett.* **82**, 1177 (1999).

FBPIC replicates this behavior well for short range…



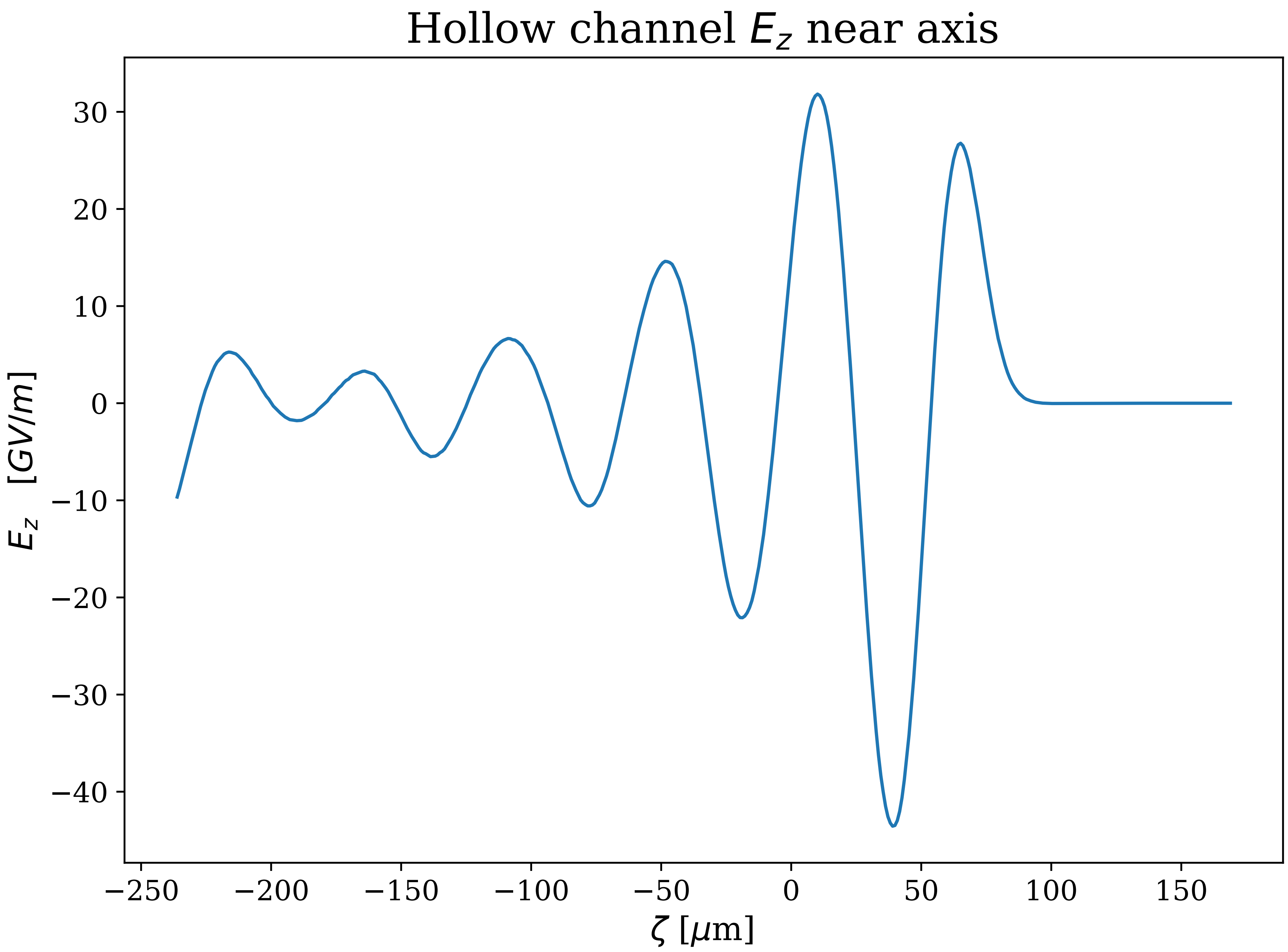
… and this discrepancy comes from the hollow channel edge





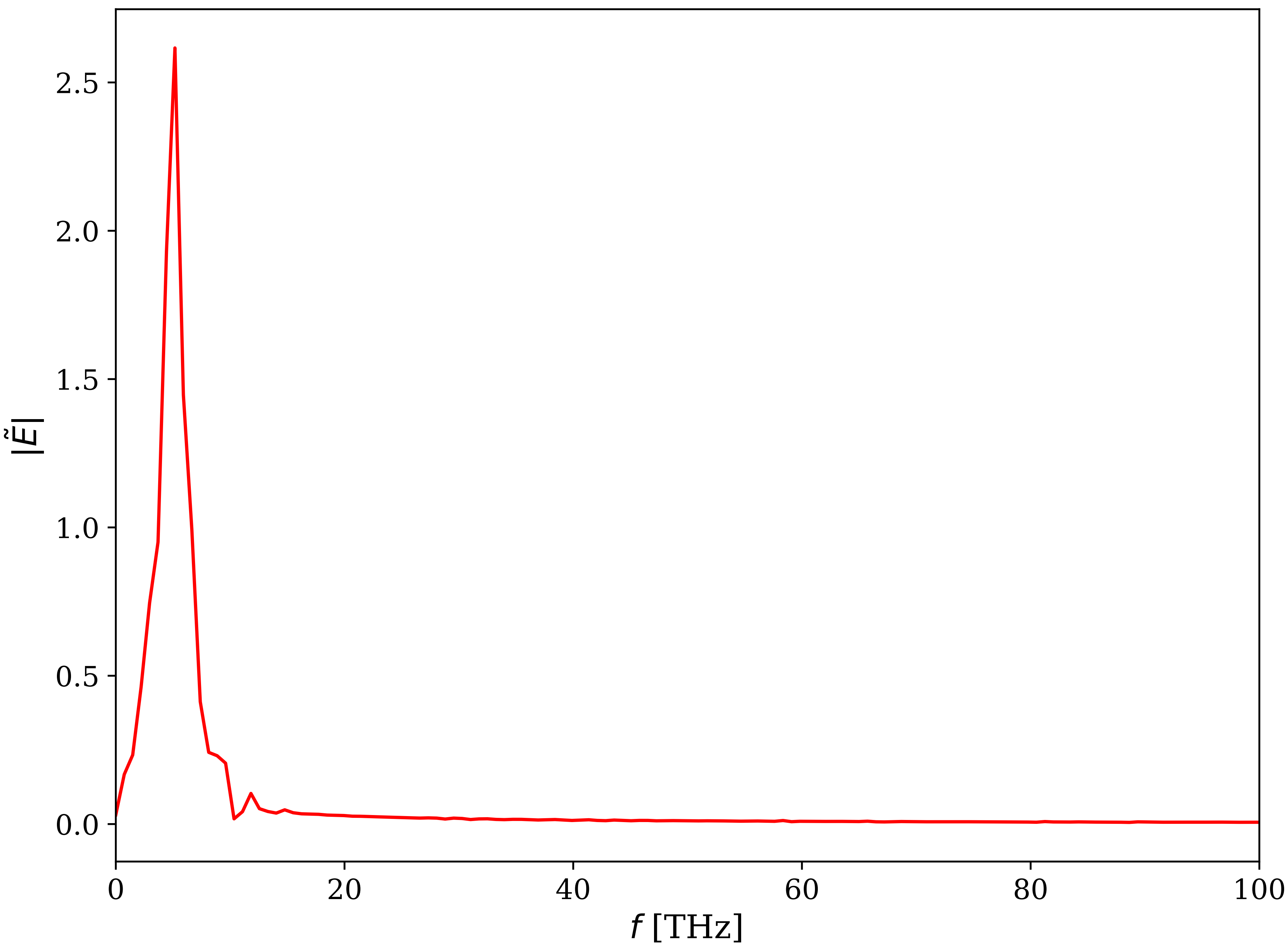
see C. A. Lindstrøm *et al.*,"Measurement of Transverse Wakefields Induced by a Misaligned Positron Bunch in a Hollow Channel Plasma Accelerator" *Phys. Rev. Lett.* **120**, 124802 (2018).

$$(F_z, \mathbf{F}_\perp)(\zeta, \mathbf{x}_\perp)$$

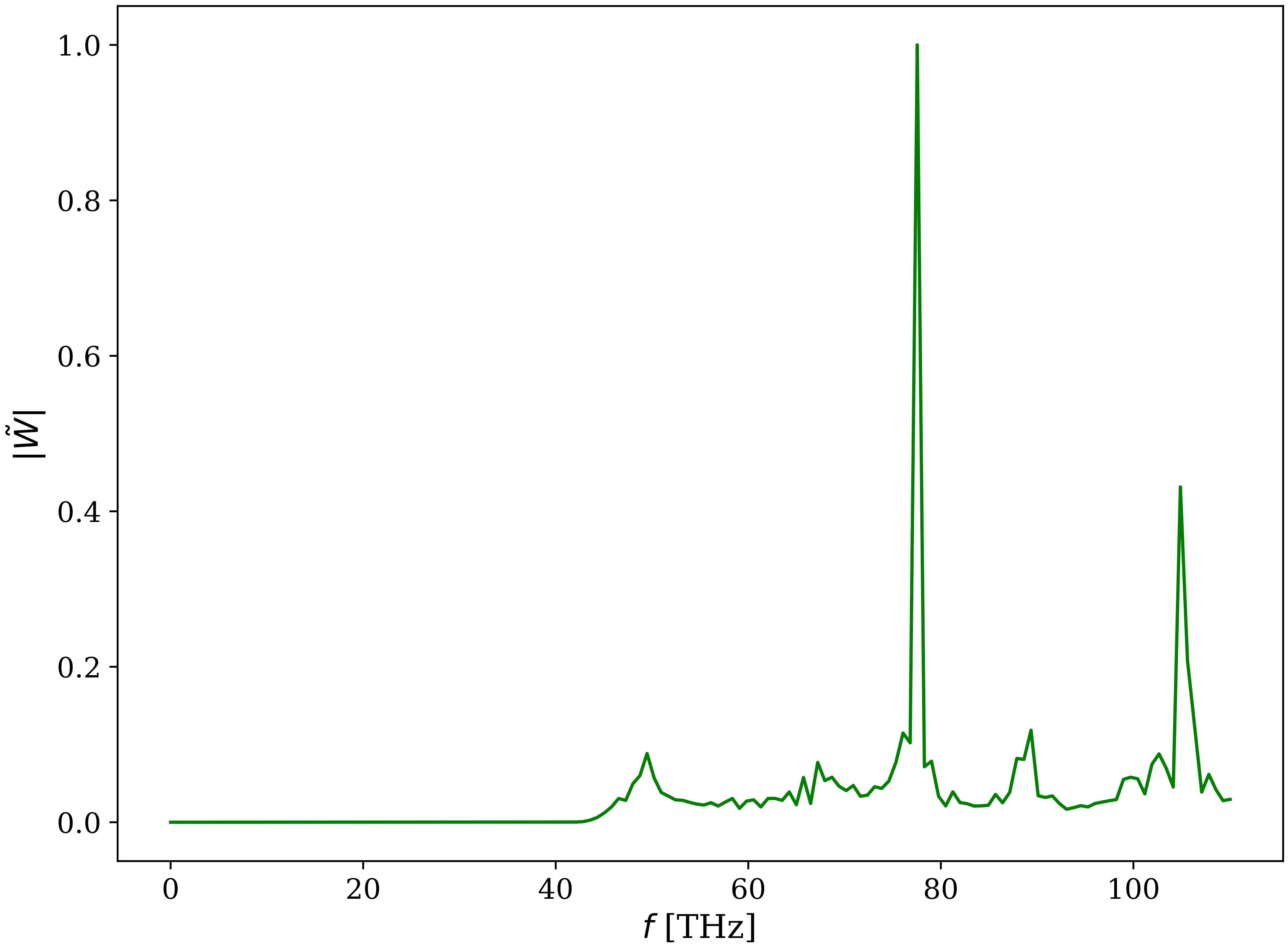


$$\mathrm{FFT}\left[(F_z, \mathbf{F}_\perp)(\zeta, \mathbf{x}_\perp)\right] =$$

$$(\tilde{F}_z, \tilde{\mathbf{F}}_\perp)(k, \mathbf{x}_\perp)$$



$$(\tilde{F}_z, \tilde{\mathbf{F}}_\perp)(k, \mathbf{x}_\perp) \propto \sum_n (Z_0, Z_\perp)_n(k)(x + iy)^n \times \langle \tilde{I}(k) \rangle_n$$



$$\mathrm{IFFT}\left[(Z_0, Z_\perp)_n(k)\right] = (W_0, W_\perp)_n$$

Thank you!

Stephen D. Webb

RadiaSoft, LLC., Boulder, CO

swebb@radiasoft.net